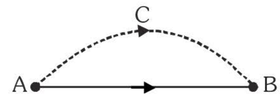
KINEMATICS

- **Distance and Displacement**

  Total length of path (ACB) covered by the particle, in definite time interval is called distance. Displacement vector or displacement is the minimum distance (AB) and directed from initial position to final position.

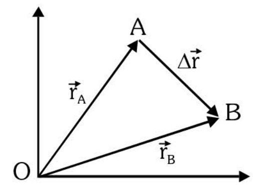


- **Displacement in terms of position vector**

  From ΔOAB $\quad \Delta\vec{r} = \vec{r}_B - \vec{r}_A$

  $\vec{r}_B = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$ and $\vec{r}_A = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

  $\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$



- **Average velocity** $= \dfrac{\text{Displacement}}{\text{Time interval}} = \vec{v}_{av} = \dfrac{\Delta\vec{r}}{\Delta t}$

- **Average speed** $= \dfrac{\text{Distance travelled}}{\text{Time interval}}$

- **For uniform motion** (Constant velocity)

  Average speed = | Average velocity | = | Instantaneous velocity | = Instantaneous speed

- **Velocity** $\vec{v} = \dfrac{d\vec{r}}{dt} = \dfrac{d}{dt}\left(x\hat{i} + y\hat{j} + z\hat{k}\right) = \dfrac{dx}{dt}\hat{i} + \dfrac{dy}{dt}\hat{j} + \dfrac{dz}{dt}\hat{k} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$

- **Average Acceleration** $= \dfrac{\text{change in velocity}}{\text{total time taken}} = \vec{a}_{av} = \dfrac{\Delta\vec{v}}{\Delta t}$

- **Acceleration**

  $$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(v_x\hat{i} + v_y\hat{j} + v_z\hat{k}\right) = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

Important points about 1D motion

- Distance $\geq$ | displacement | and Average speed $\geq$ | average velocity |
- If distance > | displacement | this implies

  The body must have retardation during the motion and atleast at one point in path, velocity is zero.
- Speed increase if acceleration and velocity both are positive or negative

  (i.e. both have same sign)

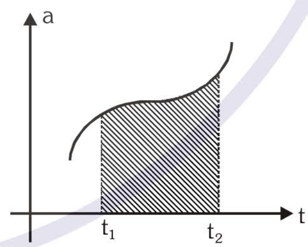
- **In 1-D motion** $a = \dfrac{dv}{dt} = v\dfrac{dv}{dx}$

- **Graphical analysis in Motion**

  - $a = \dfrac{dv}{dt} \Rightarrow \int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} a\,dt \Rightarrow v_2 - v_1 = \int_{t_1}^{t_2} a\,dt$

    $\Rightarrow$ Change in velocity

    = Area between acceleration curve and time axis, from t_1 to t_2.

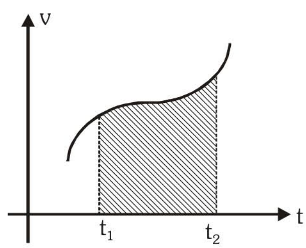


shaded area = change in velocity

  - $v = \dfrac{dx}{dt} \Rightarrow \int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} v\,dt \Rightarrow x_2 - x_1 = \int_{t_1}^{t_2} v\,dt$

    $\Rightarrow$ Change in position = displacement

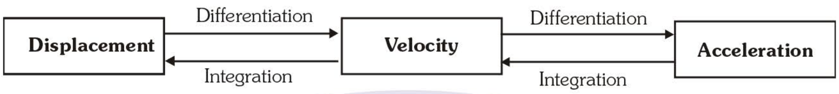
    = area between velocity curve and time axis, from t_1 to t_2.

