# Electrostatics

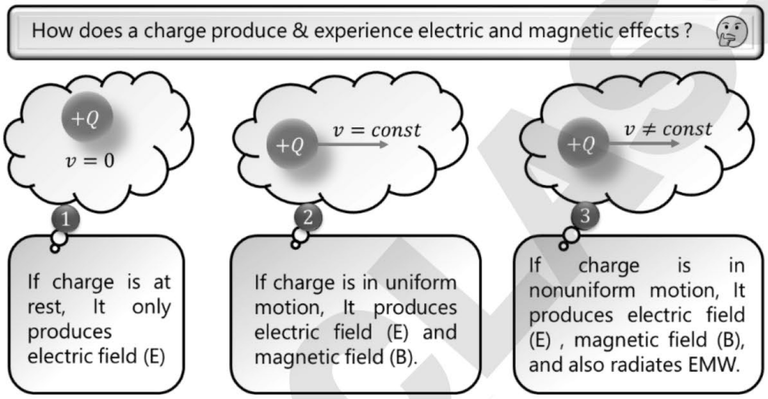
## THEORY

## Electric Charge and Field

**Electro (charge) + statics (rest) = Electrostatics :** It is a branch of physics which deals with the study of forces, fields and potential arising from static charges.

### Electric Charges and their Properties

**Electric Charge :** Charge is an intrinsic property associated with matter due to which it produces and experiences, electric and magnetic effects.




❖ A body is said to be electrified or charged whenever it gains or loses electrons.

**Types of charge :**

There exist two types of charges in nature.

(i) Positive charge : Due to deficiency of electrons (as compared to protons).

(ii) Negative charge : Due to excess of electrons (as compared to protons).

**Units of charge :**

SI $\rightarrow$ coulomb (C)

CGS $\rightarrow$ stat coulomb (stat C) or e.s.u. of charge or franklin

Another CGS $\rightarrow$ e.m.u. of charge or absolute coulomb (abC)

practical unit $\rightarrow$ faraday (F)

| | |
|---|---|
| $1C = 3\times10^9$ stat C $= \frac{1}{10}$ abC<br>$1F = 96500$ C | Smallest unit of charge $\Rightarrow$ franklin<br>Largest unit of charge $\Rightarrow$ faraday |

Dimensional formula of charge : [AT] or $[M^0L^0T^1A^1]$

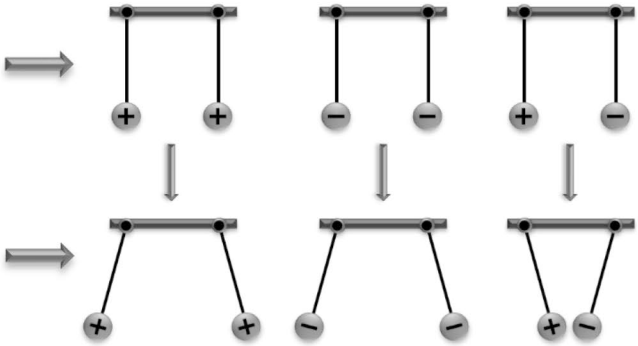
## Specific Properties of Charge

**(i)** **Charge is a scalar quantity :** It has no direction.

**(ii)** Like point charges always repel each other while unlike point charges always attract each other.

As two charged bodies or balls are brought close to each other as shown

After interaction with each other they finally stay as shown.



If the sizes of charged bodies are very small as compared to the distance between them, we treat them as point charges.

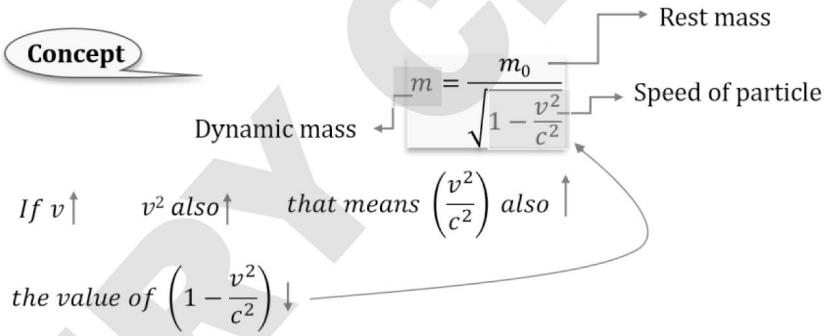
**(iii)** **Charge is transferable:** If a charged body is kept in contact with another body, then charge can be transferred to another body.

**(iv)** **Charge is always associated with mass:** Charge cannot exist without mass though mass can exist without charge.

❖ The presence of charge itself is a convincing proof of existence of mass.
Ex. (i) Photons don't have any charge as they have zero rest mass.
Ex. (ii) Rest mass of the charged body can never be zero.

❖ The mass of a body changes after being charged.
When a body is given a positive charge, its mass decreases and
When a body is given a negative charge, its mass increases.

**(v)** **Charge is realistically invariant, whereas mass is variant :** Charge is independent of frame of reference, i.e. charge on a body does not change whatever be its speed.

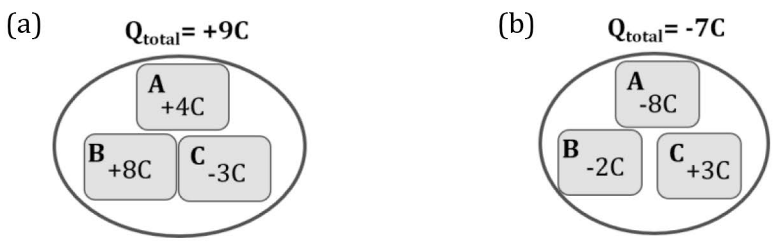



That means the particle moving with larger speed will have larger mass.

**Note :** If ($v<<c$) then m is taken as constant parameter.

**(vi)** **Charge is additive :** If a system contains n charges $q_1$, $q_2$, $-q_3$ .... $q_n$, then the total charge of the system is obtained simply by adding them algebraically (with their respective signs).

$$Q_t = \sum q = q_1 + q_2 - q_3 + .....q_n$$




Here in (a) 4+8–3 = 9 and in (b) –8–2+3 = –7

**(vii) Charge is conserved :** Total charge of an isolated system always remains constant. This is also called as "Law of conservation of charge."

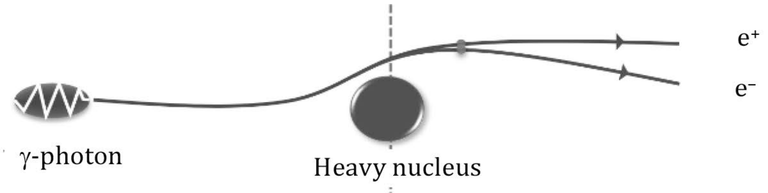
Conservation of charge holds good in all type of reactions, for example

(i) Ionization : $H(\text{atom}) \rightarrow H^+ + e^-$

(ii) Radioactive decay : $n \rightarrow p + e^- + \bar{\upsilon}$

(iii) Pair production phenomenon : conversion of energy into mass.

**Pair production phenomenon**




$Q_i = 0$

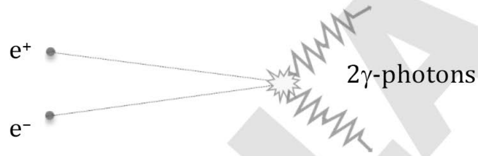
(Photon is chargeless)

$Q_f = 0$

(Positron and electron have equal and opposite charge)

Note: For this phenomenon energy of photon must be greater than 1.02 MeV.

(iv) Pair annihilation phenomenon : conversion of mass into energy

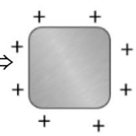



**(viii) Charge is quantized :**

Charge can have only discrete values, rather than any value.

Charge on any body must be an integral multiple of a basic unit of charge represented by e.

e = basic unit or quanta of charge or minimum transferable value of charge = $1.6\times10^{-19}$ C.

In general we can write $\boxed{Q = \pm ne} \Rightarrow$ 

where n = an integer

n = 0,1,2,3.......

**Illustration 1:**

Which of the following charges is/are not possible?

(1) $\sqrt{3}e$ (2) $48 \times 10^{-19}$ C (3) $3.2 \times 10^{-20}$ C (4) 1C

**Solution:**

(1) $\because$ q = ne $\Rightarrow \sqrt{3}$ e = ne $\Rightarrow$ n = 1.732(fraction) $\rightarrow$ not possible

(2) $48 \times 10^{-19}$C = n($1.6\times10^{-19}$)C $\Rightarrow$ n = 30(integer) $\rightarrow$ possible

(3) $3.2 \times 10^{-20}$C = n($1.6\times10^{-19}$)C $\Rightarrow$ n = 0.2(fraction) $\rightarrow$ not possible

(4) 1 = n($1.6\times10^{-19}$) $\Rightarrow$ n = $6.25 \times10^{18}$ (integer) $\rightarrow$ possible

**Illustration 2:**

How many electrons must be removed from a body to make it electrified by 3.2C of charge?

**Solution:**

From Q = ne $\Rightarrow$ $n = \frac{Q}{e} = \frac{3.2}{1.6\times10^{-19}} \Rightarrow n = 2\times10^{19}$

**Illustration 3:**

$10^{10}$ alpha particles are ejected per second from a body, then after how much time, the body will acquire a charge of 8 μC?

**Solution:**

Charge appear per second on body $Q = \left(\frac{N}{t}\right) q_{\alpha}$

$Q = 10^{10} \times (2 \times 1.6 \times 10^{-19}) = 3.2 \times 10^{-9}\, C/sec$

Let t be the time taken to acquire charge of 8 μC then

$$t = \frac{8 \times 10^{-6}}{3.2 \times 10^{-9}} = 2.5 \times 10^{3}\, sec$$

**Note:** For macroscopic charges (big charges) quantisation rule can be ignored.

**Conductors & Insulators**

**Conductors :** Those materials which allow electricity to pass through them easily are called conductors. They have electric charge (electrons) that are comparatively free to move inside the material. Ex. Metals, human and animal bodies, earth etc.

**Insulators :** Those materials which offer high resistance to the passage of electricity through them, are called insulators. Ex. Glass, rubber, plastic, nylon, wood etc.

- **Methods of Charging**

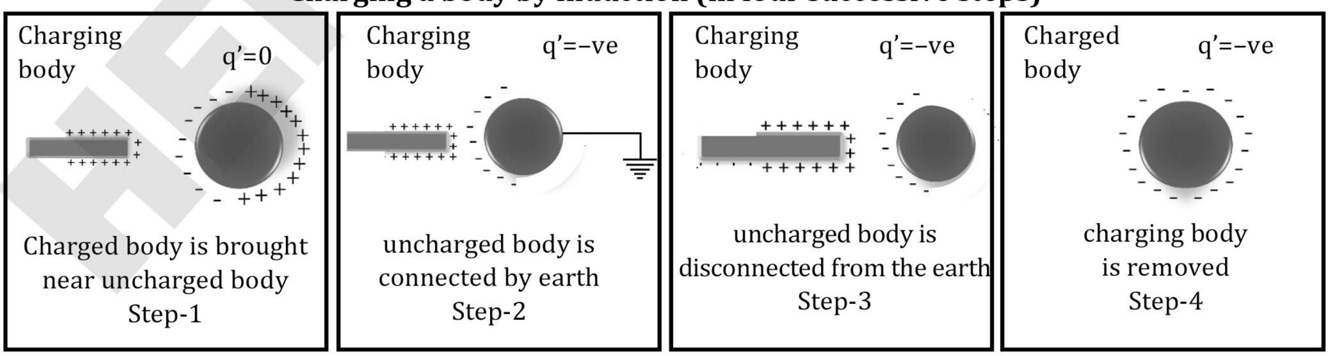
**(1)** **Charging by friction or rubbing :** If we rub one body with another body, electrons get transferred from one body to the another. Both insulators and conductors can be charged by this method. When an object in column-1 is rubbed against the object of column-2 then they acquire charges specified in the following table.

| Column-1 | Column-2 |
|---|---|
| **Positive Charge** | **Negative Charge** |
| Glass rod | Silk cloth |
| Woollen cloth | Rubber shoe, Amber, Plastic objects |
| Dry hair | Comb |

**Note:** Clouds get charged due to friction.

**(2)** **Electrostatic induction :** If a charged body is brought near a neutral body, the charged body will attract opposite charges and repel similar charges present in the neutral body. As a result of this one side of the neutral body becomes negative while the other positive, this process is called 'electrostatic induction'. Hence, “Induction is a phenomena of redistribution of charges on a body in the influence of other charged object or external field.”

**Charging a body by induction (in four successive steps)**



**In case of induction it is worth noting that :**

(i) Inducing body neither gains nor loses charge.

(ii) The nature of induced charge is always opposite to that of inducing charge.

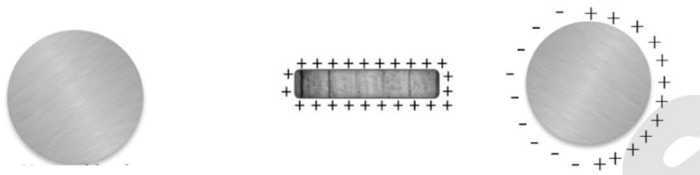
(iii) Induced charge can be lesser or equal to inducing charge (but never greater).

(iv) Induction takes place only in bodies (either conducting or non conducting) and not in particles.

**Illustration 4:**

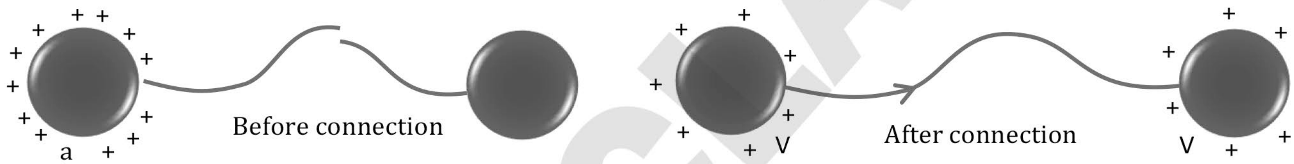
If a charged body is put near a neutral conductor, will it attract the conductor or repel it?

**Solution:**




They attract each other due to induction effect

**(3) Charging by conduction :** Whenever a charged conductor brought in contact with other conductor, charge continuously flows from one to another until their potential becomes equal. This is also called sharing of charges.




**In case of conduction it is worth noting that :**

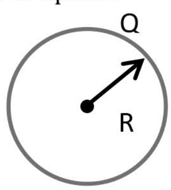
(i) Contact is necessary

(ii) Only conductors can be charged by this method

(iii) In this method both conductors will acquire same nature of charge.

(iv) Total charge of the system distributes in the ratio of their radii (only for spherical objects)

For sphere




$$v = \frac{kQ}{R}$$

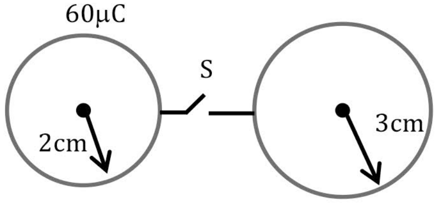
Consider two charged spheres $(q_1, R_1)$ & $(q_2, R_2)$ are connected by a wire or touched with each other.

After conduction let their final charges become $Q_1$ & $Q_2$ respectively then $q_1 + q_2 = Q_1 + Q_2$ & $\boxed{\frac{Q_1}{Q_2} = \frac{R_1}{R_2}}$

**Illustration 5:**

Find final charges on the spheres when switch S is closed.

**Solution:**



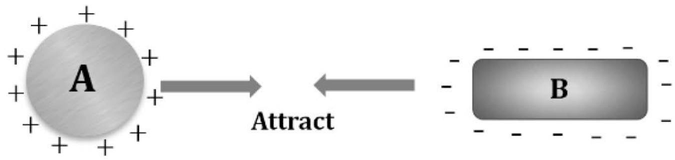

Total charge of the system = Q = 60μC +0= 60μC

Then after conduction

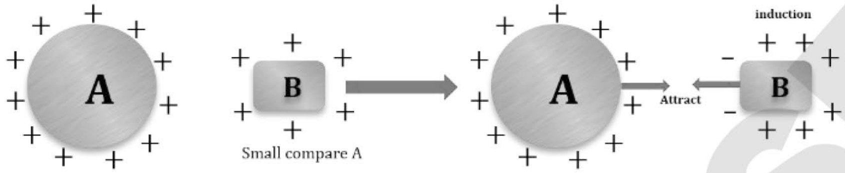
$$\frac{Q_1'}{Q_2'} = \frac{R_1}{R_2} \Rightarrow \frac{2}{3} \quad \Rightarrow \quad Q_1' = \left(\frac{2}{2+3}\right)60\mu C = 24\mu C$$

$$Q_2' = \left(\frac{3}{2+3}\right)60\mu C = 36\mu C$$

**Illustration 6:**
A positively charged body 'A' attracts a body 'B' then charge on body 'B' may be:
**Solution:**
If B is –ve

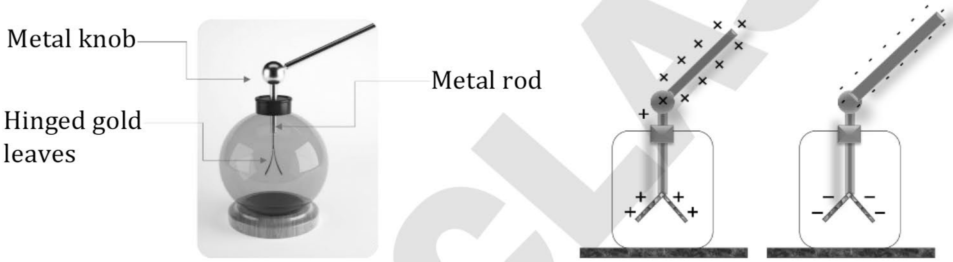



If B is +ve

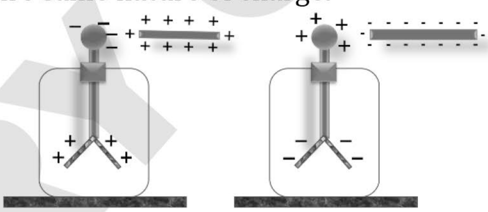



**Gold leaf electroscope**
It is used to detect the presence of charge.

→ When a conducting body touches the metal knob of an electroscope, then knob & leaves both wire acquire same nature of charge.




When a charged body is placed near the electroscope then knob will acquire opposite nature of charge & leaves will acquire same nature of charge.



**Golden Key Points**

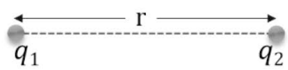
- Charge differs from mass in the following aspects :
  (i) In SI system of units, charge is a derived physical quantity while mass is a fundamental quantity.
  (ii) Charge is always conserved but mass not.
  (iii) Charge cannot exist without mass but mass can exist without charge.
  (iv) Charges are of two type (positive and negative) but mass is of only one type.
  (v) For a moving charged body, mass increases while charge remains constant.
- True test of electrification is repulsion, not attraction
- As attraction can take place between a charged and an uncharged body or between two similarly charged bodies
- For a non relativistic (i.e. v << c) charged particle, specific charge $\frac{q}{m}$ = constant.
- Charge can be detected and/or measured with the help of gold-leaf electroscope, electrometer, voltmeter etc.

## BEGINNER'S BOX-1

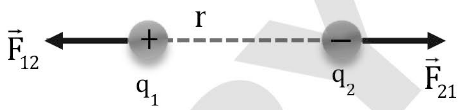
1. In a neutral sphere, $5 \times 10^{21}$ electrons are present. If 10 percent electrons are removed, then calculate the charge on the sphere.
2. Calculate the number of electrons in 100 grams of $CO_2$.
3. Can a body have a charge of (a) $0.32 \times 10^{-18}$ C (b) $0.64 \times 10^{-20}$ C (c) $4.8 \times 10^{-21}$ C ?
4. A glass tumbler contains 1 billion amoeba and two sodium ions are there on the body of each amoeba. Find out the charge contained in the glass.
5. How many electrons should be removed from a conductor so that it acquires a positive charge of 3.5 nC ?
6. It is now believed that protons and neutrons (which constitute nuclei of ordinary matter) are themselves made up of more elementary units called "quarks". A proton and a neutron consist of three quarks each. Two types of quarks, the so called 'up' quark (denoted by u) of charge +(2/3)e, and the 'down' quark (denoted by d) of charge (–1/3)e, together with electrons build up ordinary matter. (Quarks of other types have also been found which give rise to different unusual varieties of matter.) Suggest a possible quark composition of a proton and neutron.
7. A polythene piece rubbed with wool is found to have a negative charge of $3.2 \times 10^{-7}$ C. Find the
   (a) number of electrons transferred.
   (b) mass gained by the polythene.
8. Two identical metal spheres A and B placed in contact are supported on insulating stand. What kinds of charge will A and B develop when a negatively charged ebonite rod is brought near A ?

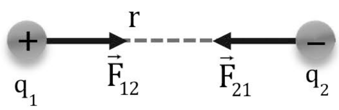
## Coulombs law, Force between two point charges

**Coulombs law :** "The electrostatic force of interaction between two point charges is directly proportional to the product of magnitude of charges and inversely proportional to the square of distance between them."




This force always works along the line joining the two centres.




$$|\vec{F}_{12}| = |\vec{F}_{21}| = F$$

i.e. $F \propto \dfrac{|q_1 q_2|}{r^2} \Rightarrow \boxed{F = \dfrac{k|q_1 q_2|}{r^2}}$

Where k = proportionality constant or coulomb's constant or electrostatic constant

❖ Value of k depends upon the choice of system of units.

