UNITS, DIMENSIONS AND MEASUREMENTS

The system of Internationally accepted units contain three classes of units :

(i) Fundamental units (ii) Derived units (iii) Supplementary units

(1) Fundamental units

Physical Quantity	***Units***	***Symbol***	***Dimensions***
Length	metre	m	L
Mass	kilogram	kg	M
Time	Second	s	T
Electric Current	Ampere	A	A
Temperature	Kelvin	K	K or θ
Luminous Intensity	candela	cd	Cd
Amount of Substance	mole	mol	mol

(2) Derived units : The units of derived quantities or the units that can be expressed in terms of the base units are called derived units.

e.g. unit of speed $= \dfrac{\text{unit of distance}}{\text{unit of time}} = \dfrac{\text{metre}}{\text{second}} = \text{ms}^{-1}$

Some derived units are named in honour of great scientists. e.g. unit of force - newton (N), unit of frequency - hertz (Hz), etc.

(3) Supplementary Units : There are two supplementary units -

Physical Quantity	***Units***	***Dimensions***
Plane angle	Radian	Dimensionless
Solid angle	Steradian	Dimensionless

(4) Dimensions : Dimensions of a physical quantity are the powers (or exponents) to which the base quantities are raised to represent that quantity.

SETS OF QUANTITIES HAVING SAME DIMENSIONS

S.No.	Quantities	Dimensions
1.	Strain, refractive index, relative density, angle, solid angle, phase, distance gradient, relative permeability, relative permittivity, angle of contact, Reynolds number, coefficient of friction, mechanical equivalent of heat, electric susceptibility, etc.	$[M^0 L^0 T^0]$
2.	Mass and inertia	$[M^1 L^0 T^0]$
3.	Momentum and impulse.	$[M^1 L^1 T^{-1}]$
4.	Thrust, force, weight, tension, energy gradient.	$[M^1 L^1 T^{-2}]$
5.	Pressure, stress, Young's modulus, bulk modulus, shear modulus, modulus of rigidity, energy density.	$[M^1 L^{-1} T^{-2}]$
6.	Angular momentum and Planck's constant (h).	$[M^1 L^2 T^{-1}]$
7.	Acceleration, g and gravitational field intensity.	$[M^0 L^1 T^{-2}]$
8.	Surface tension, free surface energy (energy per unit area), force gradient, spring constant.	$[M^1 L^0 T^{-2}]$
9.	Latent heat capacity and gravitational potential.	$[M^0 L^2 T^{-2}]$
10.	Thermal capacity, Boltzmann constant, entropy.	$[ML^2T^{-2}K^{-1}]$
11.	Work, torque, internal energy, potential energy, kinetic energy, moment of force, (q^2/C), (LI^2), (qV), (V^2C), (I^2Rt), $\dfrac{V^2}{R}t$, (VIt), (PV), (RT), (mL), $(mc\,\Delta T)$	$[M^1 L^2 T^{-2}]$
12.	Frequency, angular frequency, angular velocity, velocity gradient, radioactivity $\dfrac{R}{L}, \dfrac{1}{RC}, \dfrac{1}{\sqrt{LC}}$	$[M^0 L^0 T^{-1}]$
13.	$\left(\dfrac{\ell}{g}\right)^{1/2}, \left(\dfrac{m}{k}\right)^{1/2}, \left(\dfrac{L}{R}\right), (RC), (\sqrt{LC})$, time	$[M^0 L^0 T^1]$
14.	(VI), (I^2R), (V^2/R), Power	$[M L^2 T^{-3}]$

USES OF DIMENSIONAL EQUATIONS

(i) Conversion of one system of units into another.

(ii) To check the dimensional correctness of a given physical relation.

(iii) Deriving a correct relationship between different physical quantities.

SIGNIFICANT FIGURES OR DIGITS

The significant figures (SF) in a measurement are the figures or digits that are known with certainity plus one that is uncertain. Significant figures in a measured value of a physical quantity tell the number of digits in which we have confidence. Larger the number of significant figures obtained in a measurement, greater is its accuracy and vice versa.

Rules to find out the number of significant figures

Rule I : All the non-zero digits are significant e.g. 1984 has 4 SF.

Rule II : All the zeros between two non-zero digits are significant. e.g. 10806 has 5 SF.

