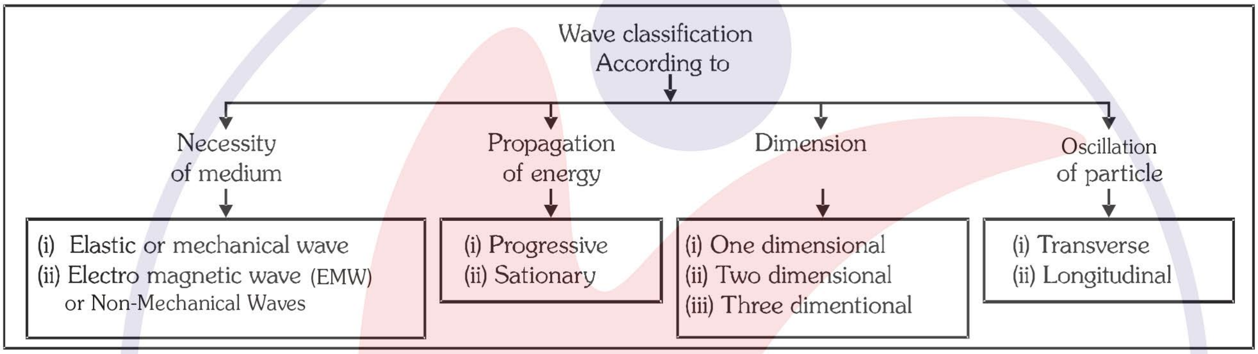
WAVE MOTION & DOPPLER'S EFFECT

WAVES & ITS CHARACTERISTIC

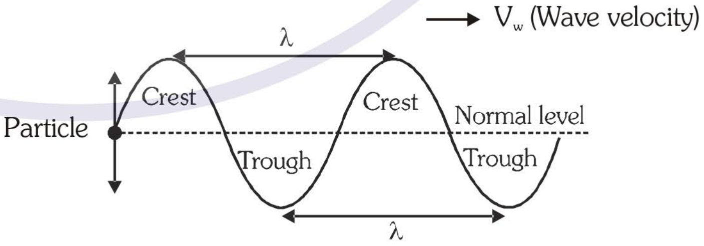
- A wave is a disturbances that propagate in space, transports energy and momentum from one point to another without the transport of matter.
- In wave motion, the disturbance travels through the medium due to repeated periodic oscillations of the particles of the medium about their mean positions.
- Each particle receives disturbance a little later than its preceding particle i.e., there is a regular phase difference between one particle and the next.
- The velocity with which a wave travels is different from the velocity of the particles with which they oscillate about their mean positions.

CLASSIFICATION OF WAVES



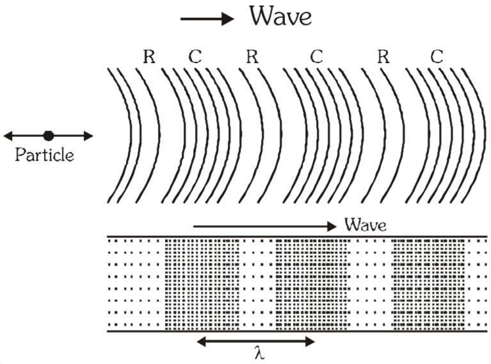
- In the propagation of mechanical waves elasticity and density of the medium plays an important role therefore mechanical waves are also known as **elastic waves**.
- A mechanical wave will be transverse or longitudinal depending on the nature of medium and mode of excitation.
- In strings, mechanical waves are always transverse when string is under a tension.
- In the bulk of gases and liquids mechanical waves are always longitudinal e.g. sound waves in air or water. This is because fluids cannot sustain shear.
- In solids, mechanical waves (may be sound) can be either transverse or longitudinal depending on the mode of excitation. The speed of the two waves in the same solid are different. (Longitudinal waves travels faster than transverse waves).

TRANSVERSE WAVE MOTION



- Mechanical transverse waves are produced in such type of medium which have shearing property, so they are known as shear wave or S-wave.
- Particles of the medium vibrate at right angles to the direction of wave motion
- Particle velocity is always perpendicular to wave velocity
- Can be polarized **e.g.** String waves, Waves on surface of solid or liquid
- A **crest** is a portion of the medium, which is raised temporarily above the normal position of rest of particles of the medium , when a transverse wave passes.
- A **trough** is a portion of the medium, which is depressed temporarily below the normal position of rest of particles of the medium , when a transverse wave passes.

LONGITUDINAL WAVES MOTION



In this type of waves, oscillatory motion of the medium particles produces regions of compression (high pressure) and rarefaction (low pressure) which propagated in space with time (see figure).