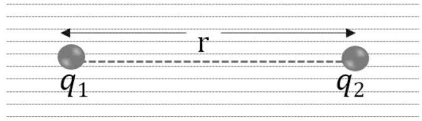
❖ In SI system, $k = 9 \times 10^9 \dfrac{N \times m^2}{C^2}$

❖ This constant is also written as, $k = \dfrac{1}{4\pi\varepsilon_0}$

$\epsilon_0$ = permittivity of free space or vacuum $= 8.85 \times 10^{-12} \dfrac{C^2}{N \times m^2}$

Then $\boxed{F = \dfrac{kq_1q_2}{r^2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1q_2}{r^2}}$

❖ When the point charge system is placed in a homogeneous dielectric medium.




$$F_m = \frac{1}{4\pi \in} \frac{q_1 q_2}{r_2}$$

$\in$ = permittivity of medium = $\in_0 \in_r$

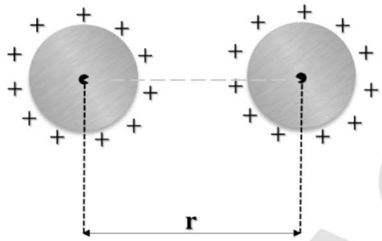
$\in_r$ = relative permittivity of medium or = Dielectric constant of medium (K)

$$F_m = \frac{1}{4\pi \in_0 \in_r} \frac{q_1 q_2}{r^2} = \frac{1}{\in_r}\left(\frac{1}{4\pi \in_0} \frac{q_1 q_2}{r^2}\right) \Rightarrow \boxed{F_m = \frac{F}{\in_r}}$$

The value of $\in_r$ or $K \geq 1$, i.e. $F_m \leq F$

**Illustration 7:**

Why we can not use Coulomb's law for large size bodies ?




**Solution:**


When large size charged conducting spheres brought close to each other their, charges moves away due to repulsion hence effective distance between their centers increases (r' > r)

$$F_{actual} < F_{calculated} = \frac{kq_1 q_2}{r^2}$$

🤔 what would have happened, if both the spheres had opposite charges?

**Important points about Coulomb's law**

❖ Coulombs law is valid for point or point like charges.

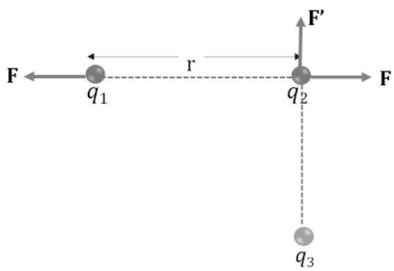
❖ It follows inverse square law.

❖ It follows the law of superposition.

❖ It is conservative so electric potential and potential energy can be define under the influence of this force

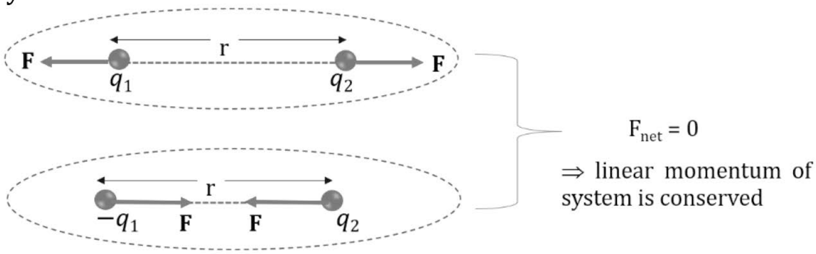
❖ Force between two charges does not depend upon presence of other neighbouring charges.

Here net force on $q_2$ will change due to presence of $q_3$, but force due to $q_1$ on $q_2$




❖ Coulomb's force doesn't depend upon the medium.

❖ For two charge system




In both the cases shown -

Net external force on the system is zero so linear momentum of system remains conserved.

❖ Angular momentum of one charge particle w.r.t. other remains conserved, here Line of action of force is passing through $q_2$ and vice versa so torque on one charge w.r.t. other will be zero.


Here torque = 0 ⇒ Angular momentum = Constant

**Illustration 8:**

Two point charges $+q_1$ and $+q_2$ are placed r meter apart in vacuum. Force on $q_1$ due to $q_2$ is F. If now a third charge $+q_3$ is placed at mid point of line joining the charges, then

(1) force on $q_1$ due to $q_2$

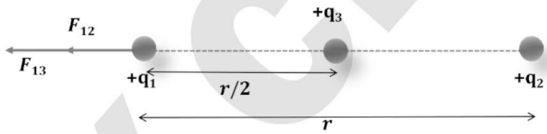
(a) Increases (b) Decreases (c) Remains same (d) Can't say

(2) net force on $q_1$

(a) Increases (b) Decreases (c) Remains same (d) Can't say

**Solution:**


When $q_3$ is placed at mid point,




Now net force on $q_1$ will definitely increase but force due to $q_2$ will remain $F_{12}$ on $q_1$.

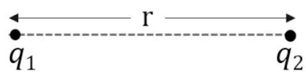
$$F_{12} = \frac{kq_1q_2}{r^2}$$

**Illustration 9:**

Calculate the force between two charges, each of 1μC separated in air by 1cm.

**Solution:**

$q_1 = q_2 = 1\mu C$ & $r = 1cm$



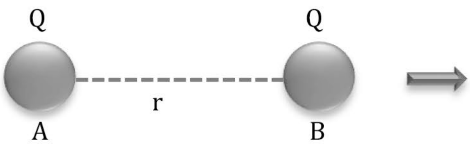

$$F = \frac{kq_1q_2}{r^2} = 9\times10^9\frac{(10^{-6})^2}{(10^{-2})^2} = 90N$$

**Illustration 10:**

Force between two identical spheres A and B carrying same charge and separated by distance r in vacuum is F. A third identical sphere C first brought in contact with A then B and finally, C is taken away. Now force between A and B becomes.

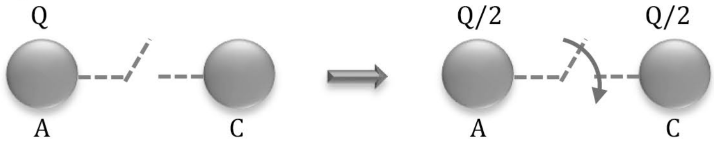
(1) F/4 (2) F/8 (3) 3F/4 (4) 3F/8

**Solution:**

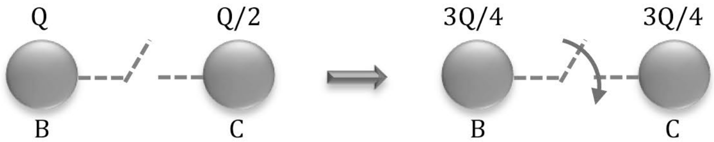



$$F = \frac{kQ^2}{r^2} \quad ...(1)$$

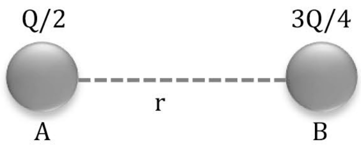
Now C (uncharged sphere) is touched with A first




Then C is touched with B




New force between A & B now is




$$F' = \frac{k\left(\frac{Q}{2}\right)\left(\frac{3Q}{4}\right)}{r^2} = \frac{3kQ^2}{8r^2} = \frac{3F}{8}$$

**Illustration 11:**

Two spheres having equal charges exert F force on each other. Now 20% charge of one sphere is transferred to another sphere. Then find new force between them in terms of F.

**Solution:**

Let us assume initially both the sphere were having equal charge q so, Force between them can be written as -

$$F = \frac{kqq}{r^2}$$

When 20% charge is transferred from one the another then new force F' -

$$F' = \frac{k(0.8q)(1.2q)}{r^2} = \frac{kqq}{r^2}\ (0.96) = 0.96F$$

**Illustration 12:**

Two spheres having equal charges are kept at long separation. If the gravitational force between two spheres is equal to electrostatic force between them. Find the ratio of specific charge $\frac{q}{m}$.

**Solution:**

Equating the gravitational and electrostatics force -

$$\frac{kq^2}{r^2} = \frac{Gm^2}{r^2}$$

$$\frac{q}{m} = \sqrt{\frac{G}{k}} = \sqrt{\frac{6.67\times10^{-11}}{9\times10^{9}}} = \sqrt{\frac{0.74}{10^{20}}} = \frac{0.86}{10^{10}} = 0.86\times10^{-10}$$

**Illustration 13:**

12C charge is to be distributed among the two spheres placed at a separation of 2m then find the charge on each sphere so that coulomb force between them is maximum also find the maximum force.

**Solution:**

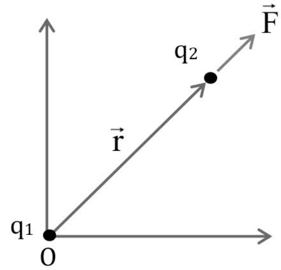
For max force divide the charge as half - half


$$F = \frac{k(6)(6)}{2^2} = \frac{9\times10^{9}\times36}{4} = 81\times10^{9}\ N$$

**Vector form of Coulomb's law –**

(i) when one of the charge is at origin

$$\vec{F} = F\hat{r} = \frac{kq_1q_2}{r^2}\hat{r} = \frac{kq_1q_2}{r^3}\vec{r} \quad \left[\because \hat{r} = \frac{\vec{r}}{r}\right]$$




(ii) when none of the charge is at origin

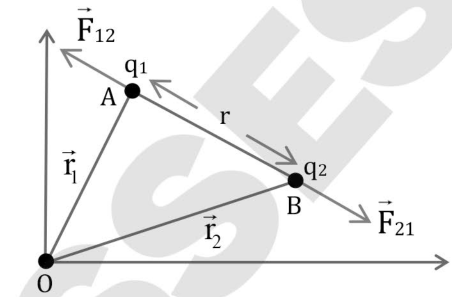
(or in terms of position vector)

$$\vec{F}_{12} = \text{force on } q_1 \text{ due to } q_2 = \frac{kq_1q_2}{r^2}\hat{r}_{21} = \frac{kq_1q_2}{r^3}\vec{r}_{21}$$

$$\vec{F}_{21} = \text{force on } q_2 \text{ due to } q_1 = \frac{kq_1q_2}{r^2}\hat{r}_{12} = \frac{kq_1q_2}{r^3}\vec{r}_{12}$$

$$\because r = |\overrightarrow{AB}| = |\overrightarrow{BA}| \Rightarrow r = |\vec{r}_2 - \vec{r}_1| = |\vec{r}_1 - \vec{r}_2|$$

$$\text{or } \vec{F}_{12} = \frac{kq_1q_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2) \text{ and } \vec{F}_{21} = \frac{kq_1q_2}{|\vec{r}_2 - \vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1)$$




## Superposition Principle

When a number of charges are interacting then, total force on a given charge is vector sum of all the individual forces exerted on it by all other charges.

$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + .....\vec{F}_{1N}$ or

$$\vec{F} = \frac{kq_0q_1}{r_1^2}\hat{r}_1 + \frac{kq_0q_2}{r_2^2}\hat{r}_2 + ..... + \frac{kq_0q_i}{r_i^2}\hat{r}_i + .... \frac{kq_0q_n}{r_n^2}\hat{r}_n$$

In vector form $\vec{F} = kq_0 \sum_{i=1}^{n} \frac{q_i}{r_i^2}\hat{r}_i$

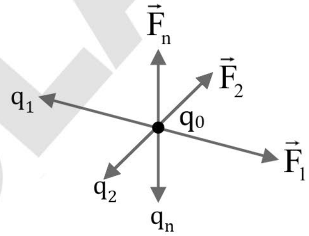
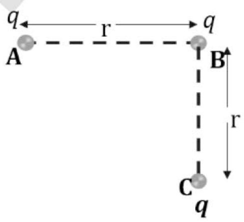
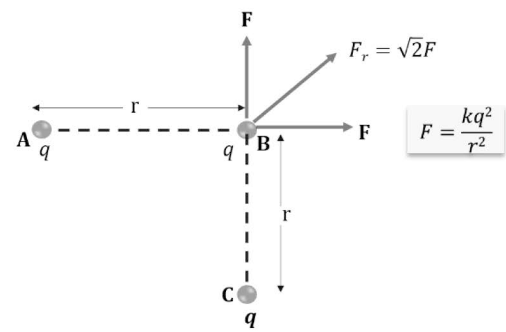



**Illustration 14:**

Find net force on charge at B.




**Solution:**




So, net force on B will be

$$F_r = \sqrt{2}\frac{kq^2}{r^2}$$

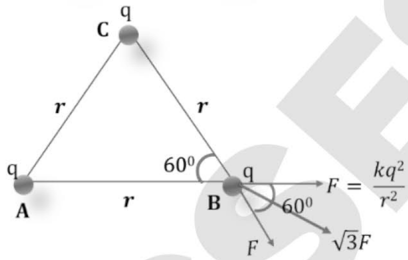
**Illustration 15:**

Find net force on the charge, placed at B, in the given arrangements


**Solution:**

(i) Required force on charge at B will be -

$$F_B = \sqrt{3}\frac{kq^2}{r^2}$$



(ii) Required force on charge at B will be -

$$F_B = \frac{kq^2}{r^2}$$

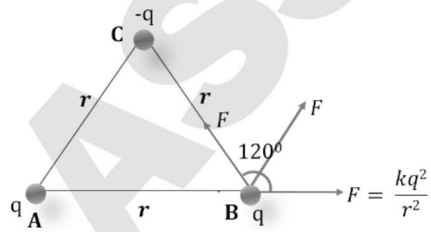
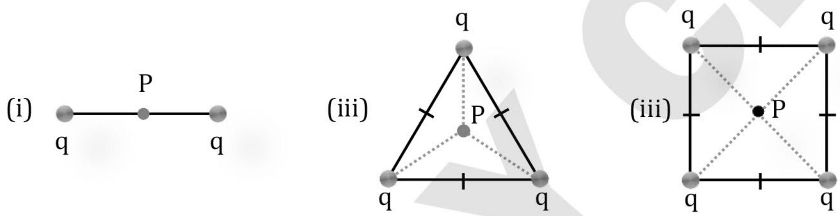


**Illustration 16:**

Find net force on the charge Q, placed at centre point P as shown :




**Solution:**

If n identical charges placed at corners of a regular polygon then net force on charge placed at center of polygon is always zero.

**Equilibrium of suspended point charge system (Pith Ball problems)**

Two identical charged pith balls are suspended by insulated strings of equal length, by a common point of suspension, if each ball has mass m then the problem related to it can be treated as,

$T \sin\theta = F_e$ ...(i)

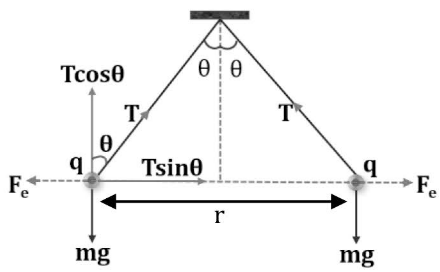
$T \cos\theta = mg$ ...(ii)

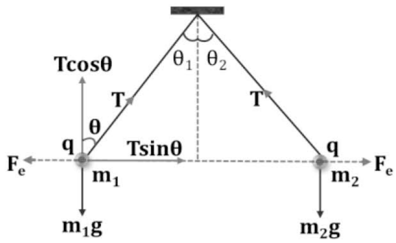
$\tan\theta = \frac{F_e}{mg}$ ...(iii)

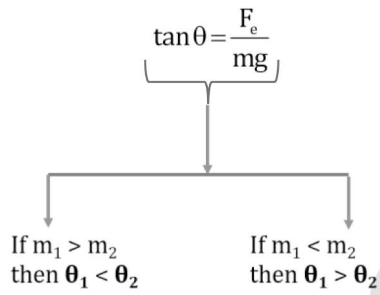
$r = 2\ell \sin\theta$ (distance between charges, where $\ell$ is length of string)

$$F_e = \frac{kq^2}{r^2} = \frac{kq^2}{4\ell^2 \sin^2\theta}$$

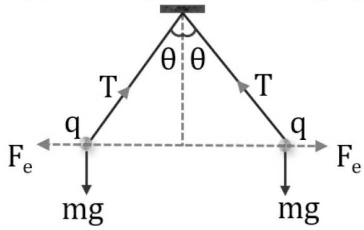
$$T = \sqrt{(F_e)^2 + (mg)^2}$$



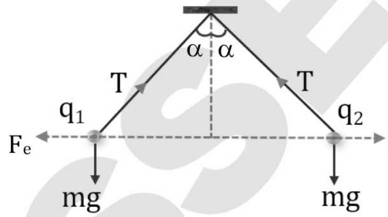


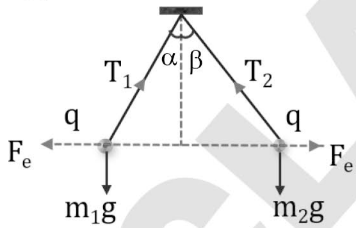

**Case -1 :** If $q_1 = q_2$, $m_1 = m_2$, then $\theta_1 = \theta_2$ (say $\theta$)




**Case -2 :** If $q_1 \neq q_2$, $m_1 = m_2$, then $\theta_1 = \theta_2$ (say $\alpha$)



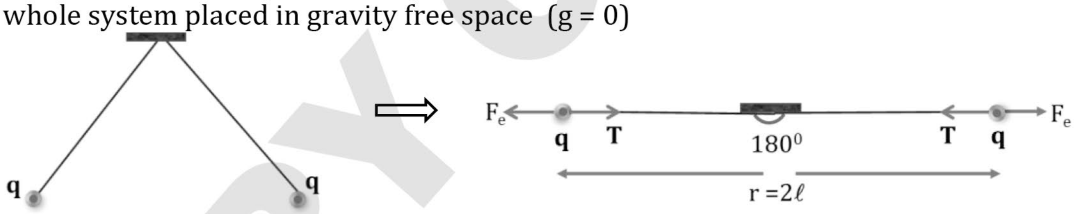

**Case -3 :** If $q_1 = q_2$, $m_1 \neq m_2$, then $\theta_1 \neq \theta_2$




$m_1 > m_2$

$\alpha < \beta$

**Case -4 :** If whole system placed in gravity free space (g = 0)

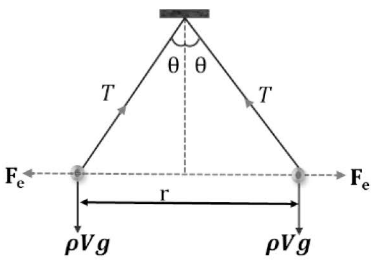



$$F_e = \frac{kq^2}{r^2} = \frac{kq^2}{4l^2}$$

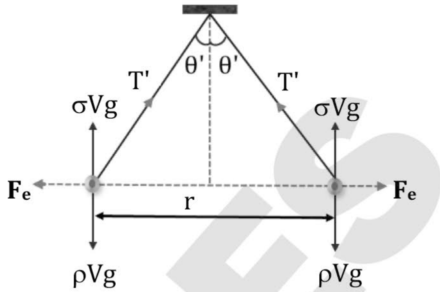
Tension in thread $T = \frac{kq^2}{4l^2}$

**Case -5 :** If density of material is $\rho$ and volume is V then -

In air, $\tan\theta = \frac{F_e}{mg} = \frac{F_e}{V\rho g}$



**Case-6 :** If experiment is done inside liquid having density $\sigma$ and dielectric constant K then (Consider density of material is $\rho$ and volume V) In medium of dielectric constant K




$$T'\sin\theta' = \frac{F_e}{K}$$

$T'\cos\theta' + F_B = mg$ ($F_B$ : Buoyant force)

$$T'\cos\theta' = \rho Vg - \sigma Vg$$

$$\tan\theta' = \frac{\frac{F_e}{K}}{(\rho Vg - \sigma Vg)}$$

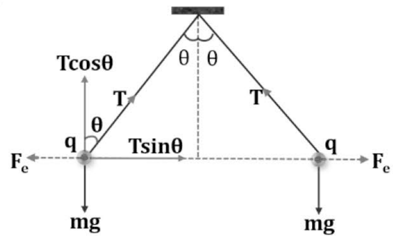
In Air $\tan\theta = \frac{F_e}{V\rho g}$

In Medium $\tan\theta' = \frac{\frac{F_e}{K}}{(\rho Vg - \sigma Vg)} = \frac{F_e}{\rho VgK\left(\frac{\rho-\sigma}{\rho}\right)} = \frac{\tan\theta}{K\left(1-\frac{\sigma}{\rho}\right)}$

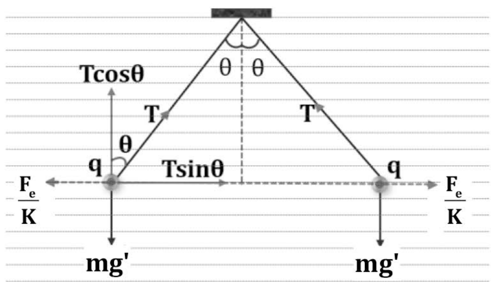
If $K\left(1-\frac{\sigma}{\rho}\right) > 1\ \theta' < \theta$ , If $K\left(1-\frac{\sigma}{\rho}\right) < 1\ \theta' > \theta$, If $K\left(1-\frac{\sigma}{\rho}\right) = 1\ \theta' = \theta$

**Illustration 17:**

Two identical charged spheres are suspended by strings of equal length. Each string makes an angle θ with the vertical. When suspended in a liquid of density σ = 0.8 gm/cc, the angle remains the same. What is the dielectric constant of the liquid? (Density of the material of sphere is ρ = 1.6 gm/cc.)




