BASIC MATHEMATICS USED IN PHYSICS AND VECTORS

ALGEBRA

Quadratic Equation and Its Solution

An algebraic equation of second order (highest power of the variable is equal to 2) is called a quadratic equation. Equation $ax^2 + bx + c = 0$ is the general quadratic equation. The solution was given by Indian mathematician shri Dhara Charya.

The general solution (Roots) of the above quadratic equation or value of variable x is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \Rightarrow \quad x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

x_1 & x_2 are the roots of the equation

Sum of the roots $\quad x_1 + x_2 = \frac{-b}{a} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$

Product of the roots $\quad x_1\, x_2 = \frac{c}{a} = \frac{\text{coefficient of constant term}}{\text{coefficient of } x^2}$

Arithmetic Progression (A.P.)

In this sequence every number is obtained by adding a certain constant value (Positive or Negative) in the preceding number, called common difference.

In general, arithmetic progression can be written as, $\quad a, (a + d), (a + 2d)$ ....

where a is the value of 1st term, d is common difference

- value of n^{th} term of A.P. is $\quad t_n = a + (n - 1)\, d$
- The sum of n terms of A.P. is $\quad S_n = \frac{n}{2}\{2a + (n - 1)d\}$

Geometric Progression

In this sequence, every number can be obtained by multiplying its preceding number by a fixed number called common ratio

- It can be written as, $\quad a, ar, ar^2$ ....

  where a is the value of 1st term, r is common ratio
- value of n^{th} term of GP is $\quad t_n = ar^{n-1}$
- Sum of n terms is given by $\quad S_n = \frac{a(1 - r^n)}{1 - r}$
- For sum of infinite terms $(r < 1)$ $\quad S_\infty = \frac{a}{1 - r}$

Binomial Theorem

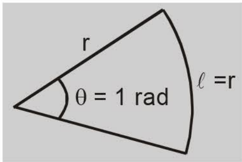
For $x << 1$

- $(1 + x)^n = 1 + nx$ where n may be a fraction
- $(1 - x)^n = 1 - nx$
- $(1 - x)^{-n} = 1 + nx$
- $(1 + x)^{-n} = 1 - nx$

TRIGONOMETRY

Angle

One radian is the angle subtended at the centre of a circle by an arc of the circle, whose length is equal to the radius of the circle.



$$1 \text{ rad} = \frac{180^\circ}{\pi} \approx 57^\circ 17' 45'' \approx 57.3^\circ$$

$$1^\circ = \frac{\pi}{180} \text{radian} \;,\; 1' = \left(\frac{1}{60}\right)^\circ, \; 1'' = \left(\frac{1}{60}\right)'$$

The T-ratios of a few standard angles ranging from 0^0 to 180^0

Angle (θ)	0°	30°	45°	60°	90°	120°	135°	150°	180°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	–1
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	–1	$-\frac{1}{\sqrt{3}}$	0

FACTS :

$\sin(90^\circ+\theta)=\cos\theta$ $\cos(90^\circ+\theta)=-\sin\theta$ $\tan(90^\circ+\theta)=-\cot\theta$	$\sin(180^\circ-\theta)=\sin\theta$ $\cos(180^\circ-\theta)=-\cos\theta$ $\tan(180^\circ-\theta)=-\tan\theta$	$\sin(-\theta)=-\sin\theta$ $\cos(-\theta)=\cos\theta$ $\tan(-\theta)=-\tan\theta$	$\sin(90^\circ-\theta)=\cos\theta$ $\cos(90^\circ-\theta)=\sin\theta$ $\tan(90^\circ-\theta)=\cot\theta$
$\sin(180^\circ+\theta)=-\sin\theta$ $\cos(180^\circ+\theta)=-\cos\theta$ $\tan(180^\circ+\theta)=\tan\theta$	$\sin(270^\circ-\theta)=-\cos\theta$ $\cos(270^\circ-\theta)=-\sin\theta$ $\tan(270^\circ-\theta)=\cot\theta$	$\sin(270^\circ+\theta)=-\cos\theta$ $\cos(270^\circ+\theta)=\sin\theta$ $\tan(270^\circ+\theta)=-\cot\theta$	$\sin(360^\circ-\theta)=-\sin\theta$ $\cos(360^\circ-\theta)=\cos\theta$ $\tan(360^\circ-\theta)=-\tan\theta$