- Particles of the medium vibrate in the direction of wave motion
- Particle velocity is parallel or antiparallel to wave velocity
- Can not be polarized **e.g.** Sound waves, Waves in gases

PLANE PROGRESSIVE WAVES

- Equation of progressive wave in positive x direction -

$$y(x,\ t) = A \sin(\omega t - kx) = A \sin \omega \left(t - \frac{x}{V_W}\right) = A \sin\left[\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right]$$

- Equation of progressive wave in negative x direction -

$$y(x,\ t) = A \sin(\omega t + kx) = A \sin \omega \left(t + \frac{x}{V_W}\right) = A \sin\left[\frac{2\pi}{T}t + \frac{2\pi}{\lambda}x\right]$$

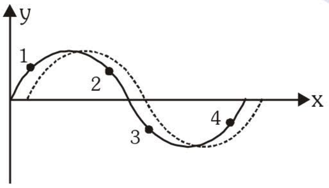
Here $y(x,\ t)$ = Displacement of medium particle   A = amplitude

ωt = time dependent phase,   kx = position dependent phase ϕ = Initial phase

k = propagation constant   T = time period   f = frequency

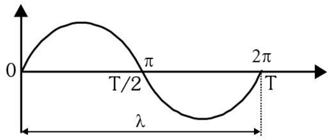
λ = wavelength V_W = wave speed $1/\lambda$ = wave number

- Differential equation : $\dfrac{\partial^2 y}{\partial x^2} = \dfrac{1}{V_w^2}\dfrac{\partial^2 y}{\partial t^2}$
- Wave velocity $V_W = \dfrac{\omega}{k} = \dfrac{\lambda}{T} = f\lambda$
- Particle velocity $V_p = \dfrac{\partial y}{\partial t} = A\omega\cos(\omega t - kx)$ $\boxed{V_p = -V_W \times \text{slope} = -V_W\left(\dfrac{\partial y}{\partial x}\right)}$
- Particle acceleration : $a_p = \dfrac{\partial^2 y}{\partial t^2} = -\omega^2 A \sin(\omega t - kx) = -\omega^2 y$



- For particle 1 : $v_p \downarrow$ and $a_p \downarrow$
- For particle 2 : $v_p \uparrow$ and $a_p \downarrow$
- For particle 3 : $v_p \uparrow$ and $a_p \uparrow$
- For particle 4 : $v_p \downarrow$ and $a_p \uparrow$

- Relation between phase difference, path difference & time difference



$$\boxed{\frac{\Delta\phi}{2\pi} = \frac{\Delta x}{\lambda} = \frac{\Delta t}{T}}$$

- **Speed of transverse wave on string :**

  $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T\ell}{M}} = \sqrt{\frac{T}{\rho\pi r^2}}$, where μ = mass/length, T = tension in the string, ρ = density, r = radius of wire, M = mass of string. Expression derived for the velocity of wave always given velocity w.r.t. medium not w.r.t. ground.

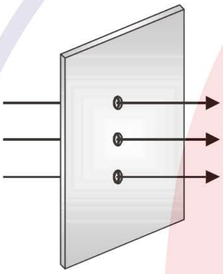
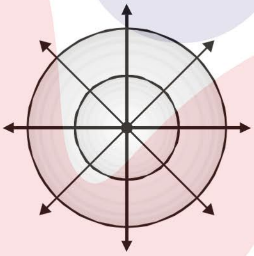
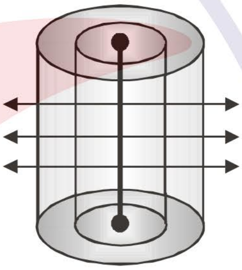
- **Energy density :** U = [Average total energy / volume] = $\frac{1}{2}\rho\omega^2A^2$

- **Power :** $P = \left(\frac{1}{2}\rho\omega^2A^2V_W(S)\right)$ [where S = Area of cross-section]

- **Intensity :** $I = \frac{\text{Power}}{\text{Area of cross-section}} = \frac{1}{2}\rho\omega^2A^2V_W$

WAVEFRONT

- An imaginary surface on which waves incident perpendicular & in same phase.