**Solution:**




$T \sin\theta = F_e$ ...(i)

$T \cos\theta = mg$ ...(ii)

$\tan\theta = \frac{F_e}{mg}$ ...(iii)

$T \sin\theta = F_e / K$ ...(iv)

$T \cos\theta = mg'$ ...(v)

$\tan\theta = \frac{F_e}{Kmg'}$ ...(vi)

According to given information angle remains same so from equations (iii) and (vi), we have

$\frac{F_e}{mg} = \frac{F_e}{Kmg'}$ or $g = Kg'$ or $g = Kg\left(1-\frac{\sigma}{\rho}\right)$

$g = Kg\left(1-\frac{\sigma}{\rho}\right)$ or $1 = K\left(1-\frac{0.8}{1.6}\right)$ or $1 = K\left(\frac{1}{2}\right)$ or $K = 2$

**Illustration 18:**
Two point charges each of +q are fixed at (a,0) & (-a,0). Another point charge $+q_0$ is free to move along y-axis is placed at (0,y). For what value of y force on $+q_0$ is maximum.

**Solution:**

$F_r$ on $+q_0$

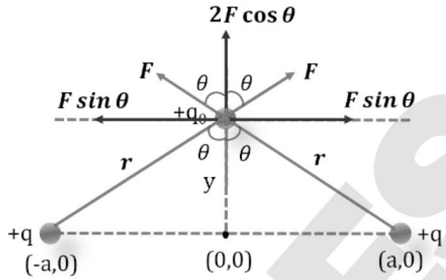
$$F_r = 2F\cos\theta$$

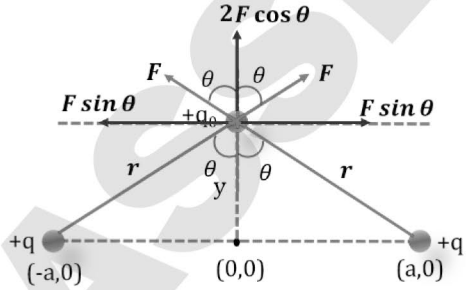
$$r = \sqrt{(a^2 + y^2)}$$

$$F_r = 2\left(\frac{kqq_0}{r^2}\right)\left(\frac{y}{r}\right)$$

$$F_r = 2\left(\frac{kqq_0}{r^3}\right)y = 2\left(\frac{kqq_0}{\left(a^2+y^2\right)^{3/2}}\right)y$$

$$F_r = 2\left(\frac{kqq_0}{r^3}\right)y = 2\left(\frac{kqq_0}{\left(a^2+y^2\right)^{3/2}}\right)y$$




For maximum value of Force

$\frac{dF}{dy} = 0$ (concept of maxima)

By doing this we obtained $y = \pm \frac{a}{\sqrt{2}}$

## Golden Key Points

- Coulomb's law is based on physical observations and is not logically derivable from any other concept. Experiments reveal its universal nature till today.
- The law is analogous to Newton's law of gravitation : $F = G\,\frac{m_1 m_2}{r^2}$
- Electric force between charged particles is much stronger than gravitational force, i.e., $F_E >> F_G$. Consequently, $F_G$ is neglected when both $F_E$ and $F_G$ are present.
- Electric force can be attractive or repulsive while gravitational force is always attractive.
- Electric force depends on the nature of medium between the charges while gravitational force does not.
- The force is conservative, i.e., work done in moving a point charge round a closed path under the action of Coulomb's force is zero.
- The law expresses the force between two point charges at rest. In applying it to the case of extended bodies of finite size care should be taken in assuming the whole charge of a body to be concentrated at its 'centre' as this is true only for spherically charged bodies, that too for external points.
- Electric force between two charges does not depend on neighbouring charges.
- The net Coulomb's force between two charged particles in free space and in an infinitely extending medium are
- $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$ and $F' = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$. So $\frac{F}{F'} = \frac{\varepsilon_0}{\varepsilon_0} = K$,
- Dielectric constant (K) of a medium is numerically equal to the ratio of the force on two point charges in free space to that in the medium extending infinitely.
- Although the net electric force on both particles change in the presence of dielectric but force due to one charged particle on another charged particle does not depend on the medium between them.

## Electrostatic equilibrium

In physics if a particle or system is in equilibrium means net force on particle or system is zero.

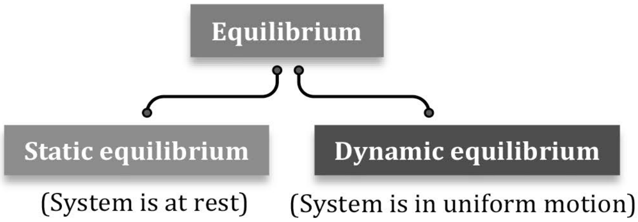
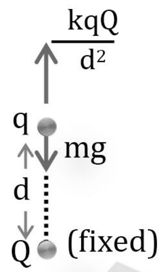



**Illustration 19:**

A small charge q of mass m is in equilibrium d distance above another charge Q then what should be the value of q in terms of other given quantities?




**Solution:**

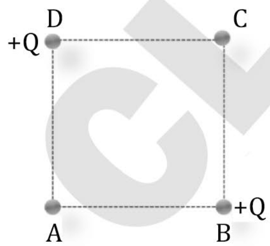
In equilibrium : $mg = F_e$

$$mg = \frac{kqQ}{d^2}$$

$$\frac{mgd^2}{kQ} = q$$

**Illustration 20:**

What equal charges should be placed at remaining two vertices so that net force on +Q charge becomes zero (Side of square is a)

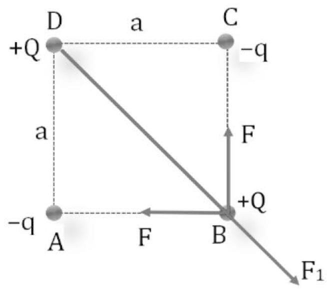



(1) $\frac{Q}{\sqrt{2}}$ (2) $-\frac{Q}{\sqrt{2}}$ (3) $\frac{Q}{2\sqrt{2}}$ (4) $-\frac{Q}{2\sqrt{2}}$

**Solution:**

To make net force on +Q charge zero, Field due to charges on vertices A & C should be opposite to that of field due to other +Q charge so there must be -q charge placed at A and C.

For net force to be zero on +Q




(i) $\sqrt{2}F = F_1$

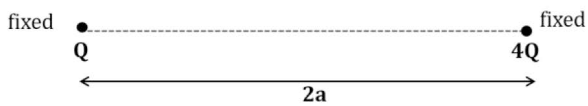
Here $F = \frac{kqQ}{a^2}$ & $F_1 = \frac{kQ^2}{(\sqrt{2}a)^2} = \frac{kQ^2}{2a^2}$

$$\sqrt{2}\frac{kqQ}{a^2} = \frac{kQ^2}{2a^2}$$

$q = \frac{Q}{2\sqrt{2}}$ (magnitude) $q = -\frac{Q}{2\sqrt{2}}$ option (4) is correct.

**Illustration 21:**

In the system of two charges Q and 4Q where should be third charge must be placed so that net force on third Charge will be zero.



**Solution:**

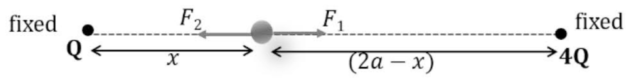
For net force to be zero on third charge q force due to Q and 4Q must be equal and opposite.

Suppose q is placed at distance x from Q.

If $F_1$ is force exerted by Q,

and $F_2$ is force exerted by 4Q on q

For net force to be zero on q $F_1 = F_2$




or $$\frac{kQq}{x^2} = \frac{4kQq}{(2a-x)^2}$$ or $$\frac{(2a-x)^2}{x^2} = \frac{4}{1}$$

or $$\frac{(2a-x)}{x} = \frac{2}{1}$$ or $$(2a-x) = 2x$$

or $$x = \frac{2a}{3}$$

**Note:** equilibrium position will be near to smaller charge.

**Method 2**

In this case we can directly calculate the value of x.

$$\Rightarrow x = \left(\frac{r}{1+\sqrt{\frac{Q_2}{Q_1}}}\right) \Rightarrow x = \left(\frac{2a}{1+\sqrt{\frac{4Q}{Q}}}\right) \Rightarrow x = \left(\frac{2a}{3}\right)$$

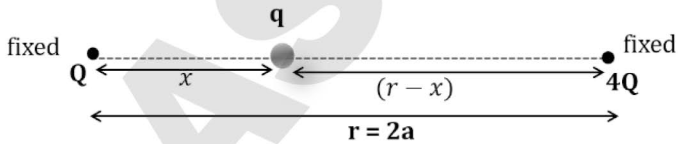
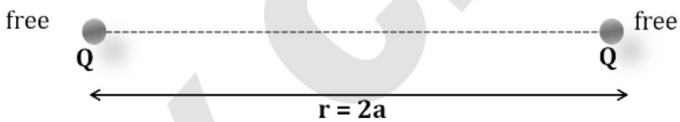


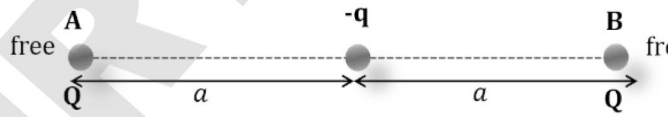

**Illustration 22:**

Where and what value of charge should be placed in between the given two charges so that whole system may remain in equilibrium.

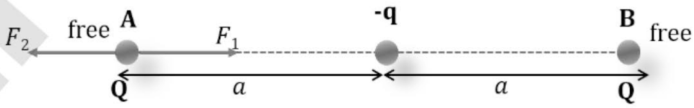



**Solution:**

As both free charges are identical so third charge will remain in equilibrium at mid point.
Third charge will be opposite in sign otherwise system's equilibrium is not possible




For equilibrium of charge placed at A, force of attraction of –q must be balanced by force of repulsion of charge placed at B




For equilibrium of Q, $F_1 = F_2$

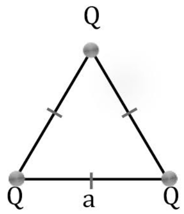
or $$\frac{kQq}{a^2} = \frac{kQ^2}{4a^2}$$

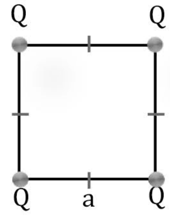
or $$\frac{q}{1} = \frac{Q}{4}$$ or $$q = \frac{Q}{4}$$ (in magnitude)

or $$q = -\frac{Q}{4}$$

**Illustration 23:**

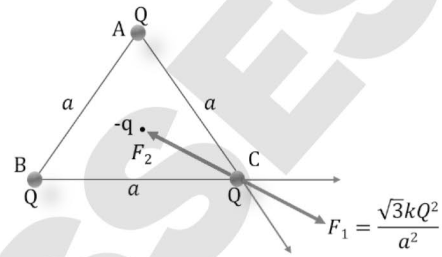
Find the value of charge, placed at the centre of given system, so that the systems will remain in equilibrium :

(i)



(ii)



**Solution:**

(i) Concept - As all charges are positive so they will repel each other. Definitely other charge must be opposite in sign.



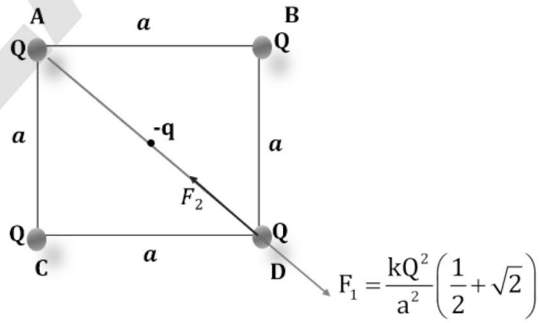

Distance b/w center to corner of triangle is $x = \frac{a}{\sqrt{3}}$

**For equilibrium of Q, $F_1 = F_2$**

or $\frac{\sqrt{3}kQ^2}{a^2} = \frac{kQq}{x^2}$ or $\frac{\sqrt{3}Q}{a^2} = \frac{3q}{a^2}$ or $q = \frac{Q}{\sqrt{3}}$ (magnitude)

or $q = -\frac{Q}{\sqrt{3}}$

(ii) Concept - As all charges are positive so they will repel each other. Definitely other charge must be opposite in sign




Distance b/w center to corner of Square is $x = \frac{a}{\sqrt{2}}$

**For equilibrium of Q, $F_1 = F_2$**

or $\frac{kQq}{x^2} = \frac{kQ^2}{a^2}\left(\frac{1}{2}+\sqrt{2}\right)$ or $\frac{2kQq}{a^2} = \frac{kQ^2}{a^2}\left(\frac{1}{2}+\sqrt{2}\right)$

or $q = \frac{Q}{4}\left(1+2\sqrt{2}\right)$ (magnitude) or $q = -\frac{Q}{4}\left(1+2\sqrt{2}\right)$

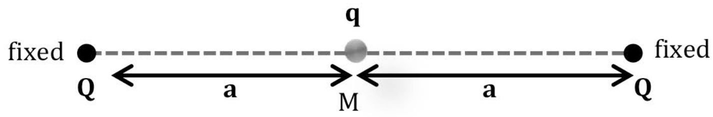
**Equilibrium of charge system**

**Types of equilibrium**

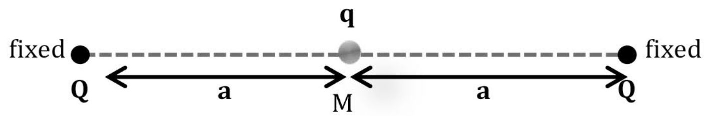
| **Stable** | **Unstable** | **Neutral** |
|---|---|---|
|  |  |  |
| A charge is initially in equilibrium position and is displaced by a small distance. If the charge tries to return back to the same equilibrium position then this equilibrium is called position of **stable equilibrium**. | If charge is displaced by a small distance from its equilibrium position and the charge has no tendency to return to the same equilibrium position. Instead it goes away from the equilibrium position, then this is called position of **unstable equilibrium**. | If charge is displaced by a small distance and it is still in equilibrium condition then it is called **neutral equilibrium**. |

**Illustration 24:**

What will be net force on charge q placed at mid point as shown

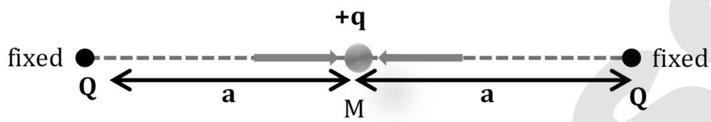



**Solution:**



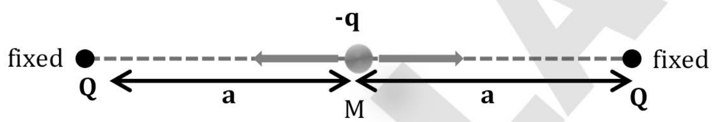

net force on q is zero whatever be sign and magnitude of q

**Case-1** when positive charge is at the mid point




$F_{net}$ on +q will be zero because it will be repel by equal and opposite force

**Case-2** when negative charge is at the mid point




$F_{net}$ on +q will be zero because it will be attract by equal and opposite force

**Illustration 25:**

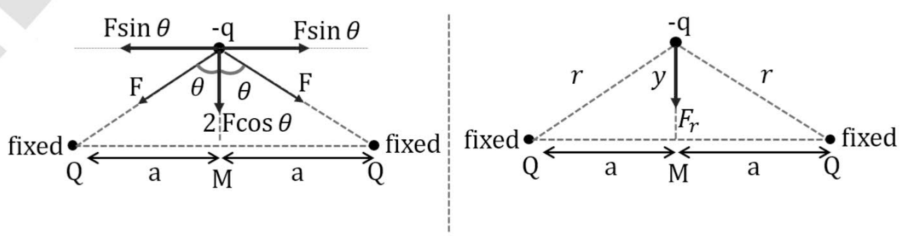
Two charges each of magnitude Q are fixed at 2a distance apart. A third charge (–q of mass 'm') is placed at the mid point of the two charges. When –q charge is slightly displaced perpendicular to the line joining the charges then find its time period.

**Solution:**

First of all we must know necessary condition for SHM. When restoring force acting on particle changes linearly w.r.t. displacement from mean position particle will perform SHM. $F_r \propto -y$

Findings time period of SHM when –q displaced in vertical direction from M

$F_r = 2F\cos\theta$




$F_r = 2\left(\frac{kQq}{r^2}\right)\frac{y}{r}$

$F_r = 2\left(\frac{kQq}{r^3}\right)\frac{y}{1}$

$F_r = 2\left(\frac{kQq}{(a^2+y^2)^{3/2}}\right)y$

$F_r = 2\left(\frac{kQq}{(a^2+y^2)^{3/2}}\right)y$ For small value of y, $y^2$ can be neglected w.r.t. $a^2$

$F_r = \left(\frac{2kQq}{a^3}\right)y$

$F_r = KY$

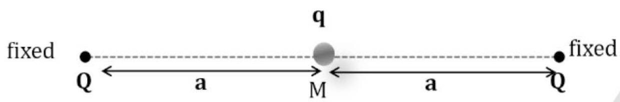
***BRAHMASTRA*** for calculation of time period.

$T = 2\pi\sqrt{\dfrac{m}{k}}$ (where k is coefficient of force, i.e. the value of all parameters except displacement in formula of restoring force)

So, $T = 2\pi\sqrt{\dfrac{ma^3}{2kqQ}}$

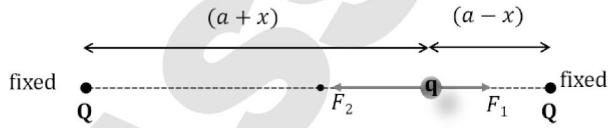
**Illustration 26:**

In the given three point charge system, final time period of central charge (q,m) when displaced slightly along the line joining the changes.




**Solution:**

As $F_2 > F_1$, So net force will be towards E.P.




$$F_r = kQq\left(\frac{4ax}{(a^2 - x^2)^2}\right)$$

For small value of x, $x^2$ can be neglected w.r.t. $a^2$

$$F_r = kQq\left(\frac{4ax}{(a^2 - x^2)^2}\right) = kQq\left(\frac{4ax}{a^4}\right) = \left(\frac{4kQq}{a^3}\right)x$$

**BRAHMASTRA** for calculation of time period

$$T = 2\pi\sqrt{\frac{m}{k}}$$

So, $T = 2\pi\sqrt{\dfrac{ma^3}{4kQq}}$

## BEGINNER'S BOX-2

1. Two identical metal spheres carry charges of + q and – 2q respectively. When the spheres are separated by a large distance r, the force between them is F. Now the spheres are allowed to touch and then moved back to the same separation. Find the new force of repulsion between them.
2. The electrostatic force of repulsion between two positive ions carrying equal charges is $4 \times 10^{-9}$ N, when their separation is 5 Å. How many electrons are missing from each ?
3. Two identical particles each of mass M and charge Q are placed a certain distance apart. If they are in equilibrium under mutual gravitational and electric force then calculate the order of $\dfrac{Q}{M}$ in SI units.
4. The force between two point charges is 100 N in air. Calculate the force if the distance between them is increased by 50%.
5. Two neutral insulating small spheres are rubbed against each other and are then kept 4 m apart. If they attract each other with a force of 3.6 N, then
   (i) calculate the charge on each sphere, and
   (ii) calculate the number of electrons transferred from one sphere to the other during rubbing.

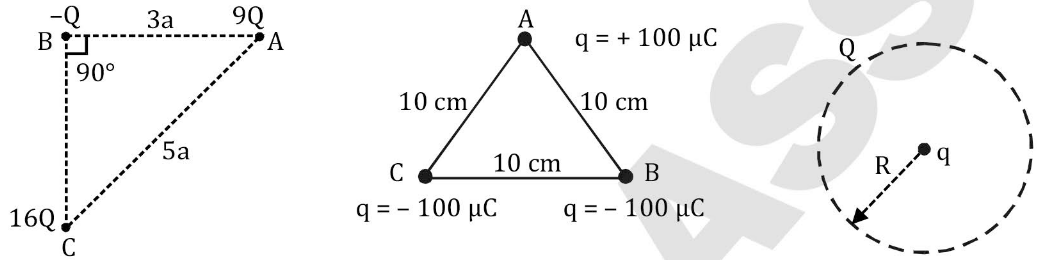
**6.** Two equal point charges Q = $\sqrt{2}$ +μC are placed at each of the two opposite corners of a square and equal point charges q at each of the other two corners. What must be the value of q so that the resultant force on Q is zero ?

**7.** In the given figure (b) three point charges are situated at the corners of an equilateral triangle of side 10 cm. Calculate the resultant force on the charge at B. What is its direction?

**8.** Two positively charged particles, each of mass $1.7 \times 10^{-27}$ kg and carrying a charge of $1.6 \times 10^{-19}$ C are placed at a distance d apart. If each experiences a repulsive force equal to its weight, find the value of d.

**9.** In figure (a) ABC is a right angled triangle. Calculate the magnitude of force on charge –Q.

**10.** In figure (c) Charge Q of mass m revolves around a point charge q due to electrostatic attraction. Show that its period of revolution is given by $T^2 = \dfrac{16\pi^3 \epsilon_0 m R^3}{Qq}$.



(a) (b) (c)

## Electric field Intensity

❖ The space around a charge or charge distribution, in which another charge experiences an electric force, is called electric field.

❖ Every charge has its own electric field.

❖ Electric field at a point can be characterized by

(i) A vector function of position, called intensity of electric field ($\vec{E}$)

(ii) A scalar function of position, called electrostatic potential(V).

(iii) Graphically, with help of electric field lines (EFL)

### Intensity of electric field

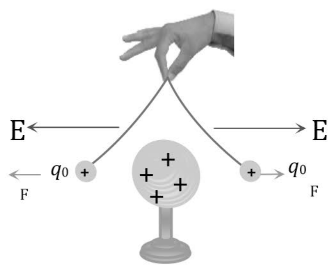
❖ It is defined as the net force experienced by a unit positive test charge.

❖ $\vec{E} = \dfrac{\vec{F}}{q_0}$ SI unit : N/C or V/m.

❖ Dimensional formula $[M^1L^1T^{-3}A^{-1}]$.