Rule III : All the zeros to the left of first non-zero digit are not significant. e.g.00108 has 3 SF.

Rule IV : If the number is less than 1, zeros on the right of the decimal point but to the left of the first non-zero digit are not significant. e.g. 0.002308 has 4 SF.

Rule V : The trailing zeroes (zeroes to the right of the last non-zero digit) in a number with a decimal point are significant. e.g. 01.080 has 4 SF.

Rule VI : The trailing zeroes in a number without a decimal point are not significant e.g. 010100 has 3 SF. But if the number comes from some actual measurement then the trailing zeroes become significant. e.g. m = 100 kg has 3 SF.

Rule VII : When the number is expressed in exponential form, the exponential term does not affect the number of S.F. For example in $x = 12.3 = 1.23 \times 10^1 = .123 \times 10^2 = 0.0123 \times 10^3 = 123 \times 10^{-1}$ each term has 3 SF only.

Rules for arithmetical operations with significant figures

Rule I : In addition or subtraction the number of decimal places in the result should be equal to the number of decimal places of that term in the operation which contain lesser number of decimal places. e.g. $12.587 - 12.5 = 0.087 = 0.1$ ($\because$ second term contain lesser i.e. one decimal place)

Rule II : In multiplication or division, the number of SF in the product or quotient is same as the smallest number of SF in any of the factors. e.g. $5.0 \times 0.125 = 0.625 = 0.62$

Rounding Off To represent the result of any computation containing more than one uncertain digit, it is *rounded off* to appropriate number of significant figures.

Rules for rounding off the numbers :

Rule I : If the digit to be rounded off is more than 5, then the preceding digit is increased by one. e.g. $6.87 \approx 6.9$

Rule II : If the digit to be rounded off is less than 5, then the preceding digit is unaffected and is left unchanged. e.g. $3.94 \approx 3.9$

Rule III : If the digit to be rounded off is 5 then the preceding digit is increased by one if it is odd and is left unchanged if it is even. e.g. $14.35 \approx 14.4$ and $14.45 \approx 14.4$

Order of Magnitude

Order of magnitude of a quantity is the power of 10 required to represent that quantity. This power is determined after rounding off the value of the quantity properly. For rounding off, the last digit is simply ignored if it is less than 5 and, is increased by one if it is 5 or more than 5.

ERRORS IN MEASUREMENT

Definition

The difference between the true value and the measured value of a quantity is known as the error of measurement.

Error = True value ~ Measured value

Classification of Errors

Systematic or Controllable Errors

- Due to the known causes like imperfect design of instruments, imperfect technique and carelessness in taking observation.
- It can be either positive or negative.
- It can be elliminated

Random Errors

- Due to unknown causes.
- Can vary in magnitude and sign.
- Can be minimised by taking several observation.

Note :- If the number of observations is made n times then the random error reduces to $(\frac{1}{n})$ times.

Accuracy and Precision : The ***accuracy*** of a measurement is a measure of how closed quantity. ***Precision*** tells us to what resolution or limit the quantity is measured. Accuracy can be checked by relative error while precision can be checked by least count.

Calculation of Errors in Mathematical Operations

Rule I : In addition / subtraction process, If $X = A + B$ or $X = A - B$

the maximum absolute error in addition / subtraction process $\Delta X = \Delta A + \Delta B$

Maximum percentage error $= \frac{\Delta X}{X} \times 100$

The result will be written as $(X \pm \Delta X)$ (in terms of absolute error) or $(X \pm \frac{\Delta X}{X} \times 100\ \%)$ (in terms of percentage error)

Rule II : In multiplication / division process, then the maximum fractional or relative error in the product or division

If $X = A \times B$ or $X = A/B$ then $\frac{\Delta X}{X} = \pm\left[\frac{\Delta A}{A} + \frac{\Delta B}{B}\right]$

Rule III : The maximum fractional error in a quantity raised to a power (n) is n times the fractional error in the quantity itself, i.e.