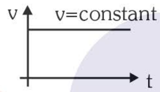
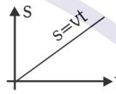
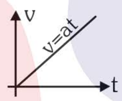
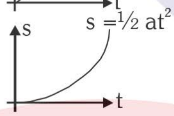
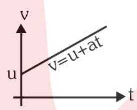
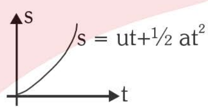
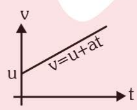
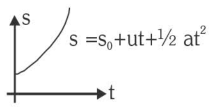
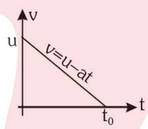
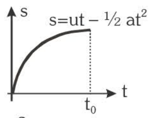
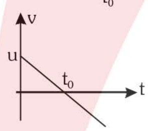
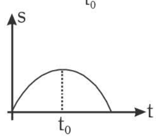


shaded area = displacement

Important point about graphical analysis of motion

- Instantaneous velocity is the slope of position time curve at a point. $\left(v = \frac{dx}{dt}\right)$
- Slope of velocity-time curve (at a point) = instantaneous acceleration $\left(a = \frac{dv}{dt}\right)$
- v-t curve area gives displacement. $\left[\Delta x = \int v\,dt\right]$
- a-t curve area gives change in velocity. $\left[\Delta v = \int a\,dt\right]$



Different Cases	v–t graph	s–t graph
1. Uniform motion		
2. Uniformly accelerated motion with u = 0 at t = 0		
3. Uniformly accelerated with u ≠ 0 at t = 0		
4. Uniformly accelerated motion with u ≠ 0 and $s = s_0$ at t = 0		
5. Uniformly retarded motion till velocity becomes zero		
6. Uniformly retarded then accelerated in opposite direction		

Motion with constant acceleration : Equations of motion

- *In vector form* :

$\vec{v} = \vec{u} + \vec{a}t$ $\quad \Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = \vec{s} = \left(\frac{\vec{u}+\vec{v}}{2}\right)t = \vec{u}t + \frac{1}{2}\vec{a}t^2 = \vec{v}t - \frac{1}{2}\vec{a}t^2$

$v^2 = u^2 + 2\vec{a}.\vec{s}$ $\quad \vec{s}_{n^{th}} = \vec{u} + \frac{\vec{a}}{2}(2n-1)$ $\quad$ [$S_{n^{th}}$ → displacement in n^{th} second]

- *In scalar form (for one dimensional motion) :*

$v = u + at$ $\quad s = \left(\frac{u+v}{2}\right)t = ut + \frac{1}{2}at^2 = vt - \frac{1}{2}at^2$

$v^2 = u^2 + 2as$ $\quad s_{n^{th}} = u + \frac{a}{2}(2n-1)$

- **RELATIVE MOTION**

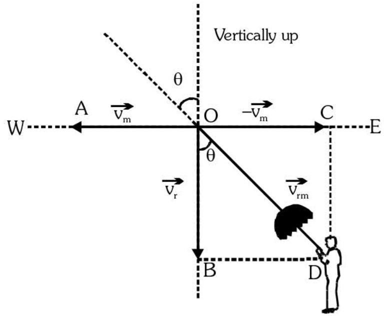
There is no meaning of motion without reference or observer. If reference is not mentioned then we take the ground as a reference of motion. Generally velocity or displacement of the particle w.r.t. ground is called actual velocity or actual displacement of the body. If we describe the motion of a particle w.r.t. and object which is also moving w.r.t. ground then velocity of particle w.r.t. ground is its actual velocity $(\vec{v}_{act})$ and velocity of particle w.r.t. moving object is its relative velocity $(\vec{v}_{rel.})$ and the velocity of moving object (w.r.t. ground) is the reference velocity $(\vec{v}_{ref.})$ then $\vec{v}_{rel} = \vec{v}_{act} - \vec{v}_{ref}$ $\quad \boxed{\vec{v}_{actual} = \vec{v}_{relative} + \vec{v}_{reference}}$

- **Relative velocity of Rain w.r.t. the Moving Man :**

A man walking west with velocity $\vec{v}_m$, represented by $\overrightarrow{OA}$.

Let the rain be falling vertically downwards with velocity $\vec{v}_r$,

represented by $\overrightarrow{OB}$ as shown in figure.