❖ **Test Charge :** It is a charge of very small magnitude which do not produce significant electric field.

So more precise definition of electric field intensity is $\vec{E} = \lim\limits_{q_0 \to 0}\left(\dfrac{\vec{F}}{q_0}\right)$.




❖ It is a vector quantity whose direction is always in the direction of force experienced by a positive test charge, and opposite to that of force experienced by a negative test charge.

❖ Electric field due to a positive charge is always away from it while due to a negative charge, this is always towards it.

❖ Force on a point charge is in the same direction as that of electric field on positive charge and in opposite direction as that of electric field on a negative charge.

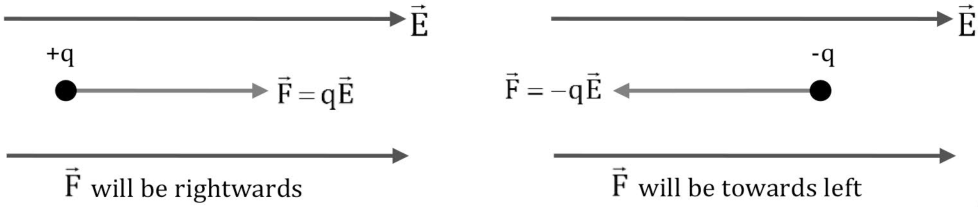
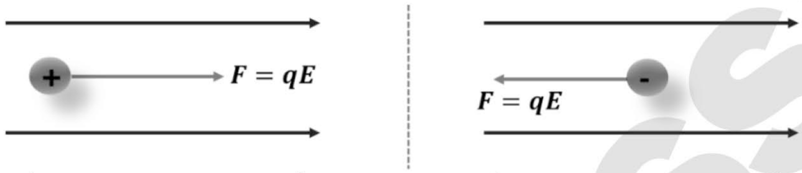



**Illustration 27:**
A charge of 10μC and -10 μC is placed in uniform electric field of $5\times10^6$ N/C directed along positive x axis, find out force acting on positive and negative charge?
**Solution:**



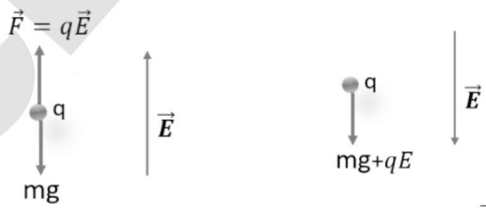

$\vec{F}=(10\times10^{-6})(5\times10^{6})\hat{i}$ $\quad$ $\vec{F}=(-10\times10^{-6})(5\times10^{6})\hat{i}$

$\vec{F}=50\hat{i}N$ $\quad$ $\vec{F}=-50\hat{i}N$

**Illustration 28:**
A positively charged oil drop is in equilibrium in a uniform electric field. If suddenly direction of electric field is reversed, then acceleration of drop becomes?

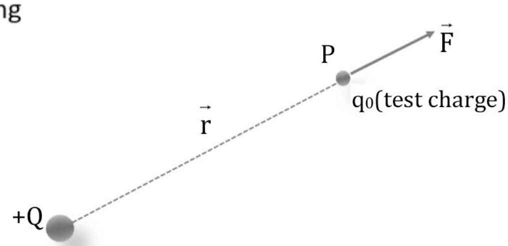
(1) g (2) 2g (3) $\frac{g}{2}$ (4) None of these

**Solution:**
For equilibrium of particle qE = mg
When electric field is reversed :
net force on particle = mg + qE = 2mg
Acceleration will be 2g (downwards)




**Electric field due to a point charge**

Force on test charge $\vec{F}=\frac{kQq_0}{r^2}\hat{r}$



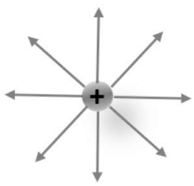

According to definition of electric field intensity

$$\vec{E}=\frac{\vec{F}}{q_0}=\frac{kQ}{r^2}\hat{r}=\frac{kQ}{r^3}\vec{r}$$

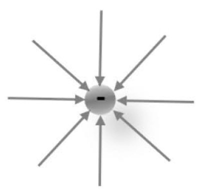
Electric field in vacuum $E=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$ and in medium $E_m=\frac{1}{4\pi\varepsilon_0 K}\frac{Q}{r^2}=\frac{E}{K}$

K is dielectric constant of medium also known as relative permittivity of medium.
(In general $K\geq1$ so $E_m\leq E$)

**Electric field due to positive and negative charge**




Electric field due to positive charge is radially outward




Electric field due to negative charge is radially inward

**Illustration 29:**

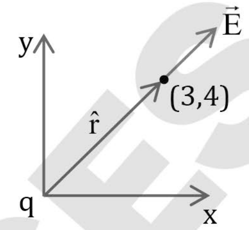
A charge particle of 1μC is placed at origin, Find intensity of electric field due to the charge at position (3,4)m.

**Solution:**

Position of particle is $\vec{r} = 3\hat{i} + 4\hat{j}$

Magnitude of r is $r = \sqrt{3^2 + 4^2} = 5m$

Electric field due to point charge $\vec{E} = \frac{kQ}{r^2}\hat{r}$




Unit vector along E, $\hat{r} = \frac{\vec{r}}{r}$ or $\hat{r} = \frac{3\hat{i} + 4\hat{j}}{5}$

$$\vec{E} = \frac{kQ}{r^2}\hat{r} = \frac{(9\times10^9)10^{-6}}{(5)^2}\left(\frac{3\hat{i}+4\hat{j}}{5}\right) = 72(3\hat{i}+4\hat{j})\ N|C$$

**Illustration 30:**

Calculate the electric field at origin due to infinite number of charges as shown in figures (a) and (b) below.

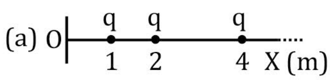


figure (a)

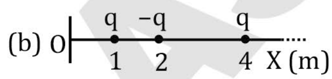


figure (b)

**Solution:**

(a) $E_0 = kq\left[\frac{1}{1} + \frac{1}{4} + \frac{1}{16} + - - - -\right] = \frac{kq.1}{(1-1/4)} = \frac{4kq}{3}$ , $[\because S_\infty = \frac{a}{1-r}$, a=1 and r = $\frac{1}{4}]$

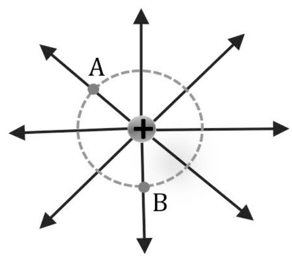
(b) $E_0 = kq\left[\frac{1}{1} - \frac{1}{4} + \frac{1}{16} - - - - - -\right] = \frac{kq.1}{(1+1/4)} = \frac{4kq}{5}$

**Illustration 31:**

There are two different points in the surrounding of point charge at same distance, electric field at those point will be same

(i) True (ii) False

**Solution:**




If distance of A and B points from point charge is r, the magnitude of electric field will be $E = \frac{kQ}{r^2}$

As electric field is vector quantity direction its direction at A and B will be different.

**Electric field due to system of charges**

- **Electric field follows superposition principle :**
  When two or more than two charges present in space than electric field at a particular point will be equal to vector sum of electric fields due to individual charges.
  According to superposition principle, net electric field at the given point will be
  $\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 ........ + \vec{E}_n$

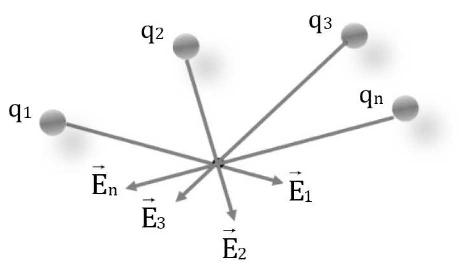
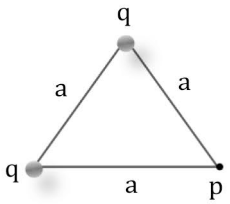
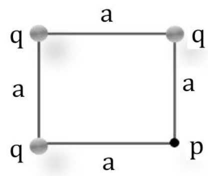
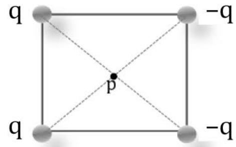


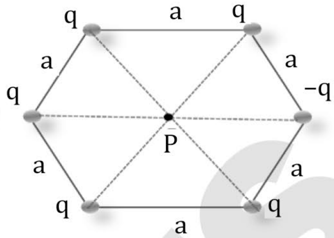
**Illustration 32:**

Find intensity of electric field at point P (as shown) in the given point charge distributions.

(i) 

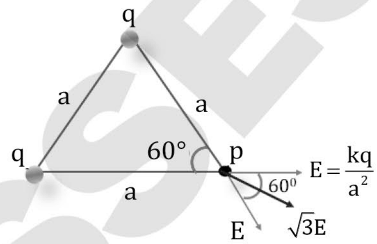

(ii) 


(iii) 

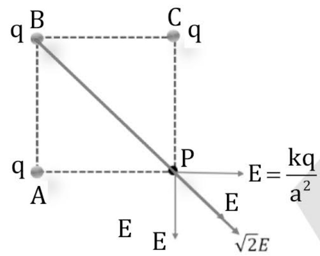

(iv) 


**Solution:**

(i) Resultant electric at p will be $E_r = \sqrt{3}E = \sqrt{3}\dfrac{kq}{a^2}$




(ii) $E_{net}$ at p will be

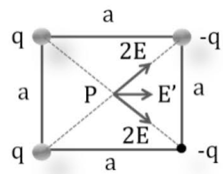



$\vec{E}_r = \vec{E}' + \sqrt{2}\vec{E}$

$E_r = \dfrac{kq}{(\sqrt{2}a)^2} + \sqrt{2}\dfrac{kq}{a^2}$

$E_r = \dfrac{kq}{a^2}\left(\dfrac{1}{2} + \sqrt{2}\right)$

(iii) Center to corner distance is $\dfrac{a}{\sqrt{2}}$ and here field $E = \dfrac{kq}{\left(a/\sqrt{2}\right)^2} = \dfrac{2kq}{a^2}$




So, net electric field $E' = 2(2E)\cos 45° = 4\sqrt{2}\dfrac{kq}{a^2}$

(iv) If all charges at corners would be q then net field at centre would be zero (Figure (a))

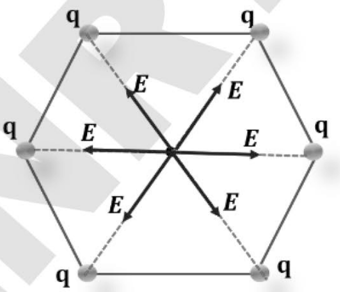



Figure (a)

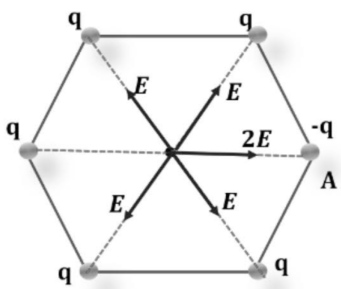
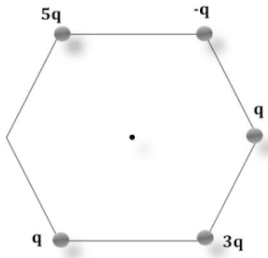



Figure (b)

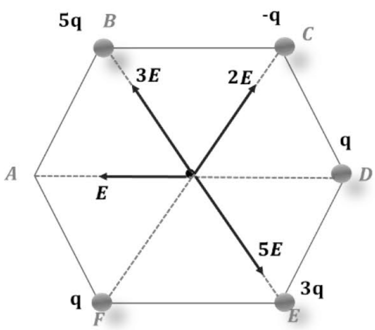
Now, if charge at point A becomes –q then we can assume it as q-2q so field at centre due to all +ve q charges will be zero and net field will be due to –2q charge at A.

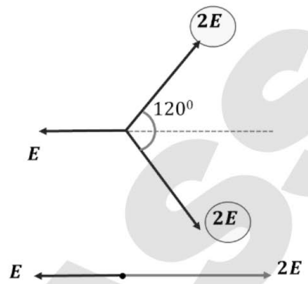
$E_{net} = \dfrac{2kq}{a^2}$ (along point A)

**Illustration 33:**
Five point charges placed on the vertices of a regular hexagon (as shown). Find net electric field at center of hexagon?




**Solution:**




Net electric field will be rightwards and it will be $E_{net} = \dfrac{kq}{r^2}$

(r is side of hexagon which is also distance b/w center to vertex)

**Important Note:**

When identical charges are placed on the corners of a regular polygon (symmetric arrangement) then resultant field at centre of the polygon is always zero.

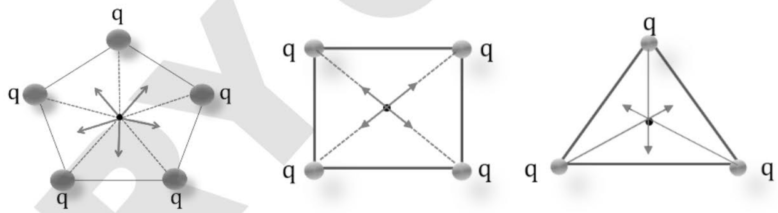
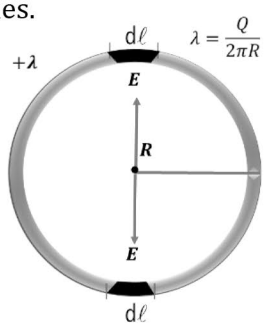
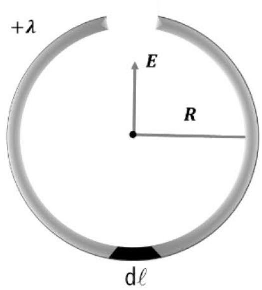



**Illustration 34**:

Charge Q is uniformly distributed over a ring of radius R. If a small portion of length dℓ is removed from ring, then find electric field at the centre of ring becomes.




**Solution:**

If dq is charge of small element then,

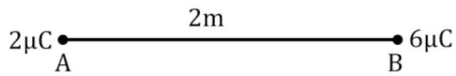
$$E = \frac{kdq}{R^2} = \frac{k\lambda d\ell}{R^2} = \frac{kd\ell}{R^2}\left(\frac{Q}{2\pi R}\right)$$

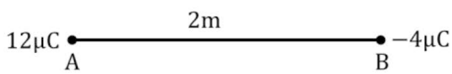
**Neutral point:**

In an electric field, a neutral point is said to be a point at which the resultant electrical field is nil or zero.

**Illustration 35**:

Find the point at which resultant $\vec{E}$ of the systems will become zero.

**(i)** 

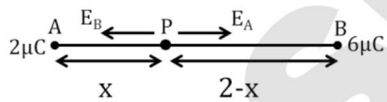

**(ii)** 


**Solution:**

(i) Let us assume at point P resultant $\vec{E}$ is zero due to A and B so

$$|\vec{E}_A| = |\vec{E}_B|$$

$$\frac{k(2\mu)}{x^2} = \frac{k(6\mu)}{(2-x)^2} \Rightarrow x = \frac{1}{2}$$




(ii) Let us assume at point P resultant $\vec{E}$ is zero due to A and B so

$$|\vec{E}_A| = |\vec{E}_B|$$

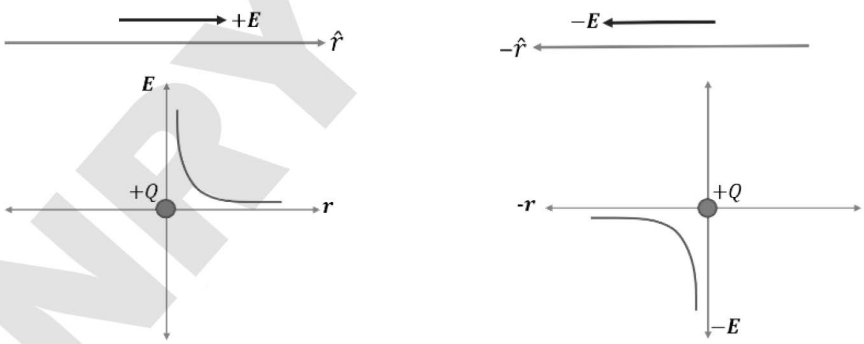
$$\frac{k(12\mu)}{(2+x)^2} = \frac{k(4\mu)}{x^2} \Rightarrow x = 1\text{m}$$


**Graphical problems**

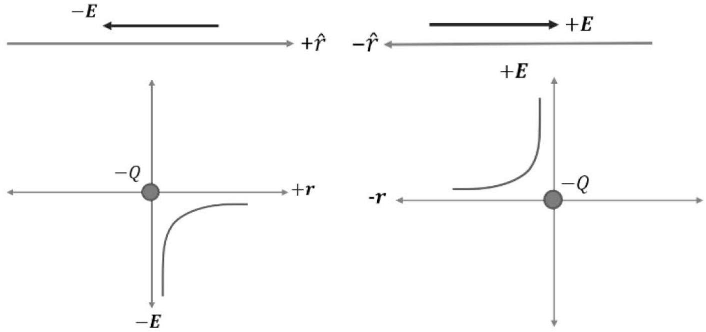
Electric field v/s distance

For point charge $E = \frac{kQ}{r^2}$ or $E \propto \frac{1}{r^2}$

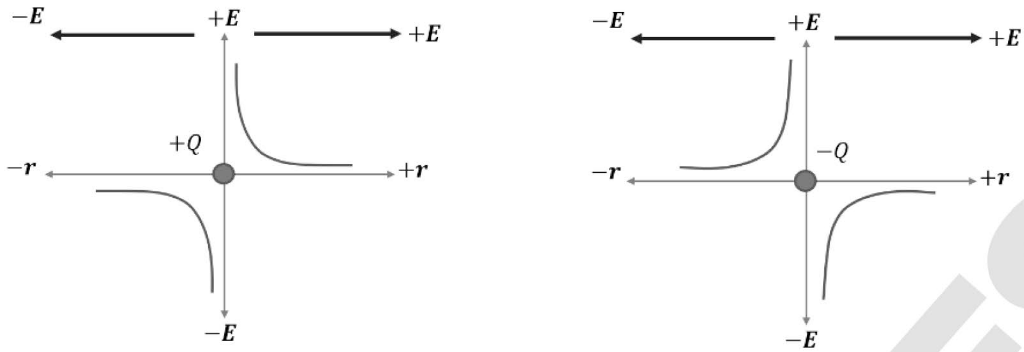
**(i) For positive charge :**



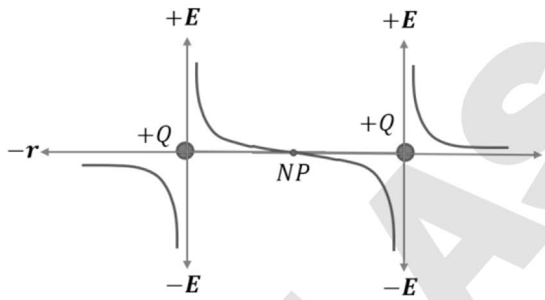

**(ii) For negative charge :**



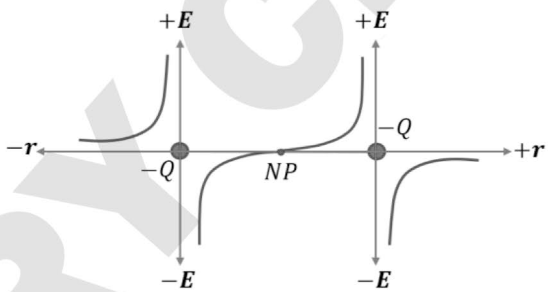
**(iii) For positive and negative charge (combined analysis) :**



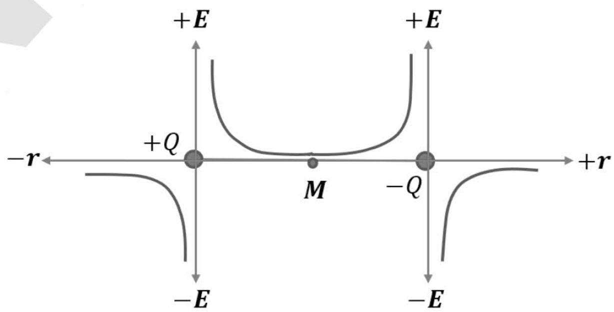

**(iv) Graph for pair of positive charges :**




**(v) Graph for pair of negative charges :**




**(vi) Graph for pair of positive and negative charges :**



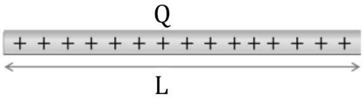

Here M is the mid point of line joining the two charges at x distance from any charge.

Here electric field will be $E = \frac{2kQ}{x^2}$ (rightward minimum electric field)

## Different type of charge density

### Types of continuous charge distributions

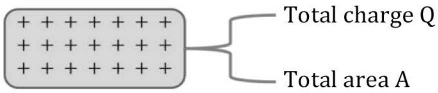
**Linear Charge distribution**




Linear charge density is,
$\lambda = \frac{Q}{L} C/m$

Used for linear objects (1-D) such as wire, thin rod, ring etc. and for non-uniform distribution of charge, charge of small element can be find by, $dQ = \lambda dl$

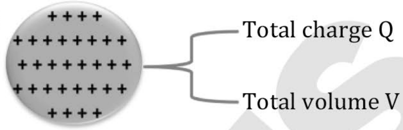
**Surface Charge distribution**




Used for flat objects (2-D) such as plate, disc, etc.
Here, $\sigma$ is surface charge density, which is given by $\sigma = \left(\frac{Q}{A}\right) C/m^2$

For non-uniform distribution of charge, charge of small element can be find by, $dQ = \sigma dA$

**Volume Charge distribution**

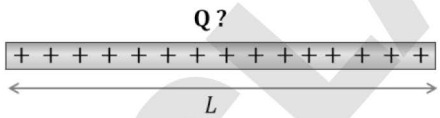



Used for 3-D objects such as sphere, cube, cylinder etc.
$\rho$ is volume charge density, which is given by $\rho = \left(\frac{Q}{V}\right) C/m^3$

For non-uniform distribution of charge, charge of small element can be find by, $dQ = \rho dV$

### Illustration 36:

Find total charge of a thin rod whose linear charge density varies according to $\lambda = \lambda_0 X$.