Wave front	**Plane**	**Spherical**	**Cylindrical**
			
Source	Source at infinite distance Sun, Torch	Point source (Bulb, small siren)	Linear source (tubelight)
Area of wavefront	$A = \ell \times b$ A = const.	$A = 4\pi r^2$ $A \propto r^2$	$A = 2\pi rl$ $A \propto r$
$I \propto \frac{1}{\text{Area}} \propto a^2$	I = constant	$I \propto \frac{1}{r^2} \propto a^2$	$I \propto \frac{1}{r} \propto a^2$
Amplitude variation	a = constant	$a \propto \frac{1}{r}$	$a \propto \frac{1}{\sqrt{r}}$

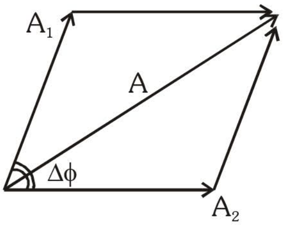
INTERFERENCE

When two coherent waves of same frequency propagate in same direction and superimpose on one another then the intensity of resultant wave becomes maximum at some points and at some points it becomes minimum. This phenomena of intensity variation w.r.t. position is known as interference.

Mathematical Analysis : At a time t, at point x two waves of equal frequency

$y_1 = A_1\sin(\omega t + \phi_1)$, $y_2 = A_2\sin(\omega t + \phi_2)$

Resultant wave :



$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\Delta\phi)}$$

as $I \propto A^2$

$$\boxed{I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos(\Delta\phi)}$$

where $\Delta\phi$ = Phase difference = $k(x_2 - x_1) = k\Delta x$

Δx = path difference

A = Resultant amplitiude

- **For constructive interference [Maximum intensity]**

  $\Delta\phi = 2n\pi$ or path difference $= n\lambda$ where $n = 0, 1, 2, 3, ...$

  $$\boxed{I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2;\ A_{max} = A_1 + A_2;\ I_{max} \propto (A_1 + A_2)^2}$$

- **For destructive interference [Minimum Intensity]**

  $\Delta\phi = (2n+1)\pi$ or path difference $= (2n+1)\dfrac{\lambda}{2}$ where $n = 0, 1, 2, 3, ...$

  $$\boxed{I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2;\ A_{min} = A_1 \sim A_2;\ I_{min} \propto (A_1 - A_2)^2}$$

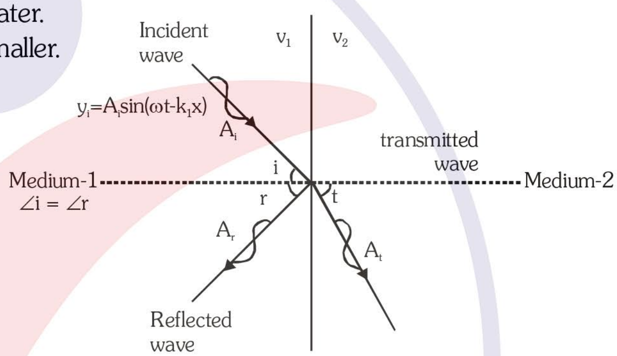
- Average intensity of interference pattern $I_{avg} = \dfrac{I_{max} + I_{min}}{2} = I_1 + I_2$

- Degree of Interference pattern $(f) = \left(\dfrac{I_{max} - I_{min}}{I_{max} + I_{min}}\right) \times 100\%$

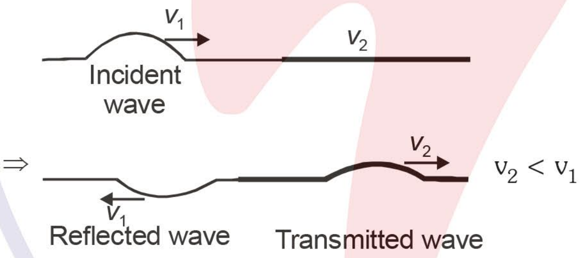
REFLECTION AND REFRACTION (TRANSMISSION) OF WAVES

Rarer Medium : A medium in which speed of wave is greater.

Denser Medium : A medium in which speed of wave is smaller.

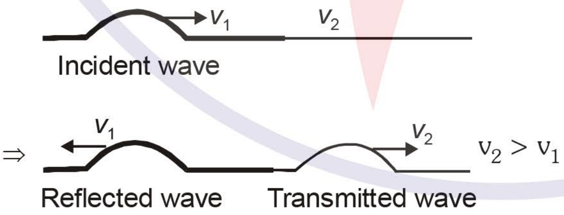


- The frequency of the wave remain unchanged.
- Amplitude of reflected wave $A_r = \left(\dfrac{v_2 - v_1}{v_1 + v_2}\right)A_i$
- Amplitude of transmitted wave $A_t = \left(\dfrac{2v_2}{v_1 + v_2}\right)A_i$
- Wave is moving from rarer to denser medium.