The relative velocity of rain w.r.t. man $\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$

will be represented by diagonal $\overrightarrow{OD}$ of rectangle OBDC.

$$\therefore v_{rm} = \sqrt{v_r^2 + v_m^2 + 2v_r v_m \cos 90^\circ} = \sqrt{v_r^2 + v_m^2}$$

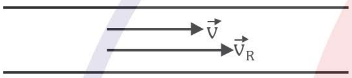
If θ is the angle which $\vec{v}_{rm}$ makes with the vertical direction then

$$\tan\theta = \frac{BD}{OB} = \frac{v_m}{v_r} \Rightarrow \theta = \tan^{-1}\left(\frac{v_m}{v_r}\right)$$

- **Swimming into the River**

A man can swim with velocity $\vec{v}$, i.e. it is the velocity of man w.r.t. still water. If water is also flowing with velocity $\vec{v}_R$ then velocity of man relative to ground $\vec{v}_m = \vec{v} + \vec{v}_R$

- If the swimming is in the direction of flow of water or along the downstream then

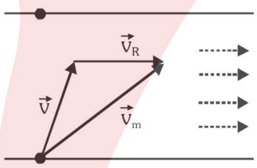


$$v_m = v + v_R$$

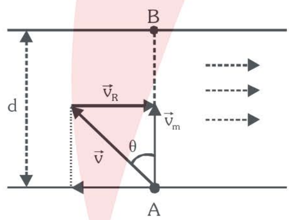
- If the swimming is in the direction opposite to the flow of water or along the upstream then

$$v_m = v - v_R$$

- If man is crossing the river as shown in the figure i.e. $\vec{v}$ and $\vec{v}_R$ not collinear then use the vector algebra $\vec{v}_m = \vec{v} + \vec{v}_R$ **(assuming $v > v_R$)**



- **For shortest path :**



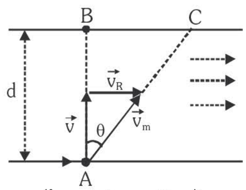
For minimum displacement

To reach at B, $v\sin\theta = v_R \Rightarrow$ $\boxed{\sin\theta = \frac{v_R}{v}}$

Time of crossing (t) = $\dfrac{d}{v\cos\theta}$

Note : If $v_R > v$ then for minimum drifting $\sin\theta = \dfrac{v}{v_R}$

- **For minimum time**

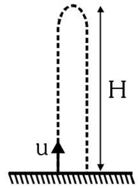


(for minimum time)

then $\boxed{t_{min} = \frac{d}{v}}$ $\boxed{\underset{(BC)}{\text{Drift}} = v_R t_{min}}$

MOTION UNDER GRAVITY

If a body is thrown vertically up with a velocity u in the uniform gravitational field (neglecting air resistance) then



(i) Maximum height attained $H = \frac{u^2}{2g}$

(ii) Time of ascent = time of descent = $\frac{u}{g}$

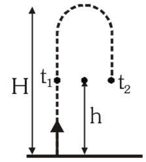
(iii) Total time of flight = $\frac{2u}{g}$

(iv) Velocity of fall at the point of projection = u (downwards)

(v) **Gallileo's law of odd numbers** : For a freely falling body ratio of successive distance covered in equal time internval 't'

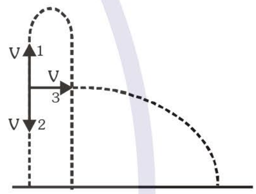
$$S_1 : S_2 : S_3 : ....S_n = 1: 3: 5 : ....: 2n-1$$

- At any point on its path the body will have same speed for upward journey and downward journey.
- If a body thrown upwards crosses a point in time t_1 & t_2 respectively then height of point $h = \frac{1}{2} g t_1 t_2$

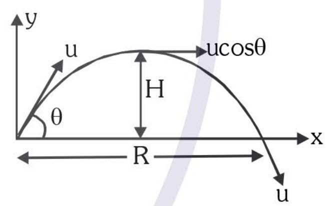


Maximum height $H = \frac{1}{8} g(t_1 + t_2)^2$

- A body is thrown upward, downward & horizontally with same speed takes time t_1, t_2 & t_3 respectively to reach the ground then $t_3 = \sqrt{t_1 t_2}$ & height from where the particle was throw is $H = \frac{1}{2} g t_1 t_2$



- **PROJECTILE MOTION**



- **Horizontal Motion**

  $u \cos\theta = u_x$

  $a_x = 0$

  $x = u_x t = (u \cos\theta)t$

- **Vertical Motion :**

  $v_y = u_y - gt$ where $u_y = u \sin\theta$; $y = u_y t - \frac{1}{2} g t^2 = u\sin\theta t - \frac{1}{2} g t^2$

  Net acceleration = $\vec{a} = a_x \hat{i} + a_y \hat{j} = -g\hat{j}$

- **At any instant** : $v_x = u\cos\theta$, $v_y = u\sin\theta - gt$

- **For projectile motion** :