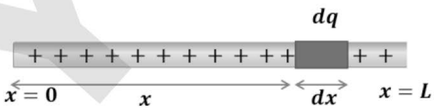



### Solution:

We consider a small element of length dx

Then $dq = \lambda dx = (\lambda_0 x)dx$

$$\int_0^Q dq = \int_0^L \lambda_0 x dx$$




$$Q = \frac{\lambda_0 L^2}{2}$$

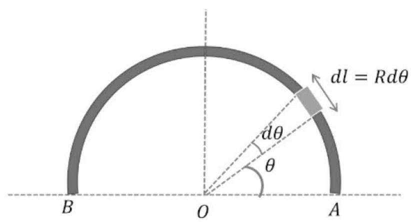
### Illustration 37:

If linear charge density of semi-circular ring is $\lambda = \lambda_0 \sin\theta$. Then find total charge on semi-circular ring.

### Solution:

For finding total charge on semi-circular ring -

Charge on small element $dq = \lambda dx$




$$dq = (\lambda_0 \sin\theta) dl = (\lambda_0 \sin\theta) R d\theta$$

$$Q = \int_0^\pi (\lambda_0 R)\sin\theta d\theta = (\lambda_0 R)[-\cos\theta]_0^\pi$$

$$Q = (-\lambda_0 R)[\cos\pi - \cos 0]$$

$$= 2\lambda_0 R$$

**Illustration 38:**

If linear charge density of a semi-circular ring is $\lambda = \lambda_0 \cos\theta$. Find total charge on the semi-circular ring?

**Solution:**

For finding total charge on semi-circular ring

$dq = \lambda dx$ or $dq = (\lambda_0 \cos\theta)dl$

or $dq = (\lambda_0 \cos\theta) R d\theta$ or $Q = \int_0^{\pi} (\lambda_0 R)\cos\theta d\theta$

or $Q = (\lambda_0 R)\,[\sin\theta]_0^{\pi}$ or $Q = (\lambda_0 R)\,[\sin\pi - \sin 0]$

or Q = 0 (i.e. total charge on ring is zero)

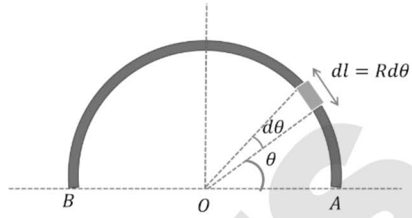



**Illustration 39:**

Find total charge on a disc of radius R, Whose surface charge density varies according to $\sigma = \sigma_0\left(1 - \frac{r}{R}\right)$ (r is radial distance from the centre of disc).

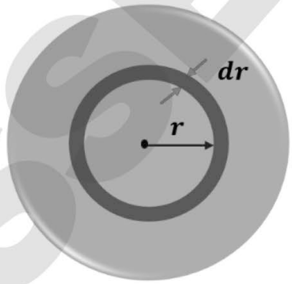
**Solution:**

Here we consider elementary ring of radius **r** and thickness **dr**

Area of elementary ring $\Rightarrow dA = (2\pi r)dr$

so Charge on elementary ring

$dq = \sigma dA = \sigma(2\pi r)dr = \sigma_0\left(1 - \frac{r}{R}\right)(2\pi r)dr$




$\therefore$ Total charge -

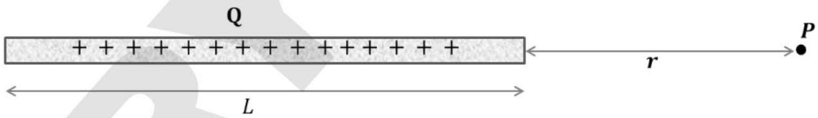
$$Q = \int_0^R \sigma_0\left(1 - \frac{r}{R}\right)(2\pi r)dr$$

$$Q = 2\pi\sigma_0 \int_0^R \left(r - \frac{r^2}{R}\right)dr = 2\pi\sigma_0\left(\frac{R^2}{2} - \frac{R^3}{3R}\right) = 2\pi\sigma_0\left(\frac{R^2}{2} - \frac{R^2}{3}\right) = \frac{2\pi\sigma_0 R^2}{6} = \frac{\pi\sigma_0 R^2}{3}$$

$Q = \frac{\sigma_0 A}{3}$, where A is Area of disc $A = \pi R^2$

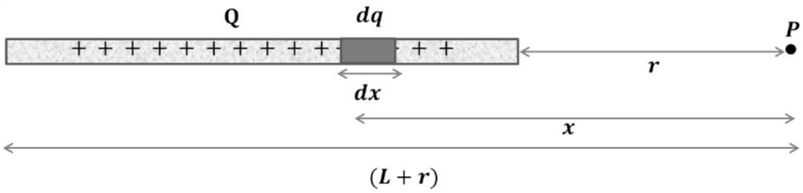
**Illustration 40:**

Find electric field at point P as shown, due to a uniformly charged thin rod (Q, L).




**Solution:**

Now consider an element as shown -




Linear charge density, $\lambda = \frac{Q}{L} C/m$

Field at point P due to the small element

$dE = \frac{k(dq)}{x^2}$ here $dq = \lambda dx = \frac{Q}{L}dx$

Total field at P, due to the whole rod

$$E = \int_r^{(r+L)} \frac{k(dq)}{x^2} = \int_r^{(r+L)} \frac{k}{x^2}\frac{Q}{L}dx = \frac{kQ}{L}\int_r^{(r+L)} \frac{1}{x^2}dx = \frac{kQ}{L}\left[-\frac{1}{x}\right]_r^{(r+L)} = \frac{kQ}{L}\left[\frac{1}{r} - \frac{1}{r+L}\right] = \frac{kQ}{r(r+L)}$$

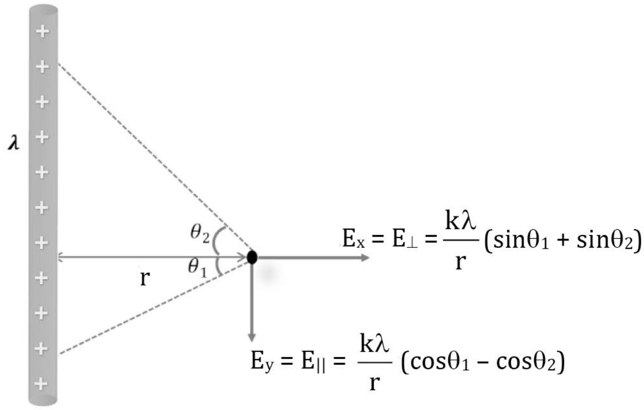
In above case if (r>>L) then L can be neglected w.r.t. r

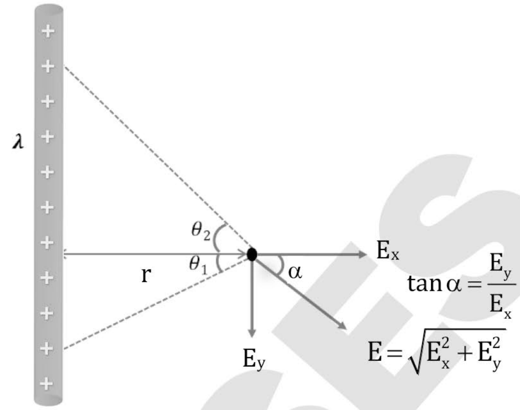
$$E = \frac{kQ}{r(r+L)} \approx \frac{kQ}{r^2}$$

i.e. . charged rod will behave like point charge.

## Electric field intensity due to different charge configurations

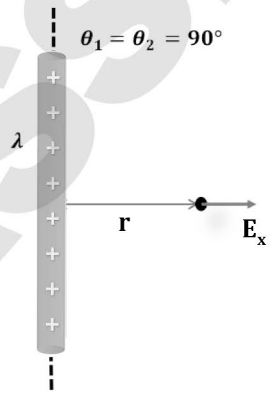
### (1) Electric field intensity due to a charged wire of finite length.




**Special Case:**

**(i) For infinite wire,** (both ends goes to infinite)




$\theta_1 = \theta_2 = 90°$

$$E_x = E_\perp = \frac{k\lambda}{r}(\sin 90° + \sin 90°)$$

$$E_x = E_\perp = \frac{2k\lambda}{r}$$

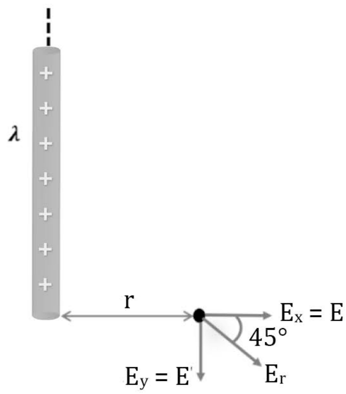
$$E_y = E_{||} = \frac{k\lambda}{r}(\cos 90° - \cos 90°) = 0$$

Net electric field will be $E_x = E_\perp = \frac{2k\lambda}{r}$ (perpendicular to wire)

**(ii) For semi – infinite wire -**

(one end goes to infinite and we find electric field at a point which is at ⊥ distance r from other end)

For the given point $\theta_1 = 0,\ \theta_2 = 90°$



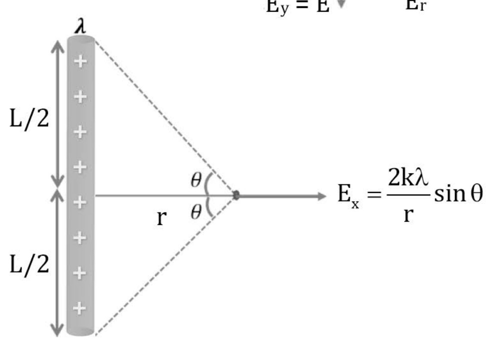

$$E_x = E_\perp = \frac{k\lambda}{r}(\sin 0° + \sin 90°) = \frac{k\lambda}{r}$$

$$E_y = E_{||} = \frac{k\lambda}{r}(\cos 0° - \cos 90°) = \frac{k\lambda}{r}$$

$$E_r = \sqrt{E_x^2 + E_y^2} = \sqrt{E^2 + E^2} = \sqrt{2}E$$

$$E_r = \frac{\sqrt{2}k\lambda}{r}$$

**(iii) Electric field due to finite wire at symmetric point :**



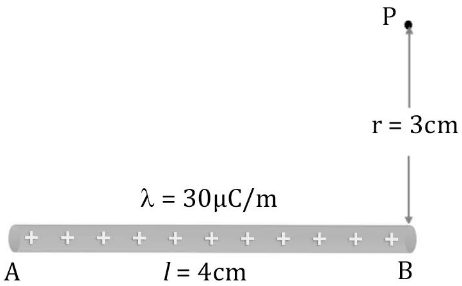

$$E_x = E_\perp = \frac{k\lambda}{r}(\sin\theta + \sin\theta) = \frac{2k\lambda}{r}\sin\theta$$

$$E_y = E_{||} = \frac{k\lambda}{r}(\cos\theta - \cos\theta) = 0$$

$$E_x = \frac{2k\lambda}{r}\left(\frac{L/2}{\sqrt{(L/2)^2 + (r)^2}}\right)$$

**Illustration 41:**

Find x component of electric field at point p (see figure).




**Solution:**

$$E = \frac{k\lambda}{r}(\cos 0° - \cos 53°)$$

$$E_x = \frac{k\lambda}{r}\left(1 - \frac{3}{5}\right) = \frac{k\lambda}{r}\left(\frac{2}{5}\right)$$

$$E_x = \frac{(9\times10^9)(30\times10^{-6})}{3\times10^{-2}}\left(\frac{2}{5}\right)$$

$$E_x = 9\times10^6\left(\frac{2}{5}\right) = 3.6\times10^6 \, N/C$$

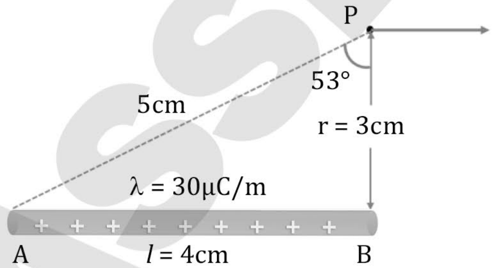



**Illustration 42:**

A negative charge particle (-q, m) is revolving around a uniformly charged long wire (+λ). Find speed of the particle.

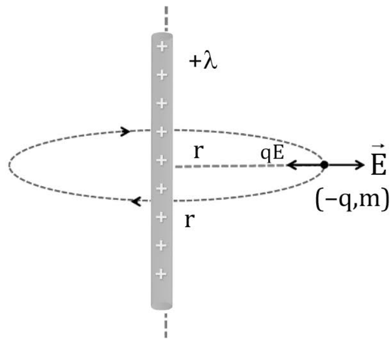
**Solution:**

Force of attraction of wire on -q will provide required centripetal force

$$qE = \frac{mv^2}{r} \qquad \text{where, } E = \frac{2k\lambda}{r}$$

E will be radially away from wire, so force on charge will be towards the wire

$$\frac{2k\lambda q}{r} = \frac{mv^2}{r}, \; v = \sqrt{\frac{2k\lambda q}{m}} \, m/s$$




**Electric field due to a uniformly charged arc**

$$E = \frac{2k\lambda}{R}\sin\left(\frac{\theta}{2}\right) \qquad \theta \rightarrow \text{angle of arc}$$

Here λ is linear charge density

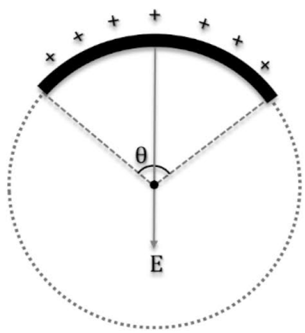
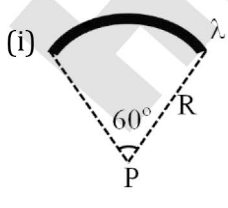
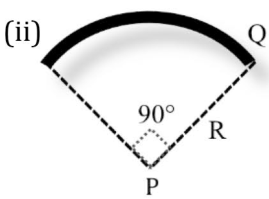



**Illustration 43:**

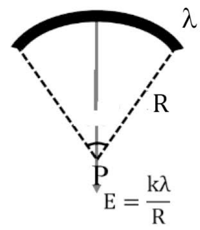
Find Electric field at point P shown in diagrams.



**Solution:**

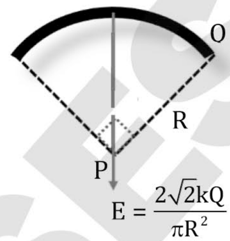
(i) From the formula of electric field due to arc




$$E=\frac{2k\lambda}{R}\sin\left(\frac{\theta}{2}\right) \qquad \theta \rightarrow \text{angle of arc}$$

$$E=\frac{2k\lambda}{R}\sin\left(\frac{60^\circ}{2}\right)=\frac{2k\lambda}{R}\times\left(\frac{1}{2}\right)=\frac{k\lambda}{R}$$

(ii) From the formula of electric field due to arc




$$E=\frac{2k\lambda}{R}\sin\left(\frac{\theta}{2}\right) \qquad \theta \rightarrow \text{angle of arc}$$

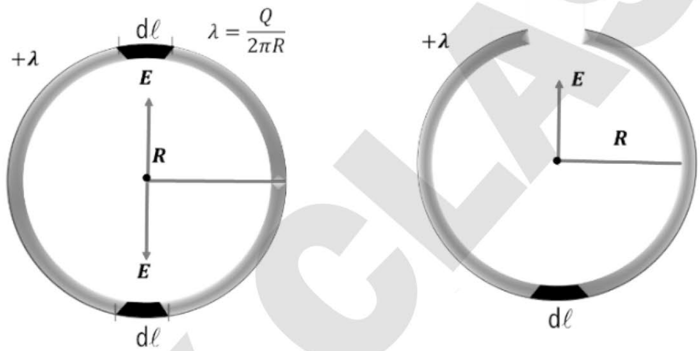
$$E=\frac{2kQ}{R}\sin\left(\frac{90^\circ}{2}\right)=\frac{2kQ}{R}\times\frac{1}{\sqrt{2}}=\frac{\sqrt{2}k\lambda}{R} \quad (\text{here } \lambda = 2Q/\pi R)$$

$$E=\frac{2\sqrt{2}kQ}{\pi R^2}$$

**Illustration 44**:

Charge Q is uniformly distributed over a ring of radius R. If a small portion of length $d\ell$ is removed from ring, then find electric field at the centre of ring.

**Solution:**

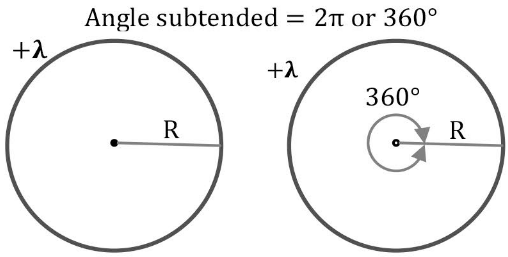



If dq is charge of small element then, $E=\frac{kdq}{R^2}=\frac{k\lambda d\ell}{R^2}=\frac{kd\ell}{R^2}\left(\frac{Q}{2\pi R}\right)$

**Electric field due to circular ring**

Electric field due to uniformly charged ring at its centre.

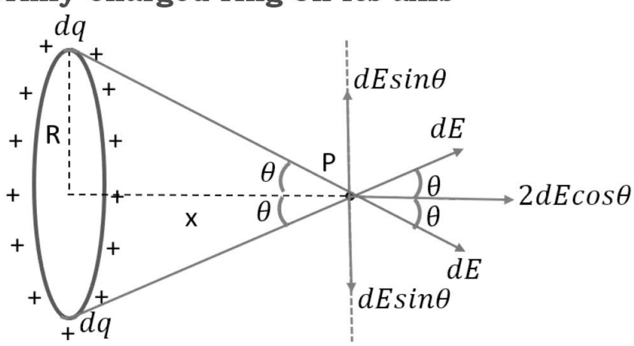
From the formula of electric field due to arc




$$E=\frac{2k\lambda}{R}\sin\left(\frac{\theta}{2}\right) \qquad \theta \rightarrow \text{angle of arc}$$

$$E=\frac{2k\lambda}{R}\sin\left(\frac{360^\circ}{2}\right)=0$$

**Electric field due to a uniformly charged ring on its axis**



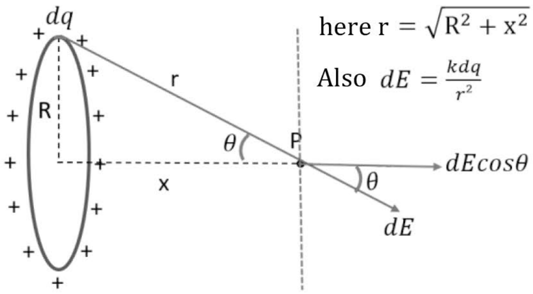
Here it is clear that dEsinθ, of all the elements will cancel out each other and sum of all dEcosθ , will give net electric field.

∴ Net electric field along x axis

$$E=\int dE\cos\theta=\int\frac{kdq}{r^2}\left(\frac{x}{r}\right)$$

Here $\int dq=Q$

So $\boxed{E=\dfrac{kQx}{(R^2+x^2)^{3/2}}}$




here $r=\sqrt{R^2+x^2}$

Also $dE=\frac{kdq}{r^2}$

**Special Cases:**

**(1) Electric field on the axis for small values of x.**

If x is very small then $x^2$ can be neglected w.r.t. $R^2$

$$E=\frac{kQ}{(R^2+x^2)^{3/2}}.x\approx\frac{kQ}{R^3}.x$$ (E changes linearly w.r.t. x)

($x^2$ neglected)

**(2) Electric field at the centre of ring.**

From $E=\dfrac{kQx}{(x^2+R^2)^{3/2}}$ ⇒ E will be zero at x=0

**(3) Electric field at the axis for larger values of x.**

If x is very large, then $R^2$ can be neglected w.r.t. $x^2$

$$E=\frac{kQ}{(R^2+x^2)^{3/2}}.x\approx\frac{kQ}{x^2}$$ (Ring behaves like a point charge)

($R^2$ neglected)

**(4) Maximum value of electric field**

For maximum value of electric field $\dfrac{dE}{dx}=0\Rightarrow\dfrac{d\left(\dfrac{kQ}{(R^2+x^2)^{3/2}}.x\right)}{dx}=0$

From here we obtained $x=\pm\dfrac{R}{\sqrt{2}}$

So, $E_{max}$ = E at $x=\dfrac{R}{\sqrt{2}}$ , Substituting values, we have $E_{max}=\dfrac{2kQ}{3\sqrt{3}R^2}$

**Variation of electric field with x for ring**

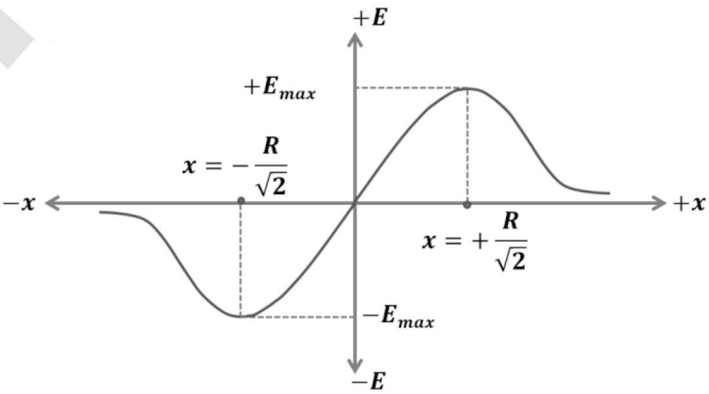
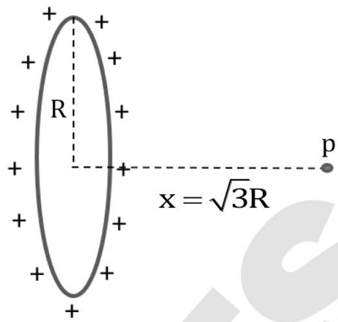


**Illustration 45:**

Find electric field on the axis of a uniformly charged ring (Q,R) at a distance $x=\sqrt{3}R$.