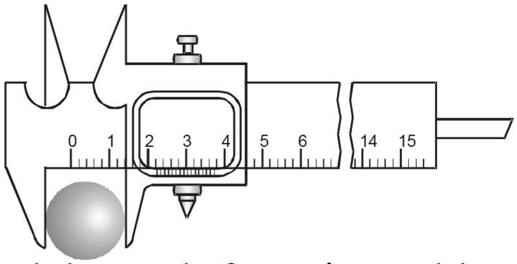
If $X = A^n$ then $\frac{\Delta X}{X} = n(\frac{\Delta A}{A})$

If $X = A^p B^q C^r$ then $\frac{\Delta X}{X} = \left[p\left(\frac{\Delta A}{A}\right) + q\left(\frac{\Delta B}{B}\right) + r\left(\frac{\Delta C}{C}\right)\right]$

If $X = \frac{A^p B^q}{C^r}$ then $\frac{\Delta X}{X} = \left[p\left(\frac{\Delta A}{A}\right) + q\left(\frac{\Delta B}{B}\right) + r\left(\frac{\Delta C}{C}\right)\right]$

Vernier Callipers

Least count = 1MSD – 1 VSD (MSD → main scale division, VSD → Vernier scale division)

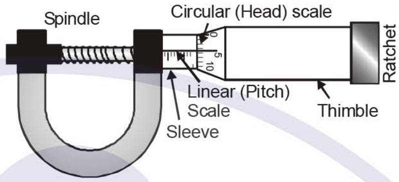


Ex. A vernier scale has 10 parts, which are equal to 9 parts of main scale having each path equal to 1 mm then

least count = 1 mm $-\frac{9}{10}$ mm = 0.1 mm [$\because$ 9 MSD = 10 VSD]

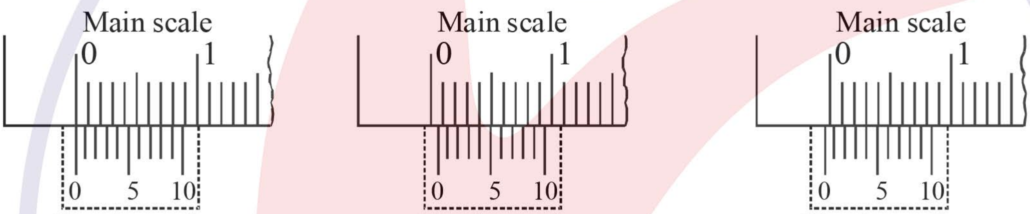
Screw Gauge

Least count = $\frac{\text{pitch}}{\text{total no. of divisions on circular scale}}$



Ex. The distance moved by spindle of a screw gauge for each turn of head is 1mm. The edge of the humble is provided with a angular scale carrying 100 equal divisions. The least count = $\frac{1\text{mm}}{100}$ = 0.01 mm

Zero Error



Vernier scale without zero error | Vernier scale with positive zero error (i) | Vernier scale with negative zero error (ii)

The zero error is always subtracted from the reading to get the corrected value.

If the zero error is positive, its value is calculated as we take any normal reading.

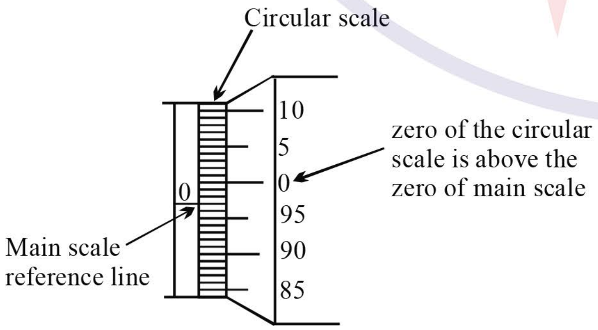
Negative zero error = – [Total no. of vsd – vsd coinciding] ×L.C.

Zero Error in Screw Gauge

If there is no object between the jaws (i.e. jaws are in contact), the screwgauge should give zero reading. But due to extra material on jaws, even if there is no object, it gives some excess reading. This excess reading is called Zero error.

Negative Zero Error

(3 division error) i.e., –0.003 cm



Positive Zero Error

(2 division error) i.e., +0.002 cm

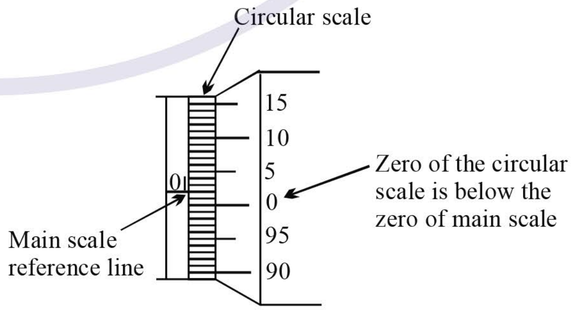


Illustration The length, breadth and thickness of a metal sheet are 4.234 m, 1.005 m and 2.01 cm respectively. Give the area and volume of the sheet to correct number of significant figures.