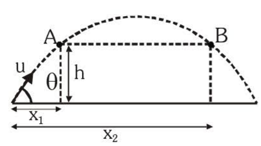
- A body crosses two points at same height in time t_1 and t_2 the points are at distance x and y from starting point then

  (a) $x_1 + x_2 = R$

  (b) $t_1 + t_2 = T$

  (c) $h = 1/2\ g t_1 t_2$

  (d) Average velocity from A to B is $u\cos\theta$



- If a person can throw a ball to a maximum distance 'x' then the maximum height to which he can throw the ball will be (x/2)

- **Velocity of particle at time t** :

$$\vec{v} = v_x\hat{i} + v_y\hat{j} = u_x\hat{i} + (u_y - gt)\hat{j} = u\cos\theta\hat{i} + (u\sin\theta - gt)\hat{j}$$

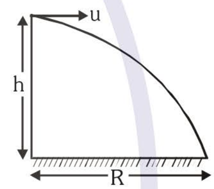
If angle of velocity $\vec{v}$ from horizontal is α, then $\tan\alpha = \frac{v_y}{v_x} = \frac{u_y - gt}{u_x} = \frac{u\sin\theta - gt}{u\cos\theta} = \tan\theta - \frac{gt}{u\cos\theta}$

- At highest point : $v_y = 0$, $v_x = u\cos\theta$
- Time of flight : $T = \frac{2u_y}{g} = \frac{2u\sin\theta}{g}$
- Horizontal range : $R = (u\cos\theta)T = \frac{2u^2\sin\theta\cos\theta}{g} = \frac{u^2\sin 2\theta}{g} = \frac{2u_xu_y}{g}$

  It is same for θ and $(90° - \theta)$ and maximum for $\theta = 45°$

- Maximum height $H = \frac{u_y^2}{2g} = \frac{u^2\sin^2\theta}{2g} = \frac{1}{8}gT^2$
- $\frac{H}{R} = \frac{1}{4}\tan\theta$
- Equation of trajectory $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} = x\tan\theta\left(1 - \frac{x}{R}\right)$

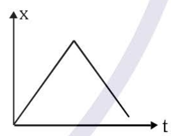
Horizontal projection from some height



- Time of flight $T = \sqrt{\frac{2h}{g}}$
- Horizontal range $R = uT = u\sqrt{\frac{2h}{g}}$
- Angle of velocity at any instant with horizontal $\theta = \tan^{-1}\left(\frac{gt}{u}\right)$

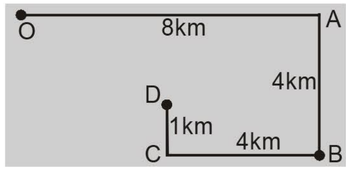
KEY POINTS :

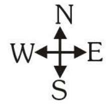
- A positive acceleration can be associated with a "slowing down" of the body because the origin and the positive direction of motion are a matter of choice.
- The x-t graph for a particle undergoing rectilinear motion, cannot be as shown in figure because infinitesimal changes in velocity are physically possible only in infinitesimal time.



- In oblique projection of a projectile the speed gradually decreases up to the highest point and then increases because the tangential acceleration opposes the motion till the particle reaches the highest point, and then it favours the motion of the particle.
- In free fall, the initial velocity of a body may not be zero.
- A body can have acceleration even if its velocity is zero at an instant.
- Average velocity of a body may be equal to its instantaneous velocity.
- The trajectory of an object moving under constant acceleration can be straight line or parabola.
- The path of one projectile as seen from another projectile is a straight line as relative acceleration of one projectile w.r.t. another projectile is zero.

Illustration A car moves from O to D along the path OABCD shown in fig. What is distance travelled and its net displacement ?