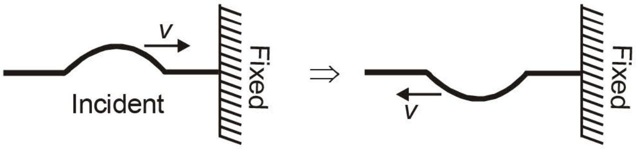
Reflection from denser medium gives phase change of π

- Wave is moving from denser to rarer medium



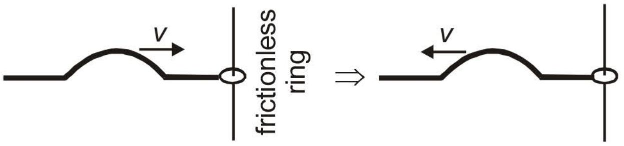
Reflection from rarer medium gives no phase change. The transmitted wave is always in phase with incident wave.

- Reflection from fixed end :



Phase change of π

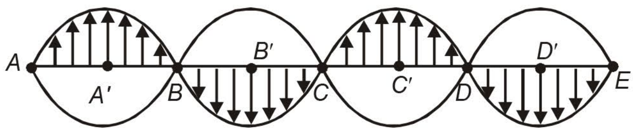
- Reflection from free end :



No phase change

STANDING WAVES

- When two waves of same frequency and amplitude travel in opposite direction at same speed, their superposition gives rise to a new type of wave, called stationary waves or standing waves.



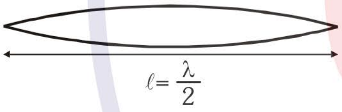
A, B, C, D & E are nodes.
A′, B′, C ′ & D′ are antinodes.

- Formation of standing wave is possible only in bounded medium.
- Let two waves are $y_1 = A\sin(\omega t - kx)$; $y_2 = A\sin(\omega t + kx)$

  By principle of superposition $\vec{y} = \vec{y}_1 + \vec{y}_2 = 2A\cos kx \sin\omega t$ ← Equation of stationary wave
- Its amplitude is not constant but varies periodically with position.
- **Nodes** → amplitude is minimum : $\cos kx = 0 \Rightarrow x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, ...$
- **Antinodes** → amplitude is maximum : $\cos kx = 1 \Rightarrow x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, ...$
- Distance between consecutive nodes = distance between consecutive antinodes = $\frac{\lambda}{2}$.
- Distance between adjacent node and antinodes = $\frac{\lambda}{4}$.
- The nodes divide the medium into segments (loops). All the particles in a segment vibrate in same phase but in opposite phase with the particles in the adjacent segment.
- As nodes are permanently at rest, so no energy can be transmitted across them, i.e. energy of one region (segment) is confined in that region.

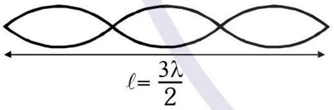
TRANSVERSE STATIONARY WAVES IN STRETCHED STRING

- **Fixed at both ends** **[fixed end →Node & free end→Antinode]**

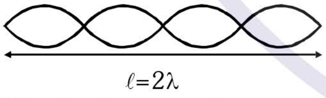


Fundamental or first harmonic $f = \frac{v}{2\ell}$

second harmonic first overtone $f = \frac{2v}{2\ell}$



third harmonic second overtone $f = \frac{3v}{2\ell}$



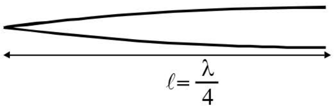
fourth harmonic third overtone $f = \frac{4v}{2\ell}$

All Harmonics are present

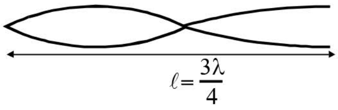
n^{th} overtone = $\frac{(n+1)V}{2\ell}$

n^{th} harmonic = $\frac{(n)V}{2\ell}$

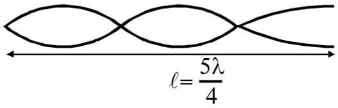
- **Fixed at one end**



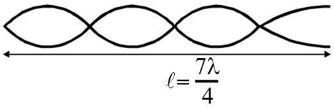
Fundamental $f = \frac{v}{4\ell}$



third harmonic first overtone $f = \frac{3v}{4\ell}$



fifth harmonic second overtone $f = \frac{5v}{4\ell}$



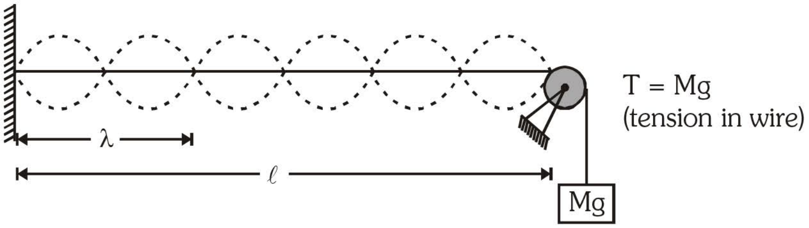
seventh harmonic third overtone $f = \frac{7v}{4\ell}$

Only Odd Harmonics are present

n^{th} overtone = $\frac{(2n+1)V}{4\ell}$

n^{th} harmonic = $\frac{(n)V}{4\ell}$

Sonometer : In this case, transverse stationary waves are formed.