**Solution:**

From $E=\frac{kQ}{(R^2+x^2)^{3/2}}.x$

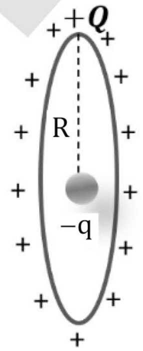
Here we put $x=\sqrt{3}R$

$$E=\frac{kQ}{(R^2+(\sqrt{3}R)^2)^{3/2}}.(\sqrt{3}R)$$

$$E=\frac{kQ}{(4R^2)^{3/2}}.(\sqrt{3}R)=\frac{kQ}{(8R^3)}.(\sqrt{3}R)\Rightarrow E=\frac{\sqrt{3}kQ}{8R^2}$$

**Illustration 46:**

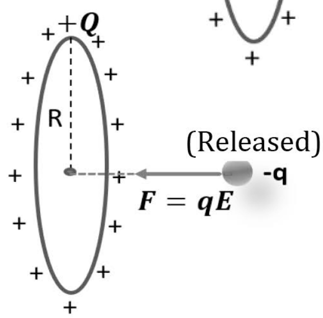
If a point charge (-q, m) is displaced slightly along the axis of a uniformly charged ring and released, find time period of oscillation of the particle.




**Solution:**

As we know electric field at centre of ring is zero, force on –q will be zero at centre so it will be in equilibrium position.

When charge particle is displaced and released, for small value of x, it will experiences force F = qE towards the centre (i.e. E.P.)




$$F=qE=\frac{kQq}{R^3}.x=\frac{1}{4\pi\epsilon_0}\frac{Qq}{R^3}.x \text{ (restoring force)}$$

Compare with equation $a = -\omega^2x$

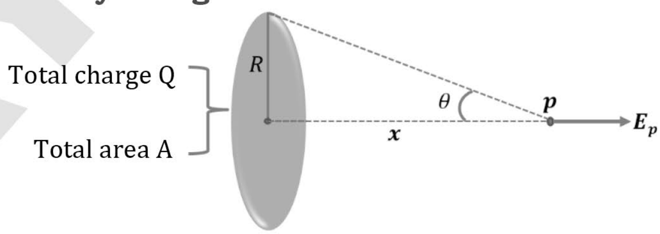
$T=\frac{2\pi}{\omega}$ hence

$$T=2\pi\sqrt{\frac{4\pi\epsilon_0 R^3m}{Qq}}$$

**Electric field on the axis of a uniformly charged disc of radius R**

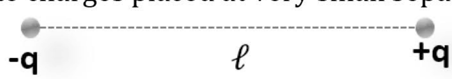
$$E_p=\frac{\sigma}{2\epsilon_0}[1-\cos\theta]$$

$$E_p=\frac{\sigma}{2\epsilon_0}\left[1-\frac{x}{\sqrt{R^2+x^2}}\right]$$




## Electric Dipole

A system of two equal and opposite charges placed at very small separation is known as electric dipole.




Every dipole has a characteristic property called dipole moment.

**Dipole moment:**

The dipole moment of a dipole is equal to product of magnitude of either charge and separation between the charges.

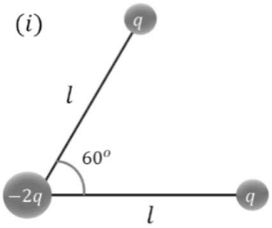
$$\boxed{\vec{p}=q\vec{\ell}}$$

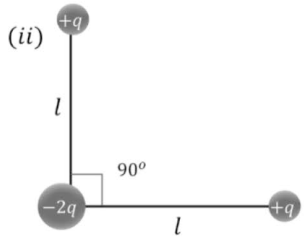
It is a vector quantity whose direction is from (-q) to (+q)

SI unit → C-m, Practical unit → debye ($3.33\times10^{-30}$ C-m)

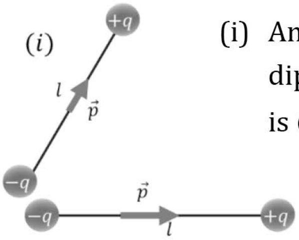
**Illustration 47:**

Find electric dipole moment of the given arrangement.



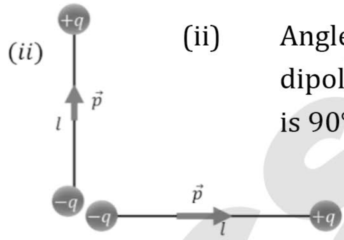




**Solution:**




(i) Angle between both the dipole moment vectors is 60°.

$P_{res} = \sqrt{3}p = \sqrt{3}(ql)$

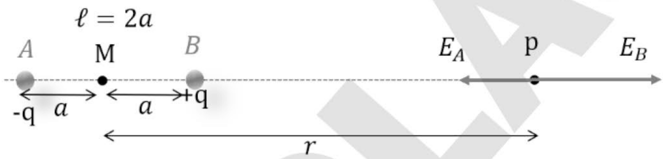



(ii) Angle between both the dipole moment vectors is 90°.

$P_{net} = \sqrt{2}p = \sqrt{2}(ql)$

**Electric field due to a dipole**

**(1) At axial / End on position :**




Here $E_A = \dfrac{kq}{(r+a)^2}$ & $E_B = \dfrac{kq}{(r-a)^2}$

Net electric field at point P on axis is

$$\text{Eaxis} = E_B - E_A = \frac{kq}{(r-a)^2} - \frac{kq}{(r+a)^2} = kq\left(\frac{4ar}{(r^2-a^2)^2}\right)$$

For r>>a ∴ $r^2 - a^2 \approx r^2$

$$E_{axis} = kq\left(\frac{4ar}{r^4}\right) = \left(\frac{2k(q2a)}{r^3}\right) = \left(\frac{2kp}{r^3}\right)$$

For axial point electric field vector and dipole moment vector both are parallel to each other or angle between both vectors is zero degree.

In vector form $\vec{E}_{axis} = \left(\dfrac{2kp}{r^3}\right)\hat{r} = \dfrac{2k\vec{p}}{r^3}$

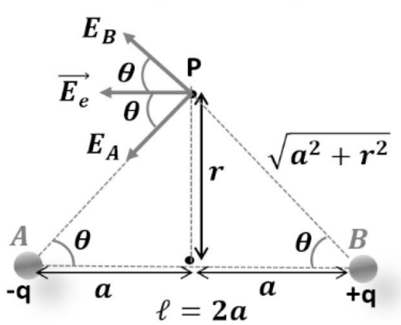

**(2) At equator / Broad side on position :**

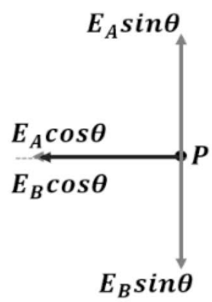
$E_A = E_B$ (in magnitude)

$$E_A = E_B = \frac{kq}{\left(\sqrt{a^2+r^2}\right)^2} = \frac{kq}{(a^2+r^2)}$$

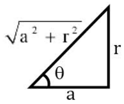
Net electric field at equator

$E_P = E_B\cos\theta + E_A\cos\theta$



$\because\ \cos\theta = \dfrac{a}{\sqrt{a^2+r^2}}$




$$E_P = 2E_B\cos\theta = 2E_A\cos\theta = 2\left(\frac{kq}{(a^2+r^2)}\right)\frac{a}{\sqrt{a^2+r^2}}$$

$$E_P = \left(\frac{kq(2a)}{(a^2+r^2)}\right)\frac{1}{\sqrt{a^2+r^2}} = \frac{kp}{(a^2+r^2)^{3/2}}$$

For r>>a $\therefore$ $r^2 + a^2 \approx r^2$

$E_P = \dfrac{kp}{r^3}$ in vector from $\vec{E}_P = \dfrac{k\vec{p}}{r^3}(-\hat{i})$

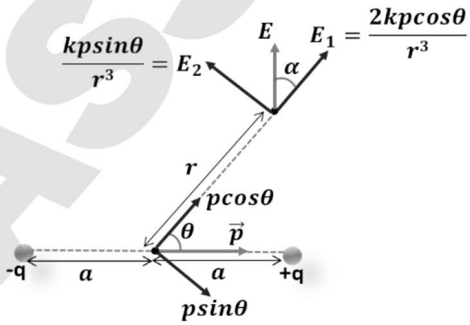
Negative sign is used because electric field vector and dipole moment vector both are opposite in direction.

**(3) At general position :**

Net electric field at that point will be $E = \sqrt{E_1^2 + E_2^2}$

$$E = \frac{kp}{r^3}\sqrt{(2\cos\theta)^2 + (\sin\theta)^2} = \frac{kp}{r^3}\sqrt{3\cos^2+1}$$

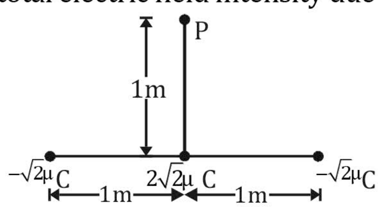
Net electric field is making angle ($\alpha+\theta$) from the direction of dipole moment vector.




$$\tan\alpha = \frac{E_2}{E_1} = \frac{\frac{kp\sin\theta}{r^3}}{\frac{2kp\cos\theta}{r^3}} = \frac{1}{2}\tan\theta \Rightarrow \boxed{\tan\alpha = \frac{1}{2}\tan\theta}$$

## BEGINNER'S BOX-3

1. Two charges of value 2 μC and – 50 μC are placed 80 cm apart. Calculate the distance of the point from the smaller charge where the intensity is zero.
2. A charged particle of mass 2 mili gram remains freely in air in an electric field of strength 4 N/C directed upward. Calculate the charge and determine its nature ($g = 10\ m/s^2$).
3. How many electrons should be added or removed from a neutral body of mass 10 mili gram so that it may remain stationary in air in an electric field of strength 100 N/C directed upwards ($g = 10\ m/s^2$) ?
4. Work out the magnitude and direction of field at point P, when a charge of 2 μC experiences an electrical force of $5 \times 10^{-2}\ \hat{j}$ N at point P.
5. Two charges 4 μC and 36 μC are placed 60 cm apart. At what distance from the larger charge is the electric field intensity is zero ?
6. Three charges of respective values $-\sqrt{2}\,\mu C$, $2\sqrt{2}\,\mu C$ and $-\sqrt{2}\,\mu C$ are arranged along a straight line as shown in the figure. Calculate the total electric field intensity due to all three charges at the point P.



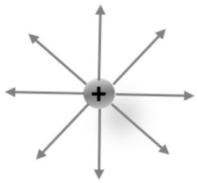
## Electric Field Lines

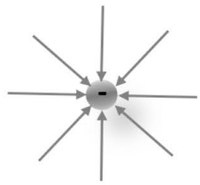
- ❖ These are imaginary lines used to represent electric field pictorially.
- ❖ These are such a smooth continuous curve that tangent drawn at any point gives direction of field at that point.

  Concept was given by MICHAEL FARADAY.

**Properties of Electric field lines:**

(i) They originate from (+) charge and terminate at (-)charge.

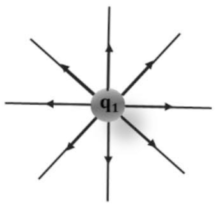


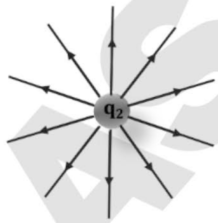



(ii) It gives an idea about the magnitude of charge.

$|q| \propto$ number of field lines

Here $|q_1| < |q_2|$



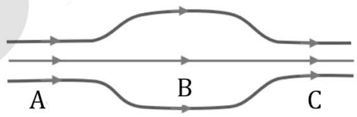



(iii) It gives an idea about the strength of electric field.

Density of electric field lines (EFL) $\propto$ strength of intensity of electric field.

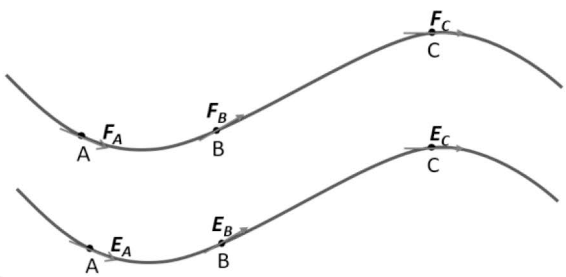
- ❖ If electric field lines are denser electric field intensity is high.
- ❖ If electric field lines are rarer electric field intensity is low.


$E_A > E_B$




$E_A = E_C > E_B$

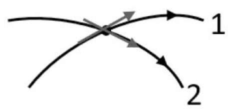
(iv) Tangent drawn at any point of the electric field line, gives the direction of force on a charge particle placed at that given point.




**Note:** It gives the direction of force not the direction of motion.

(v) Two electric field lines can never intersect each other.

**Reason:** If they do so, then at the point of intersection, there will be two tangents at same point representing two different directions of electric field at the point which is not possible.

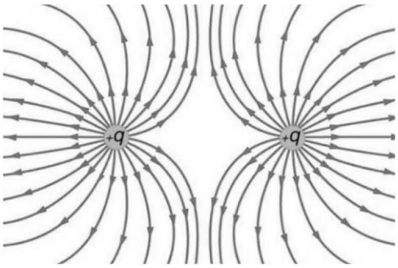



(vi) Electrostatic field lines can never form any closed loop, but induced electric field lines can form a closed loop.

Electric Field

| Electrostatic field | Induced electric field |
|---|---|
| Produced due to static charges | Produced due to change in magnetic field |
| (no close loops) | (forms close loops) |

(vii) Electric filed lines due to a pair of like nature of charges.




(viii) Electric filed lines due to a pair of equal and opposite charges.

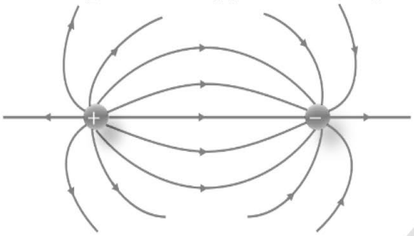
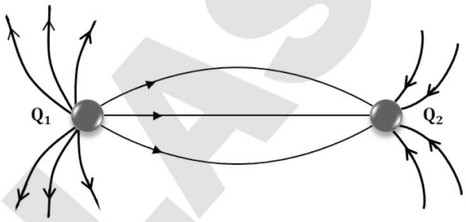


**Illustration 48:**

Identify the correct option/s




(1) $|Q_1| > |Q_2|$

(2) $|Q_1| < |Q_2|$

(3) $Q_1 \rightarrow$ +ve ; $Q_2 \rightarrow$ –ve

(4) $Q_1 \rightarrow$ –ve ; $Q_2 \rightarrow$ +ve

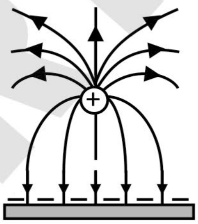
**Solution:**

(3) and (1) both correct

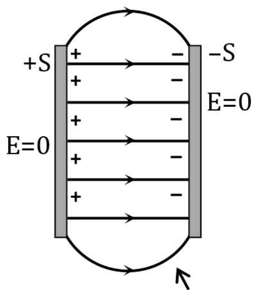
field lines starts from +ve and ends at -ve, and no. of field lines $\propto$ q

**Golden Key Points**

- Lines of force starts from (+ve) charge and end on (–ve) charge.
- Lines of force start and end normally on the surface of a conductor.

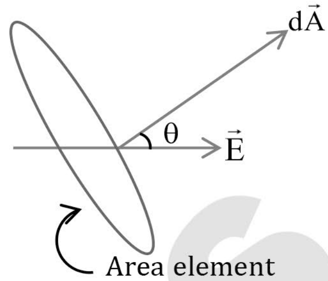



Fixed point charge
Near infinite metal plate

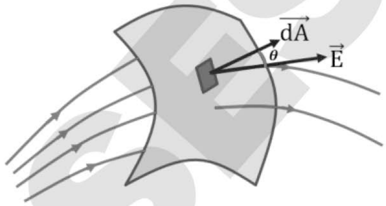



- Lines of force never intersect because, the field at a point (point of intersection) cannot have two distinct directions.
- Electric field inside a solid conductor is always zero.
- Electric field inside a hollow conductor may or may not be zero ($E \neq 0$ if net charge is present inside the sphere).
- The electric field due to a circular loop of charge and a point charge are identical provided the distance of the observation point from the circular loop is quite large as compared to its radius i.e. $x >> R$.

## Electric flux




- This physical quantity is used to measure strength of electric field (also called flux density).
- The electric flux can be understood graphically by the total number of electric field lines passing through an area.
- For an area element $d\vec{A}$, placed in an electric field $\vec{E}$ as shown, electric flux is defined as $d\phi = \vec{E}.d\vec{A}$
- Flux is a scalar quantity.
- Unit of flux : $Nm^2/C$ or Vm.
- Dimensional formula of $\phi$ is $[M^1L^3T^{-3}A^{-1}]$.
- Electric flux through a large surface.




$$\phi = \int d\phi = \int \vec{E}.d\vec{A}$$

❖ If field is uniform $\Rightarrow \vec{E}$ = constant

$\phi = \vec{E}.\int d\vec{A}$ $\qquad$ $\int d\vec{A}$ = total area vector of a plane surface

$$\Rightarrow \boxed{\phi = \vec{E}\,.\,\vec{A}}$$

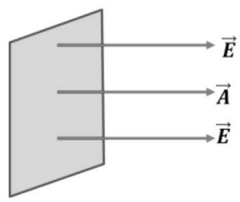
$$\Rightarrow \boxed{\phi = EA\cos\theta}$$

**Different cases:**

(i) When surface is perpendicular to the field and $\vec{A}$ is parallel to $\vec{E}$

Here θ is 0°.

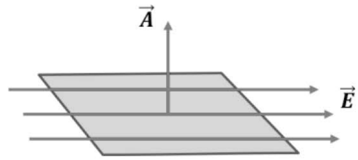
$\phi = EA\cos 0° = EA$ (positive flux means outgoing or leaving)




(ii) When surface is along the field and $\vec{A}$ is perpendicular to $\vec{E}$

Here θ is 90°

$\phi = EA\cos 90^0 = 0$ (No flux leakage)




(iii) When surface is perpendicular to the field and $\vec{A}$ is anti parallel to $\vec{E}$

Here θ is $180^0$

$\phi = EA\cos 180^0 = -EA$

Negative flux means incoming or entering

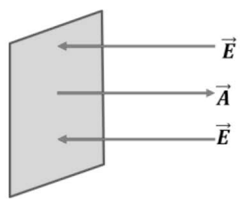



**Illustration 49:**

If $\vec{E} = \left(2\hat{i} + 3\hat{j} - 5k\right)$ N/C and $\vec{A} = \left(2\hat{i} - \hat{j} - k\right)$ $cm^2$ then find electric flux.

**Solution:**

$\phi = \vec{E}\,.\,\vec{A}$

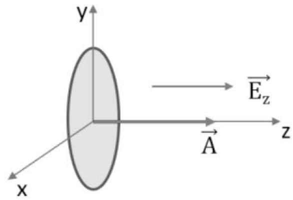
$\phi = \left(2\hat{i} + 3\hat{j} - 5\hat{k}\right).\left(2\hat{i} - \hat{j} - k\right)$

$\phi$ = 4 – 3 + 5 = 6

$\phi$ = 6N$cm^2$/C

**Illustration 50:**

A thin disc of area 2m² is placed in x-y plane in a uniform field of strength $\vec{E}=\left(2\hat{i}-\hat{j}+4k\right)N/C$, find flux of the field passing through the disc.




**Solution:**

$\vec{E}=\left(2\hat{i}-3\hat{j}+4k\right)N/C$ and $\vec{A}=\left(2k\right)m^2$

$\phi=\vec{E}.\vec{A}=(2\hat{i}-3\hat{j}+4\hat{k}).2\hat{k}=8\frac{Nm^2}{C}$

**Illustration 51:**

An uniform electric field $\vec{E}=\left(10\hat{i}+5\hat{j}\right)$ V/m is present in space. Find out flux passing through the area of 10m², if it lies in,

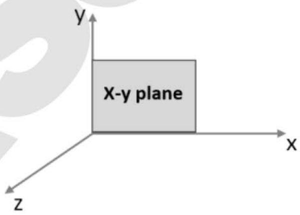
(1) x-y plane (2) x-z plane (3) y-z plane

**Solution:**

(1) Only $E_z$ field will be passed through the x-y plane

$\phi_{(x\text{-}y\ plane)}$ =0 (because $E_z$ is zero)

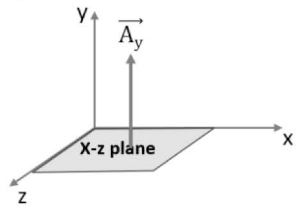
$\phi_{(x\text{-}y\ plane)}=\vec{E}.\vec{A_z}=\left(10\hat{i}+5\hat{j}\right).\left(10k\right)=0$




(2) Flux passing through x-z plane

Only $E_y$ field will be passed through the x-z plane

$\phi_{(x-z\,plane)}=\vec{E}.\vec{A_y}=\left(10\hat{i}+5\hat{j}\right).\left(10\hat{j}\right)$, $\phi_{(x-z\,plane)}$ =50 V-m




(3) Flux passing through y-z plane

Only $E_x$ field will be passed through the y-z plane

$\phi_{(y\text{-}z\ plane)}=\vec{E}.\vec{A_x}=\left(10\hat{i}+5\hat{j}\right).\left(10\hat{i}\right)$

$\phi_{(y\text{-}z\ plane)}$=100V-m

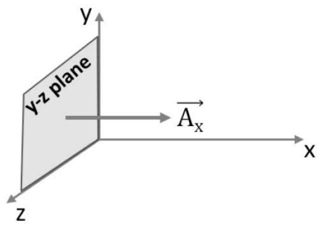
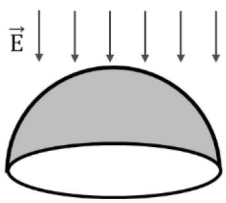



**Illustration 52:**

A hemispherical shell of radius R, is placed in a uniform field $\vec{E}$ as shown. Find flux through the hemisphere.