Solution Distance $= |\overrightarrow{OA}| + |\overrightarrow{AB}| + |\overrightarrow{BC}| + |\overrightarrow{CD}|$

$= 8 + 4 + 4 + 1 = 17$ km

Displacement $= \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}$

$= 8\hat{i} + (-4\hat{j}) + (-4\hat{i}) + \hat{j} = 4\hat{i} - 3\hat{j}$

$\Rightarrow$ | displacement | $= \sqrt{(4)^2 + (3)^2} = 5$

So, Displacement $= 5$ km, $37°$ S of E

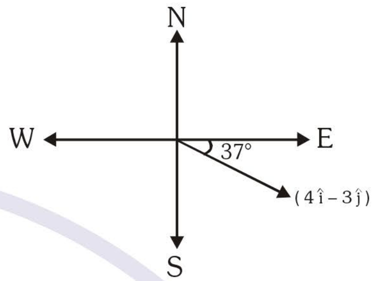
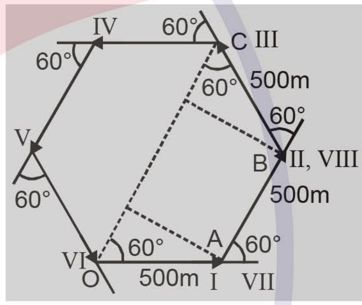


Illustration On an open ground a motorist follows a track that turns to his left by an angle of 60° after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of displacement with the total path length covered by the motorist in each case.



Solution At III turn

| Displacement | $= |\overrightarrow{OA}| + |\overrightarrow{AB}| + |\overrightarrow{BC}| + |\overrightarrow{OC}|$

$= 500 \cos 60° + 500 + 500 \cos 60°$

$= 500 \times \frac{1}{2} + 500 + 500 \times \frac{1}{2} = 1000$ m

So | Displacement | = 1000 m from O to C

Distance $= 500 + 500 + 500 = 1500$ m $\quad \therefore \frac{|\text{Displacement}|}{\text{Distance}} = \frac{1000}{1500} = \frac{2}{3}$

At VI turn

$\because$ initial and final positions are same so | displacement | = 0 and distance $= 500 \times 6 = 3000$ m

$\therefore \frac{|\text{Displacement}|}{\text{Distance}} = \frac{0}{3000} = 0$

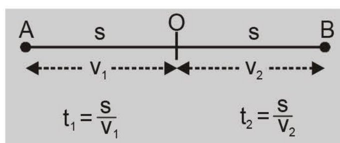
At VIII turn

| Displacement | $= 2(500)\cos\left(\frac{60°}{2}\right) = 1000 \times \cos 30° = 1000 \times \frac{\sqrt{3}}{2} = 500\sqrt{3}$ m

Distance $= 500 \times 8 = 4000$ m $\quad \therefore \frac{|\text{Displacement}|}{\text{Distance}} = \frac{500\sqrt{3}}{4000} = \frac{\sqrt{3}}{8}$

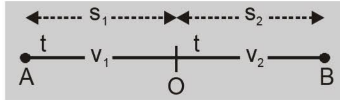
Illustration If a particle travels the first half distance with speed v_1 and second half distance with speed v_2. Find its average speed during the journey.

Solution $v_{av} = \frac{s + s}{t_1 + t_2} = \frac{2s}{\frac{s}{v_1} + \frac{s}{v_2}} = \frac{2v_1v_2}{v_1 + v_2}$



Note :- Here v_{av} is the harmonic mean of two speeds.

Illustration If a particle travels with speed v_1 during first half time interval and with speed v_2 during second half time interval. Find its average speed during its journey.



Solution $s_1 = v_1t$ and $s_2 = v_2t$

Total distance = $s_1 + s_2 = (v_1 + v_2)t$

total time = t + t = 2t then $v_{av} = \frac{s_1 + s_2}{t + t} = \frac{(v_1 + v_2)t}{2t} = \frac{v_1 + v_2}{2}$

Note :- here v_{av} is arithmetic mean of two speeds.

Illustration The displacement of a point moving along a straight line is given by

$s = 4t^2 + 5t - 6$

Here s is in cm and t is in seconds calculate

(i) Initial speed of particle

(ii) Speed at t = 4s

Solution (i) Speed, $v = \frac{ds}{dt} = 8t + 5$ Initial speed (i.e at t = 0), v = 5 cm/s

(ii) At t = 4s, v = 8(4) + 5 = 37 cm/s

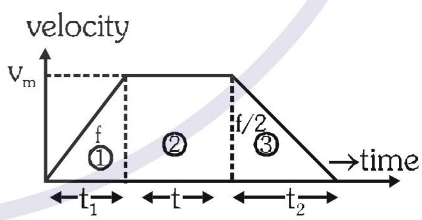
Illustration A particle moves in a straight line with a uniform acceleration a. Initial velocity of the particle is zero. Find the average velocity of the particle in first 's' distance.