- The wire vibrates in n loops, then $f = \frac{n}{2\ell}\sqrt{\frac{T}{\mu}}$
- $f \propto \sqrt{T}$ (Law of tension) $f \propto \frac{1}{\ell}$ (Law of length) $f \propto \frac{1}{\sqrt{\mu}}$ (Law of mass)
- The point where string is **plucked it is antinode**
- The point where string is **touched it is node.**
- If arm of tuning fork is filed, then its frequency increases.
- If arm of tuning fork is loaded with wax, then its frequency decreases.

SOUND WAVES

- Displacement and pressure wave

  A sound wave can be described either in terms of the longitudinal displacement suffered by the particles of the medium (called displacement wave) or in terms of the excess pressure generated due to compression and rarefaction (called pressure wave).

  Displacement wave $y = A\sin(\omega t - kx)$

  Pressure wave $p = p_0\cos(\omega t - kx)$

  where $p_0 = ABk = \rho A v \omega$

  A = displacement of amplitude, B = Bulk modulus, k = propagation constant,

  p_0 = Amplitude of pressure wave, p = Excess pressure in sound wave
- As sound-sensors (e.g., ear or mike) detect pressure changes, description of sound as pressure wave is preferred over displacement wave.
- The pressure wave is 90° out of phase w.r.t. displacement wave, i.e. displacement will be maximum when pressure is minimum and vice-versa.

INTENSITY OF SOUND

- Intensity in terms of pressure amplitude $I = \frac{p_0^2}{2\rho v}$
- Sound level in dB $= 10\log_{10}\left(\frac{I}{I_0}\right)$
- Where I_0 = threshold intensity of human ear = 10^{-12} W/m^2

CHARACTERISTICS OF SOUND

- Loudness → Sensation received by the ear due to intensity of sound.
- Pitch → Sensation received by the ear due to frequency of sound.
- Quality (or Timbre)→ Sensation received by the ear due to waveform of sound.

SPEED OF SOUND IN DIFFERENT MEDIA

(1) **For solid medium :** $V_{solid} = \sqrt{\frac{Y}{\rho}}$, where Y = Young's modulus and ρ = Density of medium

(2) **For liquid :** $V_{Liquid} = \sqrt{\frac{B}{\rho}}$, where B = Bulk modulus or Modulus of elasticity

(3) For gaseous medium : Propagation of sound in a gas is an **adiabatic process**.

$$\boxed{v_{gas} = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M_w}}}$$ where $B = \gamma P$ = Adiabatic elasticity of gas

M_W = Molecular weight or molar mass

e.g. soft iron $v_{solid} = 5150$ m/sec > For water $v_{Liquid} = 1450$m/s > For air $v_{gas} = 330$ m/s

(4) Factors effecting the speed of sound

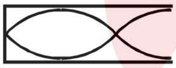
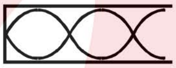
- **Effect of temperature** $v \propto \sqrt{T} \Rightarrow v_t = v_0\left[1+\frac{t}{273}\right]^{\frac{1}{2}}$

  Where
  v_t = velocity of wave at t°c
  v_0 = velocity of wave at 0°c
- **Effect of pressure :** No effect (if temperature is constant)
- **Effect of moisture** $\rho_{\text{moist air}} < \rho_{\text{dry air}}$, $v \propto \frac{1}{\sqrt{\rho}}$, [velocity in moist air is greater than velocity in dry air].
- **Effect of speed of medium –** If the direction of sound wave (v) and air medium is same than (v + w) and in case of opposite directions (v – w).
- **Effect of frequency** – There is no effect of frequency on the speed of sound, known as non dispersiveness.
- Relation between speed of sound and R.M.S. value of gas particle $v_{Sound} = \sqrt{\frac{\gamma}{3}}\, v_{rms}$