CALCULUS

Differential Calculus

- If c = constant, $\frac{d}{dx}(c) = 0$
- $y = c\,u$, where c is a constant and u is a function of x, $\frac{dy}{dx} = \frac{d}{dx}(cu) = c\frac{du}{dx}$
- $y = u \pm v \pm w$, where u, v and w are function of x, $\frac{dy}{dx} = \frac{d}{dx}(u \pm v \pm w) = \frac{du}{dx} \pm \frac{dv}{dx} \pm \frac{dw}{dx}$
- $y = u\,v$ where u and v are functions of x, $\frac{dy}{dx} = \frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
- $y = \frac{u}{v}$, where u and v are functions of x, $\frac{dy}{dx} = \frac{d}{dx}\frac{u}{v} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
- $y = x^n$, n real number, $\frac{dy}{dx} = \frac{d}{dx}(x^n) = nx^{n-1}$

Formulae for differential coefficients of trigonometric, logarithmic and exponential functions

1. $\frac{d}{dx}(\sin x) = \cos x$

2. $\frac{d}{dx}(\cos x) = -\sin x$

3. $\frac{d}{dx}(\tan x) = \sec^2 x$

4. $\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$

5. $\frac{d}{dx}(\sec x) = \sec x \tan x$

6. $\frac{d}{dx}(\text{cosec}\, x) = -\text{cosec}\, x \cot x$

7. $\frac{d}{dx}(\log_e x) = \frac{1}{x}$

8. $\frac{d}{dx}(e^x) = e^x$

Integration

Few basic formulae of integration

Following are a few basic formulae of integration :

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + c$, (not valid for $n \neq -1$)

2. $\int \sin x dx = -\cos x + c$

3. $\int \cos x dx = \sin x + c$

4. $\int \frac{1}{x} dx = \log_e x + c$ (c is constant of integration)