Solution $\because \quad s = \frac{1}{2}at^2 \quad \therefore \quad \frac{s^2}{t^2} = \frac{1}{2}as$ Average velocity $= \frac{s}{t} = \sqrt{\frac{as}{2}}$

Illustration A car starting from rest, accelerates at the rate f through a distance S, then continues at constant speed for time t and then comes to rest with retardation $\frac{f}{2}$. If the total distance travelled is 15S then calculate the value of S in term of f and t.

Solution Let constant speed be v_m

for time t_1; $v_m = ft_1$ and $S = \frac{1}{2} ft_1^2$



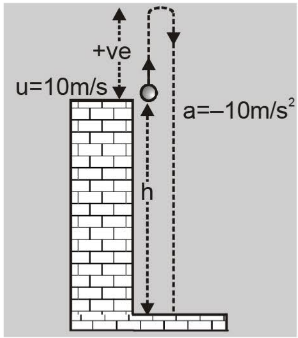
for time t_2 $\quad 0 = v_m - \frac{f}{2}t_2 \Rightarrow t_2 = 2t_1$

$S_3 = \frac{1}{2}\left(\frac{f}{2}\right)t_2^2 = \left(\frac{f}{4}\right)\left(4t_1^2\right) = ft_1^2 = 2S$

Therefore $S + v_mt + 2S = 15S \Rightarrow v_mt = 12S \Rightarrow ft_1t = 12S$

$\Rightarrow \quad f\left(\frac{2S}{f}\right)^{1/2} t = 12\,S \quad \Rightarrow 2Sf = \frac{144S^2}{t^2} \quad \Rightarrow S = \frac{ft^2}{72}$

Illustration A ball is thrown upwards from the top of a tower 40 m high with a velocity of 10 m/s, find the time when it strikes the ground ($g = 10\ m/s^2$)



Solution In the problem $u = +10\ m/s$, $a = -10\ m/s^2$

and $s = -40m$ (at the point where ball strikes the ground)

Substituting in $s = ut + \frac{1}{2}at^2$

$-40 = 10t - 5t^2$ or $5t^2 - 10t - 40 = 0$ or $t^2 - 2t - 8 = 0$

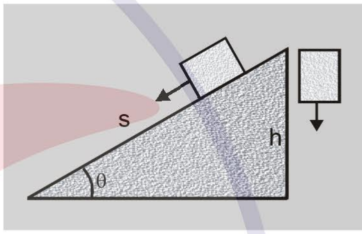
Solving this we have t = 4 s and – 2s.

Taking the positive value t = 4s.

Illustration A block slides down a smooth inclined plane when released from the top, while another falls freely from the same point. Which one of them will strike the ground: (a)earlier (b) with greater speed?

Solution In case of sliding motion on the inclined plane.

$$\frac{h}{s} = \sin\theta \Rightarrow s = \frac{h}{\sin\theta}, \quad a = g\sin\theta$$



$$t_s = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2}{g\sin\theta} \times \frac{h}{\sin\theta}} = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}} = \frac{t_F}{\sin\theta}$$

$$v_s = \sqrt{2as} = \sqrt{2g\sin\theta \times \frac{h}{\sin\theta}} = \sqrt{2gh}$$

In case of free fall $t_F = \sqrt{\frac{2h}{g}}$ and $v_F = \sqrt{2gh} = v_s$

(a) $\because$ $\sin\theta < 1$, $t_F < t_s$, i.e.,falling body reaches the ground first.

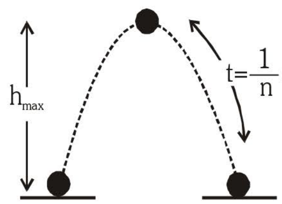
(b) $v_F = v_s$, i.e.,both reach the ground with same speed.

Special Note : (not same velocity, as for falling body direction is vertical while for sliding body along the plane downwards).

Illustration A Juggler throws balls into air. He throws one ball whenever the previous one is at its highest point. How high do the balls rise if he throws n balls each second ? Acceleration due to gravity is g.