VIBRATIONS OF ORGAN PIPES

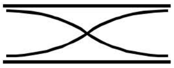
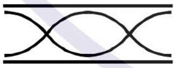
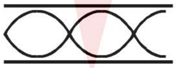
Closed end → displacement node, pressure antinode
Open end→ displacement antinode, pressure node

- **Closed end organ pipe**

		
$\ell = \frac{\lambda}{4} \Rightarrow f = \frac{v}{4\ell}$	$\ell = \frac{3\lambda}{4} \Rightarrow f = \frac{3v}{4\ell}$	$\ell = \frac{5\lambda}{4} \Rightarrow f = \frac{5v}{4\ell}$
Fundamental	Ist overtone	2nd overtone
Mode	3rd harmonic	5th harmonic

- Only odd harmonics are present
- Frequency of mth overtone $= (2m+1)\frac{v}{4\ell}$
- Frequency of mth harmonic $= (m)\frac{v}{4\ell}$

- **Open end organ pipe**

		
$\ell = \frac{\lambda}{2} \Rightarrow f = \frac{v}{2\ell}$	$\ell = \lambda \Rightarrow f = \frac{2v}{2\ell}$	$\ell = \frac{3\lambda}{2} \Rightarrow f = \frac{3v}{2\ell}$
Fundamental	Ist overtone	2nd overtone
Mode	2nd harmonic	3rd harmonic

- All harmonics are present
- Frequency of mth overtone $= (m+1)\frac{v}{2\ell}$
- Frequency of mth harmonic $= \frac{mv}{2\ell}$

- **End correction** :

  Due to finite momentum of air molecules in organ pipes reflection takes place not exactly at open end but some what above it, so antinode is not formed exactly at free end but slightly above it.

  In closed organ pipe $f_0 = \frac{v}{4(\ell+e)}$ where e = 0.6 R (R=radius of organ pipe)

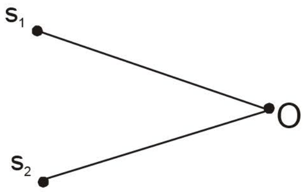
  In open organ pipe $f_0 = \frac{v}{2(\ell+2e)}$

INTERFERENCE IN TIME : BEATS

When two sound waves of same amplitude and different frequencies superimpose, then intensity at any point in space varies periodically with time. This effect is called beats.

If the equation of the two interfering sound waves emitted by s_1 and s_2 at point O are,

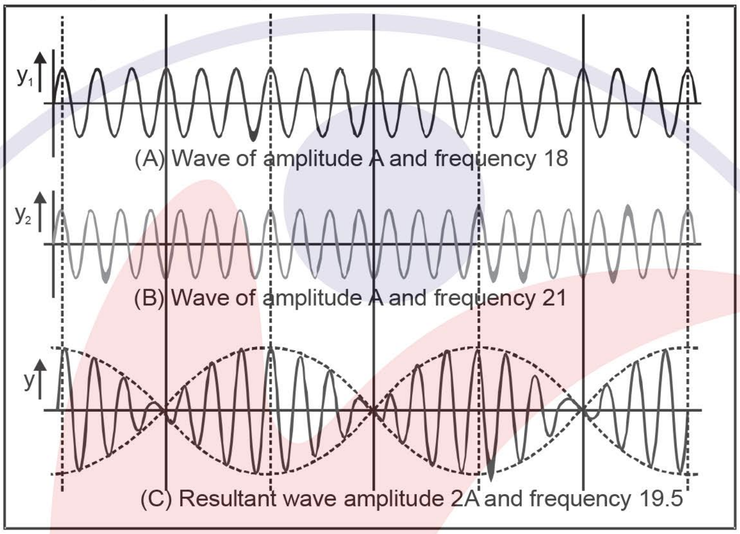
$p_1 = p_0 \sin(2\pi f_1\, t)$ $\qquad \left\{\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)\right\}$