**Solution:**

Here projected area by which flux is passing is $\pi R^2$

Flux passing out will be $\phi=E\pi R^2$

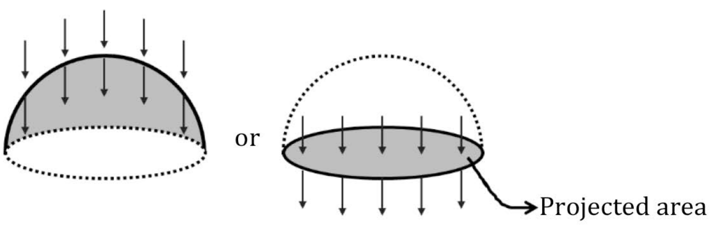
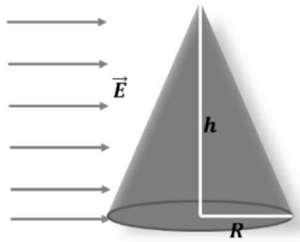


**Illustration 53:**

A cone (radius R & height h) is placed in a uniform field E as shown. Find flux through the cone.




**Solution:**

$\phi_{entering} = \phi_{leaving} = E \times$ projected area

$\phi_{entering} = \phi_{leaving} = ERh$

So, net flux through the cone is zero

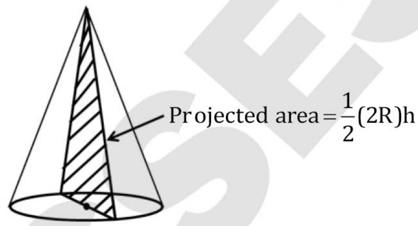
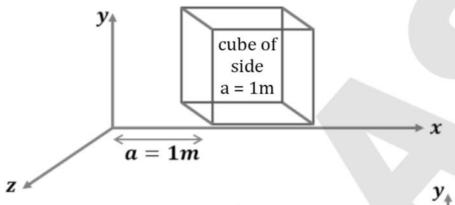



**Illustration 54:**

If $\vec{E} = x^2\hat{i}$, find net flux passing through cube

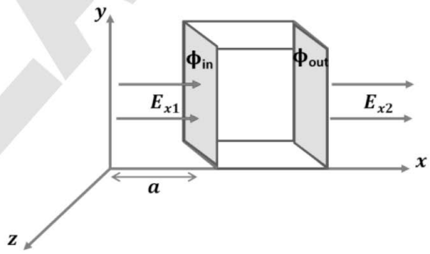



**Solution:**

$$\phi_{in} = -E_{x_1}\left(a^2\right) = -(1^2)(1^2) = -1 \text{ unit}$$

$$\phi_{out} = E_{x_2}\left(a^2\right) = 2^2 \times 1^2 = 4 \text{ unit}$$

$$\phi_{net} = 4 - 1 = 3 \text{ unit}$$



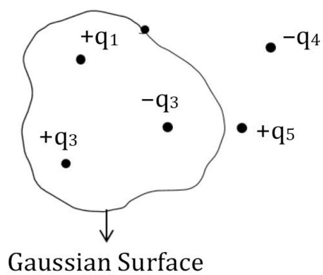

**Note:** Electric flux through a large surface (open): $\phi = \int_S \vec{E}.d\vec{A}$

Electric flux through a closed surface (having non-zero volume) $\phi = \oint_S \vec{E}.d\vec{A}$

## Gauss's Theorem & Applications of Gauss's theorem

**Gauss's Theorem:** Net electric flux passing through any closed imaginary surface (Gaussian surface) is $\frac{1}{\varepsilon_0}$ times of the total charge enclosed by it.

$$\boxed{\phi_{net} = \frac{Q_{enclosed}}{\varepsilon_0}}$$




To understand this concept, consider a closed surface as shown -

$$\phi_{net} = \frac{(q_1 - q_2 + q_3)}{\epsilon_0}$$

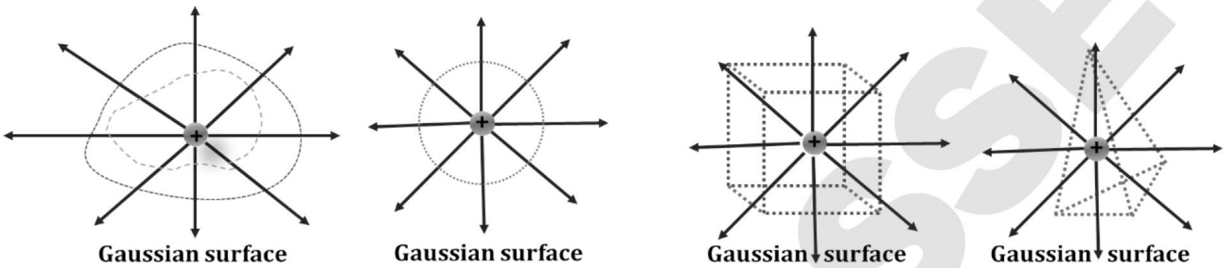
Charges present outside the closed surface $-q_4$ and $+q_5$ do not play any role in net flux passing through the surface (also called gaussian surface).

$$\boxed{\phi_{net} = \oint \vec{E}.\vec{ds} = \frac{Q_{enclosed}}{\epsilon_0}}$$

**Note:**

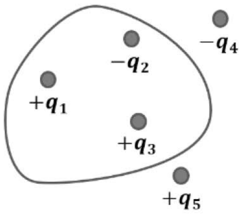
(i) Flux through Gaussian surface is independent of location of charges within Gaussian surface.


(ii) Flux through Gaussian surface is independent of shape and size of Gaussian surface.



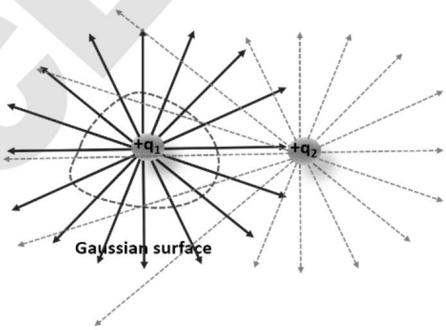

(iii) In Gauss's theorem

$E \rightarrow$ due to all the charges , but $\phi_{closed} \rightarrow$ due to enclosed charges only.



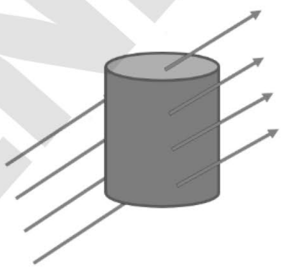

Net flux passing out is only due to $(+q_1\text{-}q_2\text{+}q_3)$ charges but field at surface is due to $(+q_1\text{-}q_2\text{+}q_3\text{-}q_4\text{+}q_5)$.

Given diagram shows that why external charges don't contribute in flux through the gaussian surface.



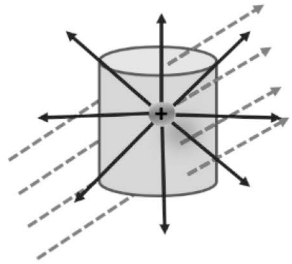

(iv) Gauss's theorem is applicable for all the forces following inverse square law.

(v) When an imaginary closed surface placed in an external electric field, net flux passing through it, will be zero and If net flux is non zero, definitely charge is enclosed by the closed surface.



$\phi$(entering) = $\phi$ (leaving)

$\Rightarrow$ Net flux is zero

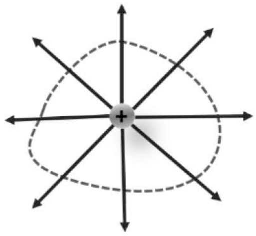


$\phi$(entering) $\neq$ $\phi$ (leaving) $\Rightarrow \phi_{net} \neq 0$

And $\phi_{net} = \dfrac{Q}{\varepsilon_0}$

(vi) For a Gaussian surface $\phi = 0$ does not imply $E = 0$, but $E = 0$ implies $\phi = 0$.

(vii) When net flux is going out then positive charge is enclosed by Gaussian surface




Gaussian surface

When net flux is coming in then negative charge is enclosed by Gaussian surface


Gaussian surface

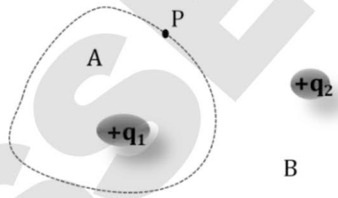
**Illustration 55:**

What effect will be observed on $\phi_{closed}$ and electric field at point P, if

(i) charge $q_1$ is shifted to A

(ii) charge $q_1$ is shifted to B and

(iii) charge $q_2$ is shifted to A

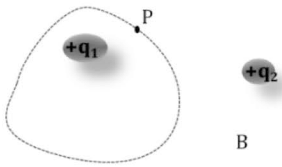



**Solution:**

(i) when $q_1$ is shifted to A

$\phi_{closed} \Rightarrow$ same

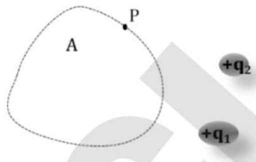
$\vec{E}_P \Rightarrow$ change




(ii) when $q_1$ is shifted to B

$\phi_{closed} \Rightarrow$ change

$\vec{E}_P \Rightarrow$ change




(iii) charge $q_2$ is shifted to A

$\phi_{closed} \Rightarrow$ change

$\vec{E}_P \Rightarrow$ change

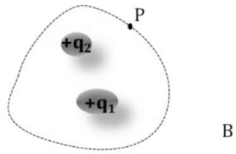
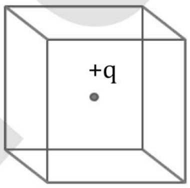



**Illustration 56:**

Find total flux through cube and flux through each face of cube and also flux through each corner.




**Solution:**

Total flux passing through cube will be

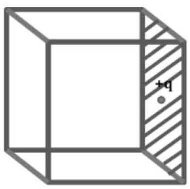
$\phi_{net} = \frac{Q_{enclosed}}{\varepsilon_0}$ (From Gauss's theorem) $\Rightarrow$ $\phi_T = \frac{q}{\varepsilon_0}$

Due to symmetry, flux passing through each face will be same $\Rightarrow \phi_{face} = \frac{q}{6\varepsilon_0}$

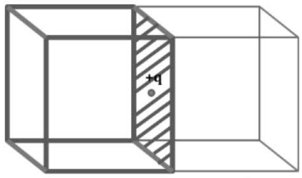
Again due to symmetry, flux through each corner, $\Rightarrow \phi_{corner} = \frac{q}{8\varepsilon_0}$

**Illustration 57:**

Find total flux through give cube.



**Solution:**



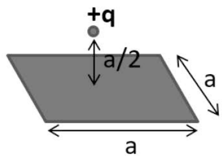

Total flux passing through both the cubes will be $\phi_T = \dfrac{q}{\varepsilon_0}$

Flux passing through each cube will be $\phi = \dfrac{q}{2\varepsilon_0}$

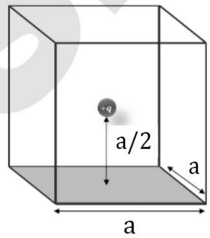
**Note:** Flux through the surface containing charge is zero.

**Illustration 58:**

Find flux passing through sheet.




**Solution:**

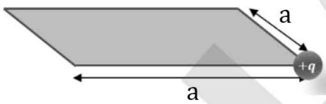



If we assume 6 similar sheets and arranged them in form of a cube in such a way that charge +q comes at centre

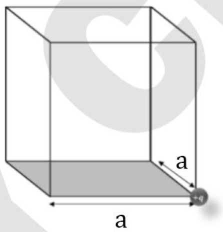
Now from gauss's theorem flux $\phi = \dfrac{q}{6\varepsilon_0}$

**Illustration 59:**

Find flux passing through sheet.

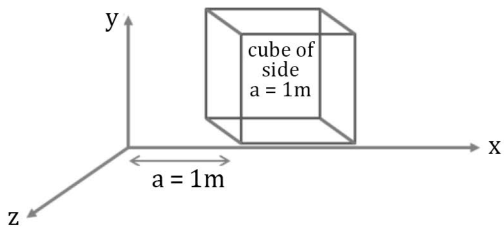



**Solution:**




Here, electric field and area vector of the sheet will be perpendicular so flux through it will be zero.

**Illustration 60:**




If $E_x$ is $5\sqrt{x}$, and $E_y$ & $E_z$ is zero then find,

(1) Net flux passing through cube

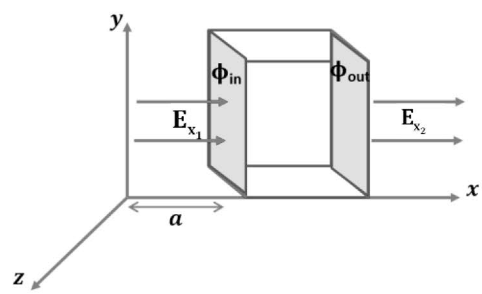
(2) Net charge enclosed by the cube

**Solution:**

$\phi_{in} = -E_{x_1}\left(a^2\right) = -5\sqrt{1}\left(1\right)^2 = -5$ unit

$\phi_{out} = E_{x_2}\left(a^2\right) = +5\sqrt{2}\left(1\right)^2 = 5\sqrt{2}$ unit

$\phi_{net} = 5\sqrt{2} - 5 = 5\left(\sqrt{2}-1\right)$ unit



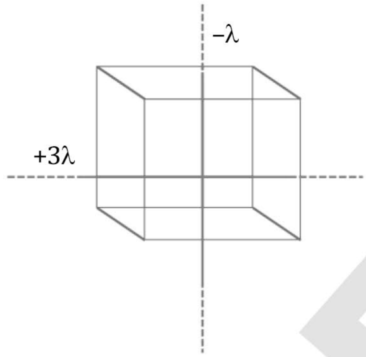

According to Gauss's theorem $\phi_{net} = \dfrac{q}{\varepsilon_0}$

Or $\quad 5\left(\sqrt{2}-1\right) = \dfrac{Q_{in}}{\varepsilon_0} \quad \Rightarrow \quad Q_{in} = 5\left(\sqrt{2}-1\right)\varepsilon_0$

Charge enclosed by the cube is $Q_{in} = 5\left(\sqrt{2}-1\right)\varepsilon_0$

**Illustration 61:**

Find flux passing out from cube of side L where linear charge densities of line charges kept in cube are $\lambda$ and $-\lambda$ as shown in the figure ?




**Solution:**

Charge enclosed by cube

$$Q_{in} = +3\lambda L - \lambda L$$

$$Q_{in} = +2\lambda L$$

According to Gauss's theorem

$$Q_{in} = \frac{Q_{in}}{\varepsilon_0} = \frac{2\lambda L}{\varepsilon_0}$$

**Applications of Gauss's theorem**

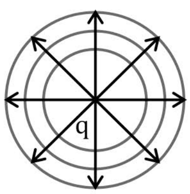
Gauss's Law is the most useful tool to find E, in situations where the charge distributions has some what symmetry. Alternatively, if we are given the field, we can use Gauss's Law to determine about the charge distributions.

To determine E for symmetric charge distributions, we select a symmetric Gaussian surface as follows :

| Charge Distributions | Direction of EF | Gaussian surface |
|---|---|---|
| Point charge | Radial (along the radius of sphere) | Concentric surface |
| Spherical charge | Radial (along the radius of sphere) | Concentric surface |
| Line charge | Radial (along the radius of sphere) | Coaxial cylinder |
| Planar charge or Charged sheets | Normal to surface | Parallel planes like cylindrical |

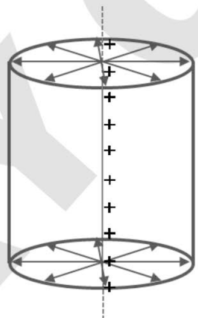
**Different shapes of Gaussian surfaces -**

Point charge and charged sphere



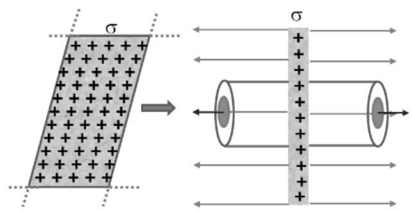

Concentric spherical Gaussian surface

Infinite linear charge



Coaxial Cylindrical Gaussian surface

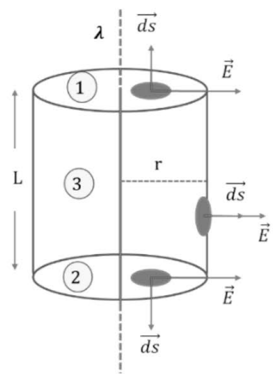
Infinite sheet




Gaussian surface

**(1) Electric field due to a uniformly charged long wire :**

Considering a coaxial gaussian cylinder of radius r and length L as shown.




Then applying Gauss's theorem

$$\oiint \vec{E}.d\vec{s} = \frac{q_{in}}{\varepsilon_0} \qquad ...(i)$$

Here, Gaussian Cylinder has three surfaces as :

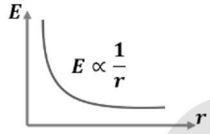
Surfaces (i) and (ii) are flat, and for these surfaces area vector is perpendicular to E

Surfaces (iii) is curved and for this surfaces area vector is along E

$$\oint \vec{E}.d\vec{s} = \int_{S_1} Eds\cos 90^\circ + \int_{S_2} Eds\cos\theta^\circ + \int_{S_2} Eds\cos 90^\circ = 0 + \int_{S_2} Eds + 0 = \int_{S2} Eds = E\int_{S2} ds = E(2\pi rL)$$

From equation No. (i)

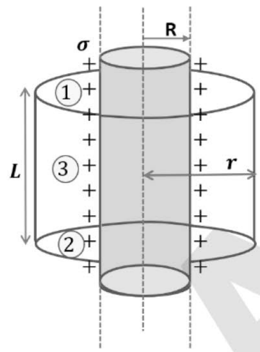
$$E(2\pi rL) = \frac{\lambda L}{\varepsilon_0} = \frac{\lambda}{2\pi\varepsilon_0 r} \text{ or } E = \frac{2k\lambda}{r}$$




Variation of E with r

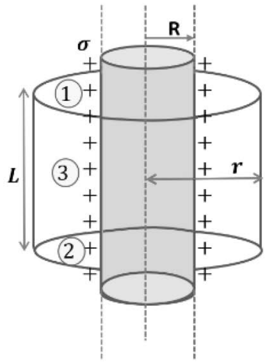
**(2) Electric field due to uniformly charged long cylindrical pipe/cylindrical shell (for students self practice) :**

(i) Electric field at any point outside the cylinder (r>R)




Considering a cylindrical Gaussian surface around charged cylinder.

from to Gauss's theorem




$$\oint_S \vec{E}.d\vec{s} = \frac{q_{in}}{\varepsilon_0}$$

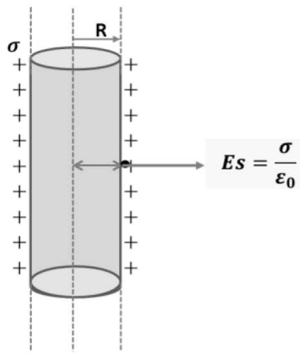
$$E(2\pi rL) = \frac{q_{in}}{\varepsilon_0}$$

$$E(2\pi rL) = \frac{\sigma 2\pi RL}{\varepsilon_0}$$

$$E = \frac{\sigma R}{r\varepsilon_0} \text{ or } E = \frac{\sigma R}{\varepsilon_0 r} \Rightarrow E \propto \frac{1}{r}$$

(ii) For the point lying on the surface $(r \approx R)$

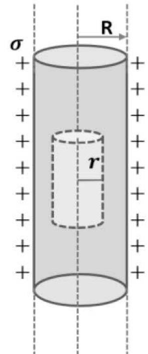
Putting r = R in $E = \dfrac{\sigma R}{\varepsilon_0 r}$ $\quad \boxed{\left(E_s = \dfrac{\sigma}{\varepsilon_0}\right)}$




(iii) For the point inside the surface (r < R)

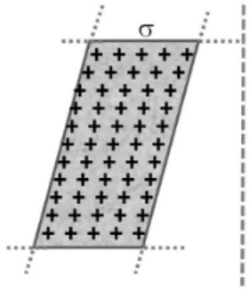
Using Gauss's theorem $\quad \phi = E(2\pi rL) = \dfrac{0}{\varepsilon_0}$