Solution Juggler throws n balls in one second so time interval between two consecutive throws is $t = \frac{1}{n}s$

each ball takes $\frac{1}{n}s$ to reach maximum height

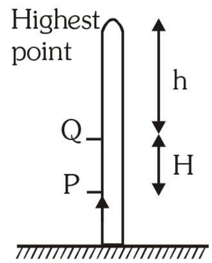


So $h_{max} = \frac{1}{2} \times gt^2 = \frac{1}{2} \times g\left(\frac{1}{n}\right)^2$

$$h_{max} = \frac{g}{2n^2}$$

Illustration A particle is thrown vertically upwards from the surface of the earth. Let T_P be the time taken by the particle to travel from a point P above the earth to its highest point and back to the point P. Similarly, let T_Q be the time taken by the particle to travel from another point Q above the earth to its highest point and back to the same point Q. If the distance between the points P and Q is H, find the expression for acceleration due to gravity in terms of T_P, T_Q and H.

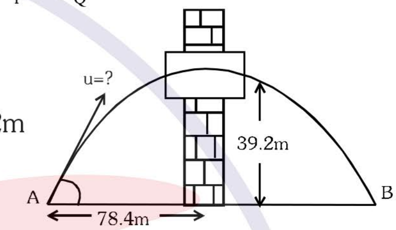
Solution Time taken from point P to point P $T_P = 2\sqrt{\frac{2(h+H)}{g}}$



Time taken from point Q to point Q $T_Q = 2\sqrt{\frac{2h}{g}}$

$$\Rightarrow T_P^2 = \frac{8(h+H)}{g} \ \& \ T_Q^2 = \frac{8h}{g} \Rightarrow T_P^2 = T_Q^2 + \frac{8H}{g} \Rightarrow g = \frac{8H}{T_P^2 - T_Q^2}$$

Illustration A boy stands 78.4 m away from a building and throws a ball which just enters a window at maximum height 39.2m above the ground. Calculate the velocity of projection of the ball.



Solution Maximum height $= \frac{u^2 \sin^2\theta}{2g} = 39.2$ m ... (i)

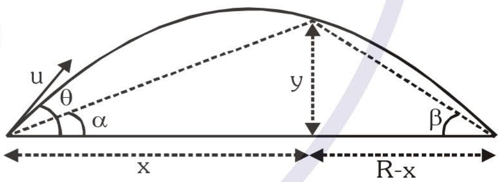
Range $= \frac{u^2 \sin 2\theta}{g} = \frac{2u^2 \sin\theta\cos\theta}{g} = 2 \times 78.4$ ... (ii)

from equation (i) divided by equation (ii) $\tan\theta = 1 \Rightarrow \theta = 45°$

from equation (ii) range $= \frac{u^2 \sin 90°}{g} = 2 \times 78.4 \Rightarrow u = \sqrt{2 \times 78.4 \times 9.8} = 39.2$ m/s

Illustration A particle thrown over a triangle from one end of a horizontal base falls on the other end of the base after grazing the vertex. If α and β are the base angles of triangle and angle of projection is θ, then prove that $\tan\theta = \tan\alpha + \tan\beta$.

Solution From triangle $y = x\tan\alpha$ and $y = (R - x)\tan\beta$



$$\tan\alpha + \tan\beta = \frac{y}{x} + \frac{y}{R-x} = \frac{yR}{x(R-x)}$$

$$\because \quad y = x\tan\theta\left[1 - \frac{x}{R}\right] \Rightarrow \tan\theta = \frac{yR}{x(R-x)}$$

$$\therefore \quad \tan\theta = \tan\alpha + \tan\beta$$

Illustration A ball is thrown from the ground to clear a wall 3 m high at a distance of 6 m and falls 18 m away from the wall, the angle of projection of ball is :-

(A) $\tan^{-1}\left(\frac{3}{2}\right)$ (B) $\tan^{-1}\left(\frac{2}{3}\right)$ (C) $\tan^{-1}\left(\frac{1}{2}\right)$ (D) $\tan^{-1}\left(\frac{3}{4}\right)$

Solution From equation of trajectory, $y = x\tan\theta\left[1 - \frac{x}{R}\right] \Rightarrow 3 = 6\tan\theta\left[1 - \frac{1}{4}\right] \Rightarrow \tan\theta = \frac{2}{3}$

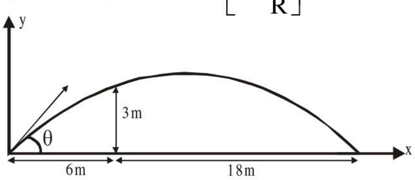
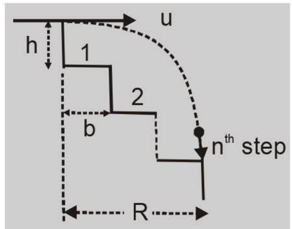


Illustration A ball rolls off the top of a stair way with a horizontal velocity u. If each step has height h and width b the ball will just hit the edge of n^{th} step. Find the value of n.