$p_2 = p_0 \sin(2\pi f_2\, t)$

By principle of superposition

$p = p_1 + p_2 = 2p_0 \cos\{\pi(f_1 - f_2)t\} \sin\{\pi(f_1 + f_2)\, t\}$

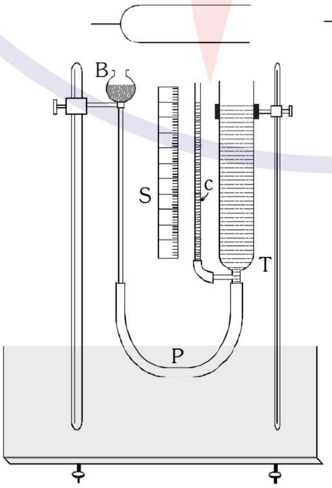


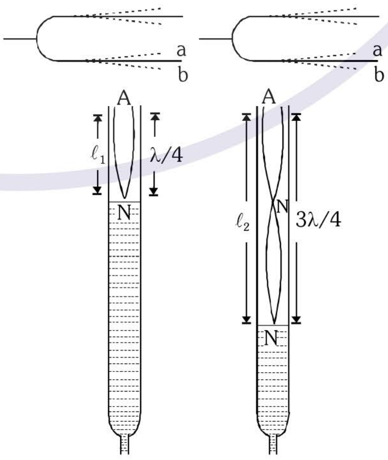
- The resultant sound at point O has frequency $\left(\frac{f_1 + f_2}{2}\right)$
- Pressure amplitude at point O varies with time with a frequency of $\left(\frac{f_1 - f_2}{2}\right)$.
- Sound intensity will vary with a frequency $f_1 - f_2$. This frequency is called beat frequency (f_B)
- The time interval between two successive intensity maxima (or minima) is called beat time period (T_B)

$$f_B = f_1 \sim f_2 \qquad T_B = \frac{1}{f_1 \sim f_2}$$

- The Beat frequency should be less than 10 Hz, for it to be audible.

RESONANCE TUBE



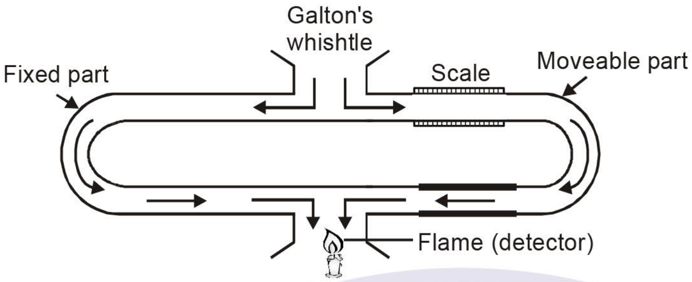


Wavelength $\lambda = 2(\ell_2 - \ell_1)$

End correction $e = \frac{\ell_2 - 3\ell_1}{2}$

QUINCKE'S TUBE

- Quincke tube is practical method for finding the speed of sound in gaseous medium



- As we slide movable part of tube by ℓ unit, path difference will become 2ℓ so $\Delta x = 2\ell$
- If vibration in flame become **maximum** to **minimum** or **minimum** to **maximum** $\Delta x \Rightarrow \frac{\lambda}{2} = 2\ell$
- If vibration in flame become **minimum** to **minimum** or **maximum** to **maximum** then $\Delta x \Rightarrow \lambda = 2\ell$

DOPPLER'S EFFECT IN SOUND

- The apparent change in frequency or pitch due to relative motion of source and observer along the line of sight is called Doppler Effect.
- Observed frequency $\boxed{f' = \left(\frac{v \pm v_m \pm v_0}{v \pm v_m \mp v_s}\right)f}$ v_0 = velocity of observer, v_s = velocity of source, v_m = velocity of medium

  f' = Apparent frequency & f = frequence of source
- If source and observer approach each other, observed frequency increases.
- If source and observer move away from each other, observed frequency decreases.

SPECIAL CASES

Case-I If medium moves in a direction opposite to the direction of propagation of sound, then

$$f' = \left(\frac{v - v_m \pm v_O}{v - v_m \pm v_S}\right)f$$

Case-II Source in motion towards the observer. Both medium and observer are at rest.

$$f' = \left(\frac{v}{v - v_S}\right)f; \quad \text{Clearly} \quad f' > f$$

Case-III Source in motion away from the observer. Both medium and observer are at rest.

$$f' = \left(\frac{v}{v + v_S}\right)f; \quad \text{Clearly} \quad f' < f$$

Case-IV Observer in motion towards the source. Both medium and source are at rest.

$$f' = \left(\frac{v + v_O}{v}\right)f; \quad \text{Clearly} \quad f' > f$$

Case-V Observer in motion away from the source. Both medium and source are at rest.

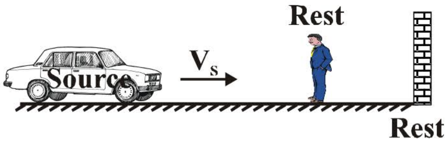
$$f' = \left(\frac{v - v_O}{v}\right)f; \quad \text{Clearly} \quad f' < f$$

Case-VI Both source and observer are moving away from each other. Medium at rest.

$$f' = \left(\frac{v - v_O}{v + v_S}\right)f; \quad \text{Clearly} \quad f' < f$$

Case-VII When source moves towards stationary target.