Solution length (ℓ) = 4.234 m breadth (b) = 1.005 m

thickness (t) = 2.01 cm = 2.01×10^{-2} m

Therefore area of the sheet = $2(\ell \times b + b \times t + t \times \ell)$

$= 2(4.234 \times 1.005 + 1.005 \times 0.0201 + 0.0201 \times 4.234)\ m^2 = 8.7209478\ m^2$

Since area can contain a maximum of 3 SF (Rule II of article 4.2) therefore, rounding off,

we get Area = $8.72\ m^2$

Like wise volume = $\ell \times b \times t$ = $4.234 \times 1.005 \times 0.0201\ m^3 = 0.0855289\ m^3$

Since volume can contain 3 SF, therefore, rounding off, we get

Volume = $0.0855\ m^3$

Illustration The inital and final temperatures of water as recorded by an observer are (40.6 ± 0.2)°C and (78.3 ± 0.3)°C. Calculate the rise in temperature with proper error limits.

Solution Given $\theta_1 = (40.6 \pm 0.2)$°C and $\theta_2 = (78.3 \pm 0.3)$°C

Rise in temp. $\theta = \theta_2 - \theta_1 = 78.3 - 40.6 = 37.7$°C.

$\Delta\theta = \pm(\Delta\theta_1 + \Delta\theta_2) = \pm(0.2 + 0.3) = \pm 0.5$°C

$\therefore$ rise in temperature = (37.7 ± 0.5)°C

Illustration The length and breadth of a rectangle are (5 ± 0.1) cm and (3 ± 0.2) cm. Calculate area of the rectangle with error limits.

Solution Given $\ell = (5 \pm 0.1)$ cm and $b = (3 \pm 0.2)$ cm

Area $A = \ell \times b = 5 \times 3 = 15\ cm^2 \Rightarrow \frac{\Delta A}{A} = \pm\left(\frac{\Delta \ell}{\ell} + \frac{\Delta b}{b}\right) = \pm\left(\frac{0.1}{5} + \frac{0.2}{3}\right) = \pm(0.02 + 0.06)$

or $\Delta A = \pm 0.08 \times 15 = \pm 1.2$ $\therefore$ Area = (15 ± 1.2) sq. cm²

Illustration A body travels uniformly a distance (16 ± 0.2) m in a time (4.0 ± 0.2) s. Calculate its velocity with error limits. What is the percentage error in velocity?

Solution Given distance $s = (16 \pm 0.2)$ m and time $t = (4.0 \pm 0.2)$ s

velocity $v = \frac{s}{t} = \frac{16}{4} = 4\ ms^{-1} \Rightarrow \frac{\Delta v}{v} = \pm\left(\frac{\Delta s}{s} + \frac{\Delta t}{t}\right) = \pm\left(\frac{0.2}{16} + \frac{0.2}{4}\right) = \pm(0.0125 + 0.05)$

or $\Delta v = 0.0625 \times v = \pm 0.0625 \times 4 = \pm 0.25$ $\therefore$ $v = (4 \pm 0.25)\ ms^{-1}$

percentage error in velocity = $\frac{\Delta v}{v} \times 100 = +\frac{0.25}{4} \times 100 = 6.25\%$

Illustration A thin copper wire of length L increase in length by 2% when heated from T_1 to T_2. If a copper cube having side 10 L is heated from T_1 to T_2 what will be the percentage change in

(i) area of one face of the cube and. (ii) volume of the cube.

Solution (i) Area $A = 10\,L \times 10\,L = 100\,L^2$.