5. $\int e^x dx = e^x + c$

6. $\int \frac{1}{ax+b} dx = \frac{1}{a} \log_e (ax+b) + c$

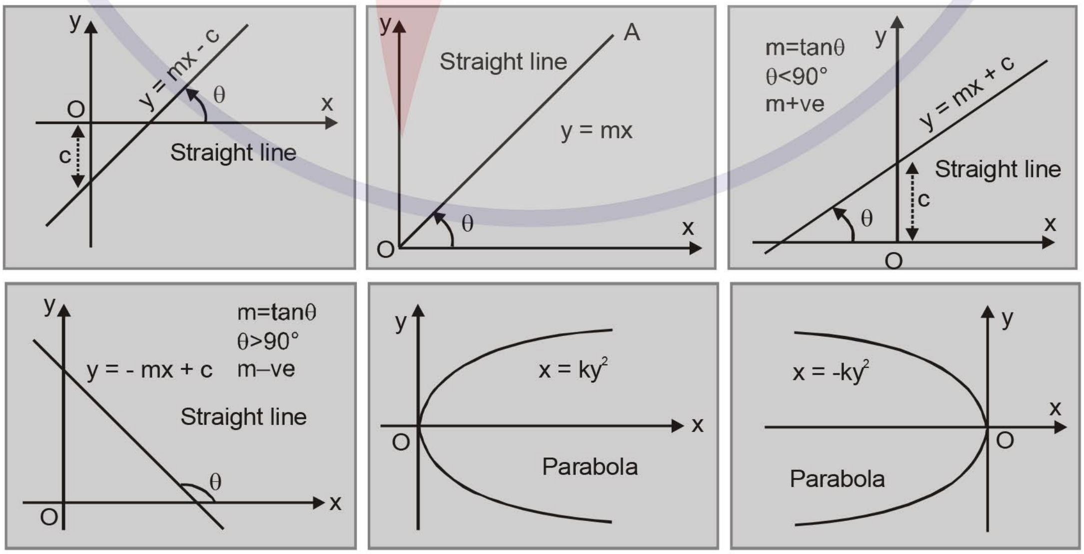
- ***Definite Integrals***

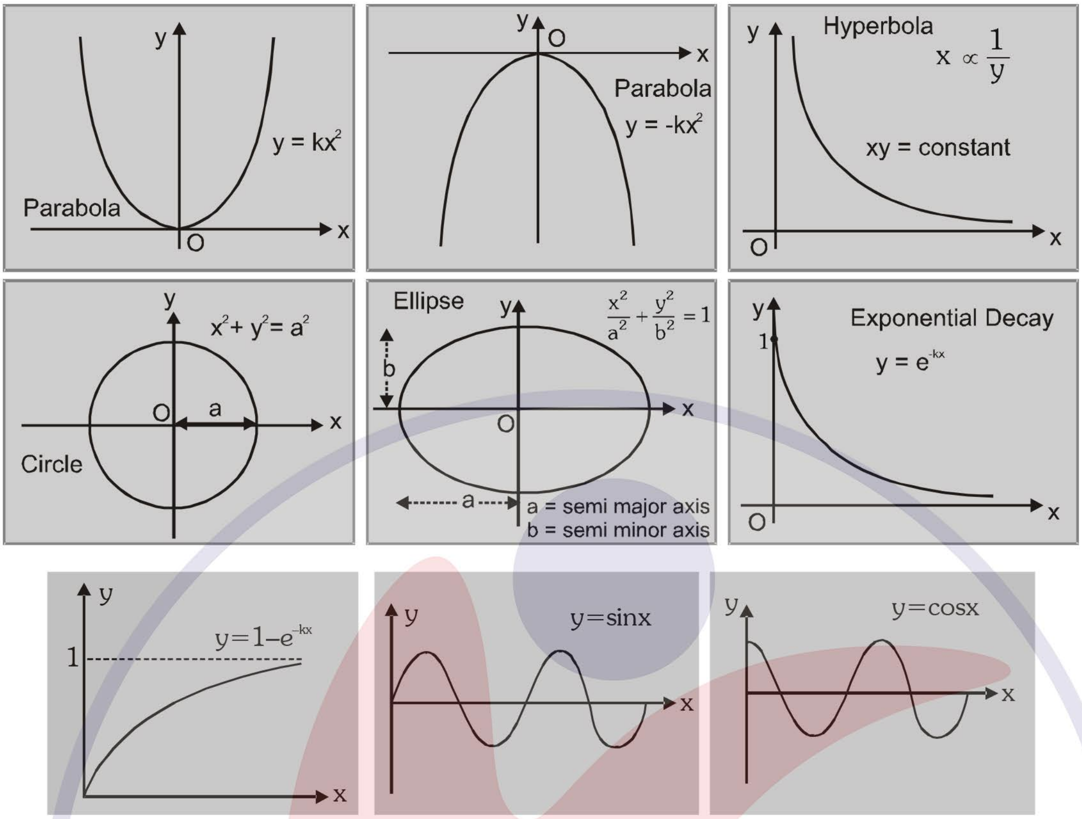
When a function is integrated between a lower limit and an upper limit, it is called a definite integral.

$$\int_a^b f'(x) dx = |f(x)|_a^b = f(b) - f(a)$$

Without any limit integration is called indefinite integral

Graphs : Some Standard graphs and their equations





- **Maxima and Minima (Use of Differential Calculus)**

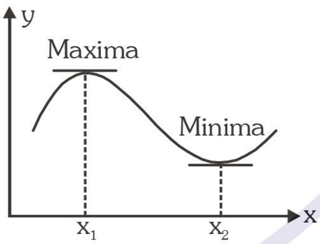
For maxima or minima, consider a function $y = f(x)$,

Step - 1 Differentiate given function y with respect to x and equate it with zero. $\frac{dy}{dx} = 0$

Step - 2 Again differentiate $\frac{dy}{dx}$ with respect to x and verify whether it has a positive value or negative value.