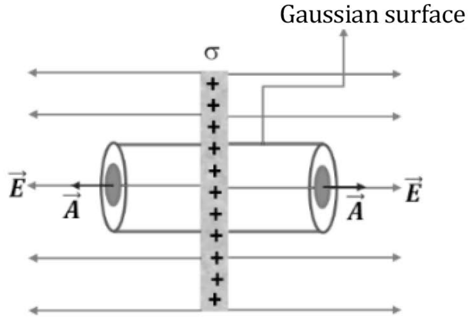
$E_{in} = 0$



**Electric field intensity due to uniformly charged infinite sheet**

(i) Non conducting sheet -

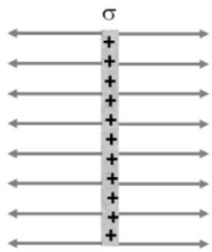


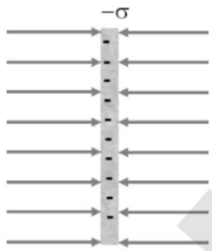

According to Gauss's theorem

$$\phi_{closed} = EA + EA + 0 = \frac{Q_{in}}{\varepsilon_0} \quad \text{or} \quad 2EA = \frac{\sigma A}{\varepsilon_0} \quad \text{or} \quad E = \frac{\sigma}{2\varepsilon_0}$$

Electric field due to non conducting sheet is uniform




$\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{n}$



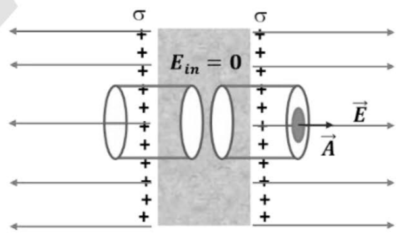

$\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{n}$

Variation of E with r

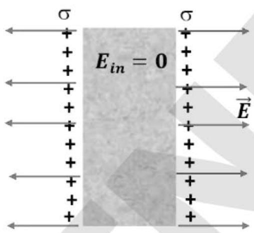

(ii) Conducting sheet or metal plate -

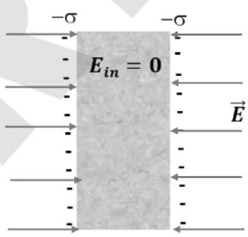
According to Gauss's theorem

$$\phi_{closed} = EA + 0 + 0 = \frac{q_{in}}{\varepsilon_0} \Rightarrow EA = \frac{\sigma A}{\varepsilon_0} \Rightarrow E = \frac{\sigma}{\varepsilon_0}$$




Electric field due to conducting sheet is uniform




Variation of E with r

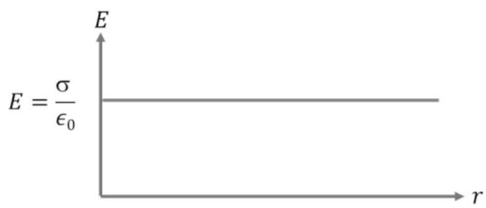
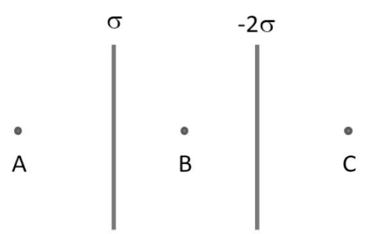


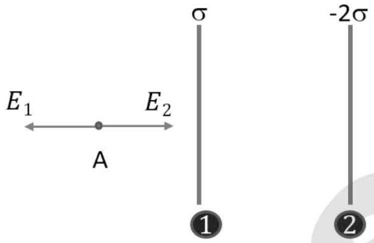

**Illustration 62:**

Find $E_A$, $E_B$ and $E_C$



**Solution:**

**At point A -**

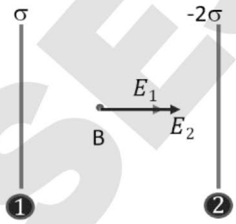



Electric field due to sheet 1 will be leftwards $E_1 = \frac{\sigma}{2\varepsilon_0}$

Electric field due to sheet 2 will be rightwards $E_2 = \frac{2\sigma}{2\in_0} = \frac{\sigma}{\in_0}$

Net electric field will be rightwards $E_A = E_2 - E_1 = \frac{\sigma}{\in_0} - \frac{\sigma}{2\in_0} = \frac{\sigma}{2\in_0}$

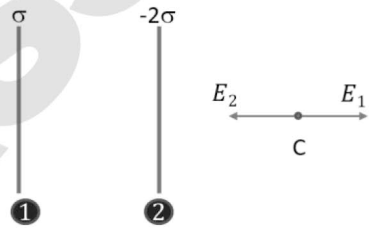
**At point B –**




Electric field due to both sheets will be rightwards

Net electric field will be rightwards $E_B = E_1 + E_2 = \frac{\sigma}{2\in_0} + \frac{2\sigma}{2\in_0} = \frac{3\sigma}{2\in_0}$

**At point C –**



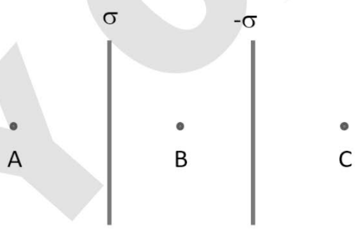

Electric field due to sheet 1 will be rightwards = $E_1 = \frac{\sigma}{2\in_0}$

Electric field due to sheet 2 will be leftwards $E_2 = \frac{2\sigma}{2\in_0} = \frac{\sigma}{\in_0}$

Net electric field will be leftwards $E_A = E_2 - E_1 = \frac{2\sigma}{2\in_0} - \frac{\sigma}{2\in_0} = \frac{\sigma}{2\in_0}$

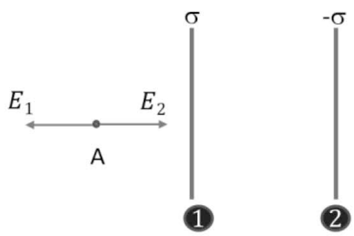
**Illustration 63:**

Find $E_A$, $E_B$ and $E_C$




**Solution:**

**At point A -**

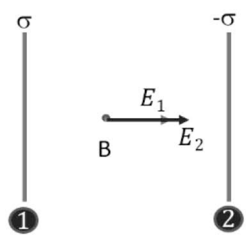



Electric field due to sheet 1 will be leftwards $E_1 = \frac{\sigma}{2\in_0}$

Electric field due to sheet 2 will be rightwards $E_2 = \frac{\sigma}{2\in_0}$

Net electric field will be zero $E_A = E_2 - E_1 = \frac{\sigma}{2\in_0} - \frac{\sigma}{2\in_0} = 0$

**At point B -**




Electric field due to both sheets will be rightwards

Net electric field will be rightwards

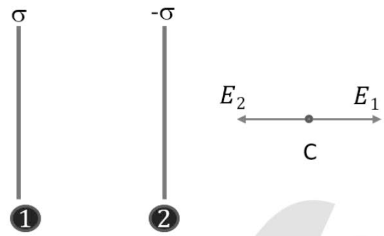
$E_B = E_1 + E_2 = \frac{\sigma}{2\in_0} + \frac{\sigma}{2\in_0} = \frac{\sigma}{\in_0}$

**At point C -**

Electric field due to sheet 1 will be rightwards $E_1 = \dfrac{\sigma}{2\in_0}$

Electric field due to sheet 2 will be leftwards $E_2 = \dfrac{\sigma}{2\in_0}$

Net electric field will be zero $E_A = E_2 - E_1 = \dfrac{\sigma}{2\in_0} - \dfrac{\sigma}{2\in_0} = 0$



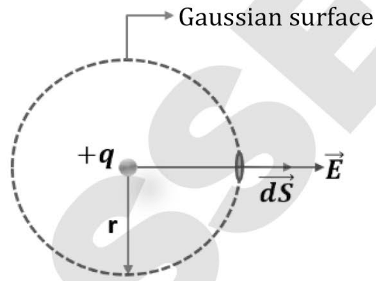

**Electric field due to the charged conducting sphere and non conducting sphere**

**(1) Electric field due to point charge.**

According to Gauss's theorem

$$\phi = \oint \vec{E}.\overrightarrow{ds} = \frac{q_{in}}{\varepsilon_0}$$

Angle between E and ds is $0^0$




$$\oint Eds = \frac{q}{\varepsilon_0}$$

$$E\oint ds = \frac{q}{\varepsilon_0} \text{ or } E\left(4\pi r^2\right) = \frac{q}{\varepsilon_0}$$

$$E = \frac{q}{4\pi\varepsilon_0\left(r^2\right)} \text{ or } E = \frac{kq}{r^2}$$

**Electric field due to a point charge**

Considering a concentric spherical gaussian surface of radius r, as shown,

We now apply gauss's theorem as :

$$\oint \vec{E}.d\vec{s} = \frac{q_{in}}{\varepsilon_0} \quad ...(i)$$

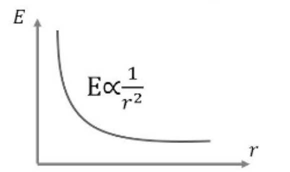
Angle between $\vec{E}$ and $d\vec{s}$ at every point of the Gaussian surface is 0°

$$\Rightarrow \oint \vec{E}.d\vec{s} = \oint Eds\cos\theta = \oint Eds$$

Since all points on spherical Gaussian surface are equidistant from the point charge :

$$\therefore \oint Eds = E\oint ds = E(4\pi r^2)$$

From $E\left(4\pi r^2\right) = \dfrac{q}{\varepsilon_0} \Rightarrow E = \dfrac{q}{4\pi\varepsilon_0 r^2}$ or $\dfrac{kq}{r^2}$



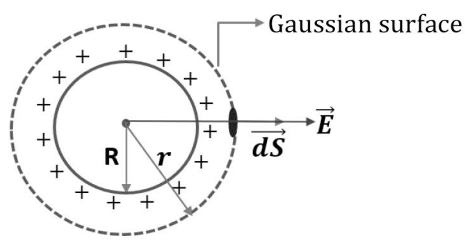

**variation of E with r**

**(2) Electric field due to uniformly charged thin shell or conducting sphere.**

(i) EF at any point outside the sphere (r>R)

Considering a Gaussian surface and applying Gauss's theorem

$$\oint \vec{E}.\overrightarrow{ds} = \frac{q_{in}}{\varepsilon_0}$$




For spherical surface, we can directly write

$$\oint \vec{E}.\overrightarrow{ds} = E(4\pi r^2)$$

$$\therefore E(4\pi r^2) = \frac{q}{\varepsilon_0} \quad \text{or} \quad E = \frac{q}{4\pi\varepsilon_0\left(r^2\right)} \text{ or } E = \frac{kq}{r^2}$$

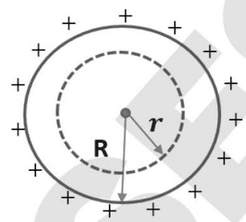
Thus for point lying outside the sphere, sphere behaves as point charge centered at centre.

(ii) EF at any point lying on the surface of sphere (r = R)

For the point on surface, we putting r = R in $E = \frac{kq}{r^2}$, we have

$$\boxed{E_S = \frac{kq}{R^2}}$$ (max. value of electric field)

$$E_S = \frac{q}{4\pi \in_0 R^2} = \frac{\sigma}{\in_0} \quad \left(\because \sigma = \frac{q}{4\pi R^2}\right)$$

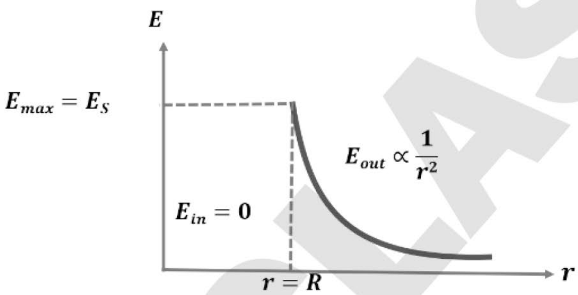



(iii) EF at any point inside the sphere (r < R)

In this case, charge enclosed by the GS is zero, so using Gauss's theorem

$$\oint \overrightarrow{E_{in}}.\overrightarrow{ds} = \frac{0}{\varepsilon_0} \Rightarrow E_{in} = 0$$

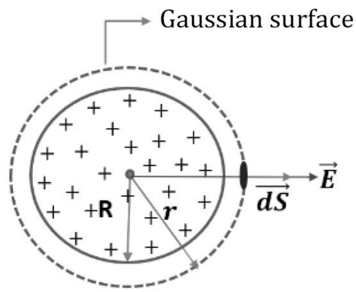
(iv) Variation of E with r




**(3) Electric field due to uniformly charged nonconducting sphere.**

(i) EF at any point outside the sphere (r > R)

According to Gauss's theorem




$$\oint \vec{E}\overrightarrow{ds} = \frac{q_{in}}{\varepsilon_0}$$

$$\oint E ds = \frac{q}{\varepsilon_0} \Rightarrow E\oint ds = \frac{q}{\varepsilon_0}$$

$$\Rightarrow E(4\pi r^2) = \frac{q}{\varepsilon_0} \Rightarrow E = \frac{kq}{r^2}$$

Thus for point lying outside the sphere, sphere behaves as point charge centered at centre.

(ii) EF at any point lying on the surface of sphere (r=R)

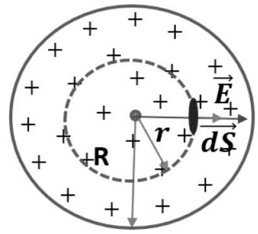
$$E_S = \frac{kq}{R^2}$$ (max. value of electric field)

$$E_S = \frac{q}{4\pi\varepsilon_0 R^2} = \frac{qR}{3\left(\frac{4}{3}\pi R^3\right)\varepsilon_0} = \frac{\rho R}{3 \in_0}$$

[$\therefore$ ρ is volume charge density]

(iii) EF at any point inside the sphere (r<R)

If $q_{in}$ is the charge enclosed by G.S., then




$$\Rightarrow q_{in} = \rho\left(\frac{4}{3}\pi r^3\right) = \frac{q}{\frac{4}{3}\pi R^3}\left(\frac{4}{3}\pi r^3\right)$$

According to Gauss's theorem

$$\oiint \vec{E}.\vec{ds} = \frac{q_{in}}{\varepsilon_0}$$

$$\oiint Eds = \frac{q_{in}}{\varepsilon_0} \Rightarrow E\oiint ds = \frac{q_{in}}{\varepsilon_0} \Rightarrow E\left(4\pi r^2\right) = \frac{q}{R^3}\left(r^3\right)\frac{1}{\varepsilon_0} \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^3}(r) = \frac{kq}{R^3}(r)$$

Inside the nonconducting sphere electric field is directly proportional to r.

$$E_{in} \propto r \qquad \Rightarrow E_{in} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^3}(r) = \frac{q}{3\left(\frac{4}{3}\pi R^3\right)}(r) = \frac{\rho}{3\varepsilon_0}(r)$$

For r = 0, E = 0 $\Rightarrow$ electric field intensity at the centre of the sphere is zero.

(iv) Variation of E with r

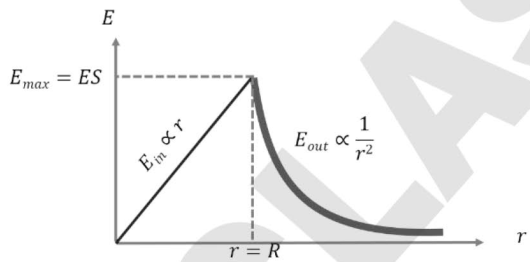



**Illustration 64:**

Electric field at a distance 20 cm from a charged metallic sphere of radius 10 cm is E. Find E at a distance :

(1) r = 5cm (2) r = 10cm (3) r =30cm, from centre.

**Solution:**

(1) 5cm (r <R), so E = 0 (for conductor E= 0 at inside point)

(2) 10cm (r = R), $E_s = \frac{kQ}{R^2}$ As $E = \frac{kQ}{(20)^2} = \frac{kQ}{4R^2}$ so $E_s = \frac{kQ}{R^2} = 4E$

(3) 30cm (r > R), $E_{out} = \frac{kQ}{9R^2}$ so $E_{out} = \frac{4E}{9}$

**Illustration 65:**

Electric field at a distance 30 cm from a uniformly charged non conducting sphere of radius 15 cm is E. Find E at a distance :

(1) r = 5 cm (2) r = 15 cm (3) r =20 cm, from centre.

**Solution:**

(1) 5 cm (r <R), $E_{out} = \frac{kQ}{r^2}$

$E_{out} = \frac{kQ}{r^2} = E$ (given) or $kQ = (30)^2E$

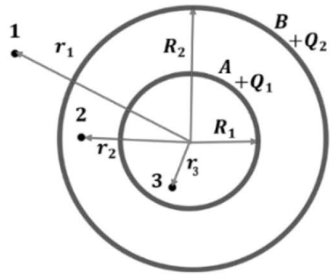
$$E_{in} = \frac{kQ}{R^3}(r) = \frac{(30)^2 E}{(15)^3}(5) = 4/3E$$

(2) 15cm (r = R), $E_S = \frac{kQ}{R^2} \Rightarrow E_s = \frac{(30)^2 E}{(15)^2} = 4E$

(3) 20cm (r > R), $E_{out} = \frac{kQ}{r^2} = \frac{(30)^2 E}{(20)^2} = \frac{9E}{4}$

**Illustration 66:**

Two concentric conducting spherical shells are shown in figure. Find electric field at points 1,2 & 3.




**Solution:**

For $GS_1$ charge enclosed is $+Q_1$ and $+Q_2$ i.e.

$$E_1 = \frac{k(Q_1 + Q_2)}{r_1^2}$$

For $GS_2$ charge enclosed is $+Q_1$, i.e.

$$E_2 = \frac{kQ_1}{r_1^2}$$

For $GS_3$ charge enclosed is zero, i.e.

$E_3 = 0$

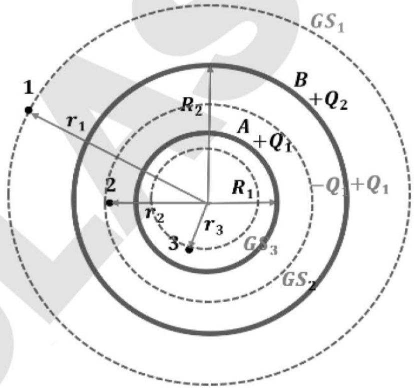
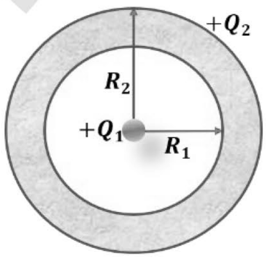



**Illustration 67:**

A point charge $+Q_1$ is placed at the centre of a thick conducting shell of inner radius $R_1$ and outer radius $R_2$ as shown then find. (1) $\sigma_{\text{inner surface}}$ (2) $\sigma_{\text{outer surface}}$

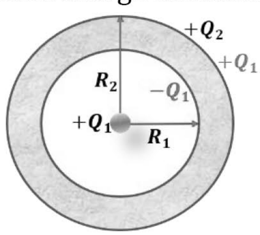



**Solution:**

due to charge at centre $-Q_1$ & $Q_2$ will be induced charge on inner & outer surface respectively of shell.

(1) $\sigma_{\text{inner surface}} = \frac{-Q_1}{4\pi R_1^2}$

(2) $\sigma_{\text{outer surface}} = \frac{(Q_1 + Q_2)}{4\pi R_2^2}$



## BEGINNER'S BOX-4

**1.** Two point charges Q and 4Q are 12 cm apart. Sketch the lines of force and calculate the distance of neutral point from 4Q charge.

**2.** A charge Q is uniformly distributed over a large plastic (non-conducting) sheet. The electric field at a point close to the centre of the plate near the surface is 20 V/m. If the plate is replaced by a copper plate of the same geometrical dimensions and carrying the same charge Q, then what is the electric field at that point?

**3.** What is the net flux of a uniform electric field $\vec{E} = 3\times 10\,\hat{i}$ $NC^{-1}$ through a cube of side 20 cm oriented such that its faces are parallel to the coordinate planes ?

**4.** Charges $Q_1$ and $Q_2$ lie inside and outside a closed surface S respectively. Let E be the field at any point on S and $\phi$ be the flux of E over S. Which statement is wrong?

(A) If $Q_1$ changes, both E and $\phi$ will change.

(B) If $Q_2$ changes, E will change but $\phi$ will not change.

(C) If $Q_1=0$ and $Q_2\neq 0$ then $E \neq 0$ but $\phi = 0$

(D) If $Q_1\neq 0$ and $Q_2=0$ then $E=0$ but $\phi \neq 0$

**5.** A charge 'q' is placed at the centre of a cube whose top face is open (it has only 5 faces). Calculate the total electric flux passing through the cube.

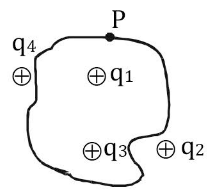
**6.** A point charge of 2.0 μC is at the centre of a cubic Gaussian surface of edge 9.0 cm. What is the net electric flux through the surface ?

**7.** An electric flux of $-6 \times 10^{-3}$ $Nm^2/C$ passes normally through a spherical Gaussian surface of radius 10 cm, due to a point charge placed at its centre.

(a) What is the charge enclosed by the Gaussian surface ?

(b) If the radius of the Gaussian surface is doubled, how much flux would pass through the surface ?

**8.** A Gaussian surface encloses two of the four positively charged particles as shown in figure. Which of the particles contribute to the electric field at a point P on the surface ?




**9.** Is it possible to have flux associated with an imaginary closed surface to be zero even when electric field on this surface is non–zero. If yes, then give one example.

**10.** Two large, thin metal plates are parallel and close to each other. The plates have surface charge densities of opposite signs and of magnitude $17.0 \times 10^{-12}$ $C/m^2$ on their inner faces. What is electric field,

(a) in the outer region of the first plate ?

(b) between the plates ?

**11.** Plot the following graphs –

(a) Electric field inside a conducting sphere with distance from its centre.

(b) E versus (1/r) where E is electric field due to a point charge and r is the distance from the charge.