GENERAL EQUATION OF WAVE MOTION

$$\frac{d^2y}{dt^2} = v^2\frac{d^2y}{dx^2}$$

$$y(x,t) = f\left(t \pm \frac{x}{v}\right)$$

where, $y(x,t)$ should be finite everywhere.

$f\left(t + \frac{x}{v}\right)$ represents wave travelling in –ve x-axis.

$f\left(t - \frac{x}{v}\right)$ represents wave travelling in +ve x-axis.

$y = A\sin(\omega t \pm kx + \phi)$

TERMS RELATED TO WAVE MOTION

(For 1-D Progressive Sine Wave) Wave Number (or Propagation Constant) (k)

$k = 2\pi/\lambda = \frac{\omega}{v}(rad\ m^{-1})$

Phase of Wave

The argument of harmonic function $(\omega t \pm kx + \phi)$ is called phase of the wave. Phase difference $(\Delta\phi)$: difference in phases of two particles at any time t.

$\Delta\phi = \frac{2\pi}{\lambda}\Delta x$ where Δx is path difference.

Also $\Delta\phi = \frac{2\pi}{T}.\Delta t$

Speed of Transverse Wave Along the string

$v = \sqrt{\frac{T}{\mu}}$

where T = Tension

μ = mass per unit length

Power Transmitted Along the string

Average Power $< P > = 2\pi^2 f^2 A^2 \mu v$

Intensity $I = \frac{<P>}{s} = 2\pi^2 f^2 A^2 \rho v$

REFLECTION OF WAVES

If we have a wave

$y_i(x,t) = \text{a}\sin(kx - \omega t)$ then,

(i) Equation of wave reflected at a rigid boundary

$(y_r(x,t) = \text{a}\sin(kx + \omega t + \pi)$

or $y_r(x,t) = -\text{a}\sin(kx + \omega t)$

i.e. the reflected wave is 180° out of phase.

(ii) Equation of wave reflected at an open boundary

$y_r(x,t) = \text{a}\sin(kx + \omega t)$

i.e. the reflected wave is in phase with the incident wave.

STANDING/STATIONARY WAVES

$y_1 = A\sin(\omega t - kx + \theta_1)$

$y_2 = A\sin(\omega t - kx + \theta_2)$

$y_1 + y_2 = 2\,A\cos\left(kx + \frac{\theta_2 - \theta_1}{2}\right)\sin\left(\omega t + \frac{\theta_1 + \theta_2}{2}\right)$

The quantity $2\,A\cos\left(kx + \frac{\theta_2 - \theta_1}{2}\right)$ represents resultant amplitude at x. At some position resultant **amplitude is zero** these are called **nodes**. At some positions resultant **amplitude is 2A**, these are called **antinodes**.

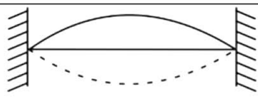
Distance between successive nodes or antinodes $= \frac{\lambda}{2}$

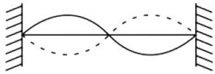
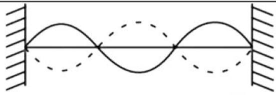
Distance between adjacent nodes and antinodes$= \frac{\lambda}{4}$.

All the particles in same segment (portion between two successive nodes) vibrate in same phase. Since nodes are permanently at rest so energy cannot be transmitted across these.

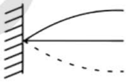
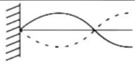
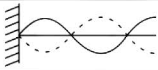
VIBRATIONS OF STRINGS (STANDING WAVE)

Fixed at Both Ends

First harmonics or Fundamental frequency	$L = \frac{\lambda}{2}, f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$	

Second harmonics or First overtone	$L = \frac{2\lambda}{2}$, $f_2 = \frac{2}{2L}\sqrt{\frac{T}{\mu}}$	
Third harmonics or Second overtone	$L = \frac{3\lambda}{2}$, $f_3 = \frac{3}{2L}\sqrt{\frac{T}{\mu}}$	
n^{th} harmonics or $(n-1)^{th}$ overtone	$L = \frac{n\lambda}{2}$, $f_n = \frac{n}{2L}\sqrt{\frac{T}{mu}}$	

String Free at One End

First harmonics or Fundamental frequency	$L = \frac{\lambda}{4}, f_1 = \frac{1}{4L}\sqrt{\frac{T}{\mu}}$	
Third harmonics or First overtone	$L = \frac{3\lambda}{4}, f_3 = \frac{3}{4L}\sqrt{\frac{T}{\mu}}$	
Fifth harmonics or Second overtone	$L = \frac{5\lambda}{4}, f_5 = \frac{5}{4L}\sqrt{\frac{T}{\mu}}$	
(2n + 1)th harmonic or nth overtone	$L = \frac{(2n+1)\lambda}{4}$, $f_{2n+1} = \frac{(2n+1)}{4L}\sqrt{\frac{T}{\mu}}$