At maxima $\frac{d^2y}{dx^2} < 0$

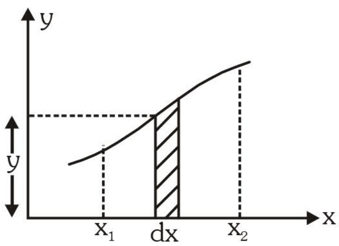
At minima $\frac{d^2y}{dx^2} > 0$



$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$ is called second derivative

Area of graph (Use of Integral Calculus)

Let $y = f(x)$ is a function, then area of graph with x axis is given by



$$A = \int_{x_1}^{x_2} y.dx$$

VECTORS

Scalar Quantities : A physical quantity which can be described completely by its magnitude only and does not require a direction is called a scalar.

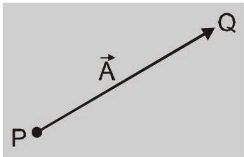
Ex : Distance, mass, time, speed, density, volume, temperature, current etc.

Vector Quantities : A physical quantity which requires magnitude and a particular direction, when it is expressed. i.e. Displacement, velocity, acceleration, force etc.

Representation of vector

A vector is represented by a line headed with an arrow.

Magnitude of $\vec{A} = |\vec{A}|$ or A $\longrightarrow$ Length of the arrow



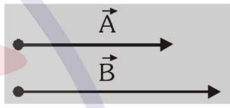
Direction of $\vec{A}$, $\quad \hat{A} = \dfrac{\vec{A}}{|\vec{A}|}$ $\longrightarrow$ Tip of the arrow

TYPES OF VECTOR

- **Parallel Vectors :-**

  Those vectors which have same direction are called parallel vectors.

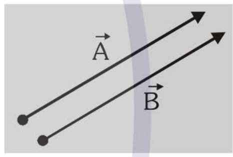
  Angle between two parallel vectors is always 0°



- **Equal Vectors**

  Vectors which have equal magnitude and same direction are called equal vectors.

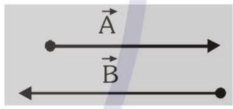
  $$\vec{A} = \vec{B}$$



- **Anti–parallel Vectors :**

  Those vectors which have opposite direction are called anti–parallel vector.

  Angle between two anti–parallel vectors is always 180°

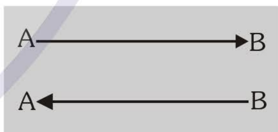


- **Negative (or Opposite) Vectors**

  Vectors which have equal magnitude but opposite direction are called negative vectors of each other.

  $\overrightarrow{AB}$ and $\overrightarrow{BA}$ are negative vectors

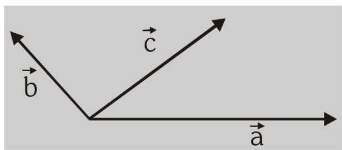
  $$\overrightarrow{AB} = -\overrightarrow{BA}$$



- **Co-initial vector**

  Co-initial vectors are those vectors which have the same initial point.

  In figure $\vec{a}, \vec{b}$ and $\vec{c}$ are co-initial vectors.



- **Collinear Vectors :**

  The vectors lying in the same line are known as collinear vectors.

  Angle between collinear vectors is either 0° or 180°.

  Example.

  (i) $\longleftarrow \quad \longleftarrow$ $(\theta = 0°)$ (ii) $\longrightarrow \quad \longrightarrow$ $(\theta = 0°)$

  (iii) $\longleftarrow \quad \longrightarrow$ $(\theta = 180°)$ (iv) $\longrightarrow \quad \longleftarrow$ $(\theta = 180°)$