Solution If the ball hits the n^{th} step, the horizontal and vertical distances traversed are nb and nh respectively. Let t be the time taken by the ball for these horizontal and vertical displacements. Velocity along horizontal direction = u (remains constant) and initial vertical velocity = zero.

$\therefore$ $nb = ut$ and $nh = 0 + \frac{1}{2}gt^2$

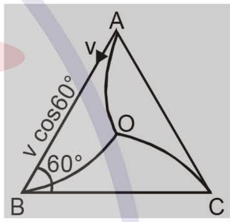


Eliminating t from the equation

$$nh = \frac{1}{2}g\left(\frac{nb}{u}\right)^2 \Rightarrow n = \frac{2hu^2}{gb^2}$$

Illustration Three boys A, B and C are situated at the vertices of an equilateral triangle of side d at t = 0. Each of the boys move with constant speed v. A always moves towards B , B towards C and C towards A. When and where will they meet each other ?

Solution By symmetry they will meet at the centroid of the triangle.



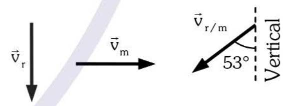
Approaching velocity of A and B towards each other is $v + v\cos 60°$ and they cover distance d when they meet. So that time taken, is given by

$$\therefore \quad t = \frac{d}{v + v\cos 60^\circ} = \frac{d}{v + \frac{v}{2}} = \frac{2d}{3v}$$

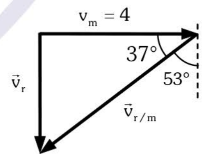
Illustration A man at rest observes the rain falling vertically. When he walks at 4 km/h, he has to hold his umbrella at an angle of 53° from the vertical. Find the velocity of raindrops.

Solution Assigning usual symbols $\vec{v}_m$, $\vec{v}_r$ and $\vec{v}_{r/m}$ to velocity of man, velocity of rain and velocity of rain relative to man, we can express their relationship by the following equation

$$\vec{v}_r = \vec{v}_m + \vec{v}_{r/m}$$



The above equation suggests that a standstill man observes velocity $\vec{v}_r$ of rain relative to the ground and while he is moving with velocity $\vec{v}_m$, he observes



velocity of rain relative to himself $\vec{v}_{r/m}$. It is a common intuitive fact that umbrella must be held against $\vec{v}_{r/m}$ for optimum protection from rain. According to these facts, directions of the velocity vectors are shown in the adjoining figure.

Therefore $v_r = v_m \tan 37° = 3$ km/h

Illustration A boat can be rowed at 5 m/s in still water. It is used to cross a 200 m wide river from south bank to the north bank. The river current has uniform velocity of 3 m/s due east.

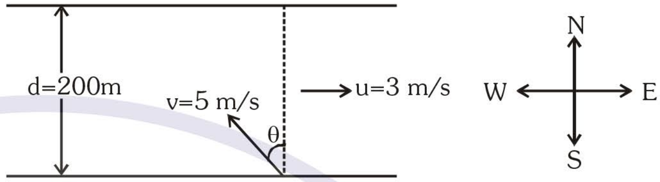
(a) In which direction must it be steered to cross the river perpendicular to current?

(b) How long will it take to cross the river in a direction perpendicular to the river flow?

(c) In which direction must the boat be steered to cross the river in minimum time? How far will it drift?

Solution (a) To cross the river perpendicular to current i.e. along shortest path

$$\sin\theta = \frac{u}{v} = \frac{3}{5} \Rightarrow \theta = 37°$$



(b) Time taken by boat,

$$t = \frac{d}{v\cos\theta} = \frac{200}{5 \times \frac{4}{5}} = 50s$$

(c) To cross the river in minimum time, $\theta = 0°$

Therefore $t_{min} = \frac{d}{v} = \frac{200}{5} s = 40s$

Drift = $u(t_{min})$ = 3(40)m = 120 m