Percentage change in area = $\frac{\Delta A}{A} \times 100 = 2 \times \frac{\Delta L}{L} \times 100 = 2 \times 2\% = 4\%$

$\left(\because \frac{\Delta A}{A} = \frac{\Delta 100}{100} + 2\frac{\Delta L}{L} = 0 + 2\frac{\Delta L}{L} = 2\frac{\Delta L}{L}\right)$

(ii) Volume $V = 10\,L \times 10\,L \times 10\,L = 1000\,L^3$

Percentage change in volume = $\frac{\Delta V}{V} \times 100 = 3\frac{\Delta L}{L} = 3 \times 2\% = 6\%$

Conclusion (i) Constants do not have any error in them.

(ii) The maximum percentage change will be observed in volume, lesser in area and the least (minimum) change will be observed in length or radius.

Illustration If $\alpha = \frac{F}{v^2}\sin\beta t$, find dimensions of α and β. Here v = velocity, F = force and t = time.

Solution Here sin βt and βt must be dimensionless

$$\text{So } [\beta t] = 1 \Rightarrow [\beta] = \left[\frac{1}{t}\right] = [T^{-1}];\ [\alpha] = \left[\frac{F}{v^2}\sin\beta t\right] = \left[\frac{F}{v^2}\right] = \left[\frac{MLT^{-2}}{L^2T^{-2}}\right] = [ML^{-1}]$$

Illustration Following observations were taken with a vernier callipers while measuring the length of a cylinder.
3.29 cm, 3.28 cm, 3.29 cm, 3.31 cm, 3.28 cm, 3.27 cm, 3.29 cm, 3.30 cm
Then find
(a) Most accurate length of the cylinder. (b) Absolute error in each observation.
(c) Mean absolute error (d) Relative error
(e) Percentage error
Express the result in terms of absolute error and percentage error.

Solution (a) Most accurate length of the cylinder will be the mean length $(\bar{\ell})$

$$\bar{\ell} = \frac{3.29+3.28+3.29+3.31+3.28+3.27+3.29+3.30}{8} = 3.28875 \text{ cm or } \bar{\ell} = 3.29 \text{ cm}$$

(b) Absolute error in the first reading = 3.29 – 3.29 = 0.00 cm
Absolute error in the second reading = 3.29 – 3.28 = 0.01 cm
Absolute error in the third reading = 3.29 – 3.29 = 0.00 cm
Absolute error in the forth reading = 3.29 – 3.31 = –0.02 cm
Absolute error in the fifth reading = 3.29 – 3.28 = 0.01 cm
Absolute error in the sixth reading = 3.29 – 3.27 = 0.02 cm
Absolute error in the seventh reading = 3.29 – 3.29 = 0.00 cm
Absolute error in the last reading = 3.29 – 3.30 = –0.01 cm

(c) Mean absolute error $= \overline{\Delta\ell} = \frac{0.00+0.01+0.00+0.02+0.01+0.02+0.00+0.01}{8} = 0.01$ cm

(d) Relative error in length $= \frac{\overline{\Delta\ell}}{\bar{\ell}} = \frac{0.01}{3.29} = 0.0030395 = 0.003$

(e) Percentage error $= \frac{\overline{\Delta\ell}}{\bar{\ell}} \times 100 = 0.003 \times 100 = 0.3\%$

So length $\ell = 3.29 \text{ cm} \pm 0.01 \text{ cm}$ (in terms of absolute error)
or $\ell = 3.29 \text{ cm} \pm 0.30\%$ (in terms percentage error)

Illustration One cm on the main scale of vernier callipers is divided into ten equal parts. If 20 divisions of vernier scale coincide with 8 small divisions of the main scale. What will be the least count of callipers ?

Solution 20 division of vernier scale = 8 division of main scale $\Rightarrow$ 1 VSD$=\left(\frac{8}{20}\right)$MSD = 0.4 MSD

Least count = 1 MSD – 1 VSD = 1 MSD – 0.4 MSD = 0.6 MSD

$$= 0.6 \times 0.1 \text{ cm} = 0.06 \text{ cm} \quad (\because 1 \text{ MSD} = \frac{1}{10} \text{ cm} = 0.1 \text{ cm})$$

Illustration A spherometer has 100 equal divisions marked along the periphery of its disc, and one full rotation of the disc advances on the main scale by 0.01 cm. Find the least count of the system.

Solution Given Pitch = 0.01 cm

$$\therefore \text{ Least count} = \frac{\text{Pitch}}{\text{Total no. of divisions on the the circular scale}} = \frac{0.01}{100} \text{ cm} = 10^{-4} \text{ cm.}$$