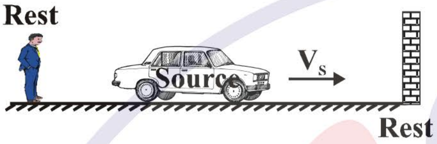
f'_D = direct apparent frequency, f' = apparent frequency at reflector, f'_R = reflected apparent frequency

- 

$$n'_D = \left(\frac{v}{v - v_S}\right)n$$

$$n' = \left(\frac{v}{v - v_S}\right)n$$

$$n'_R = n' \Rightarrow \text{Beats} = \Delta n = n'_R - n'_D = 0$$

- 

$$n'_D = \left(\frac{v}{v + v_S}\right)n$$

$$n' = \left(\frac{v}{v - v_S}\right)n$$

$$\text{Beats} = \Delta n = \left(\frac{v}{v - v_S} - \frac{v}{v + v_S}\right)n = \frac{2v_S vn}{v^2 - v_S^2} \cong \left(\frac{2v_S}{v}\right)n$$

DOPPLER'S EFFECT IN LIGHT :

Doppler effect holds also for EM waves. As speed of light is independent of relative motion between source and observer, the formula are different from that of sound. Here when either source or observer (detector) or both are in motion, only two cases are possible (approach or recession)

Case I : In case of approach

$$\text{Frequency} \quad \nu' = \left(\sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}}\right)\nu \approx \left(1 + \frac{v}{c}\right)\nu$$

$$\text{Wavelength} \quad \lambda' = \left(\sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}}\right)\lambda \approx \left(1 - \frac{v}{c}\right)\lambda$$

$\Big\}$ Blue Shift $\Delta\lambda = \lambda' - \lambda = -\left(\frac{v}{c}\right)\lambda$

Case II : In case of recession

$$\text{Frequency} \quad \nu' = \left(\sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}}\right)\nu \approx \left(1 - \frac{v}{c}\right)\nu$$

$$\text{Wavelength} \quad \lambda' = \left(\sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}}\right)\lambda \approx \left(1 + \frac{v}{c}\right)\lambda$$

$\Big\}$ Red Shift $\Delta\lambda = \lambda' - \lambda = +\left(\frac{v}{c}\right)\lambda$

Illustration A progressive wave of frequency 500 Hz is travelling with a velocity of 360 m/s. How far apart are two points 60° out of phase.

Solution We know that for a wave $v = f\lambda$ So $\lambda = \frac{v}{f} = \frac{360}{500} = 0.72$ m

Phase difference $\Delta\phi = 60^\circ = (\pi/180) \times 60 = (\pi/3)$ rad,

so path difference $\Delta\lambda = \frac{\lambda}{2\pi}(\Delta\phi) = \frac{0.72}{2\pi} \times \frac{\pi}{3} = 0.12$ m

Illustration A man generates a symmetrical pulse in a string by moving his hand up and down. At $t = 0$ the point in his hand moves downward. The pulse travels with speed of 3 m/s on the string & his hands passes 6 times in each second from the mean position. Then the point on the string at a distance 3m will reach its upper extreme first time at time t =

Solution Frequency of wave $= \frac{6}{2} = 3 \Rightarrow T = \frac{1}{3}s\,;\ \lambda = VT = (3)\left(\frac{1}{3}\right) = 1m$

Total time taken $= \frac{3}{3} + \frac{3T}{4} = 1.25$ sec

Illustration The equation of a wave is, $y(x,t) = 0.05\sin\left[\frac{\pi}{2}(10x - 40t) - \frac{\pi}{4}\right]m$

Find : (a) The wavelength, the frequency and the wave velocity

(b) The particle velocity and acceleration at x=0.5 m and t=0.05 s.

Solution (a) The equation may be rewritten as, $y(x,t) = 0.05\sin\left(5\pi x - 20\pi t - \frac{\pi}{4}\right)m$

Comparing this with equation of plane progressive harmonic wave,

$y(x,t) = A\sin(kx - \omega t + \phi)$ we have, wave number $k = \frac{2\pi}{\lambda} = 5\pi\,\text{rad/m}\quad \therefore \lambda = 0.4m$

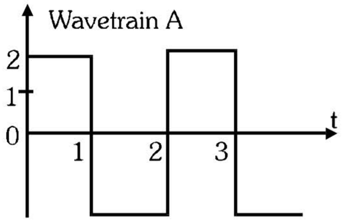
The angular frequency is, $\omega = 2\pi f = 20\pi\,\text{rad/s} \therefore f = 10Hz$

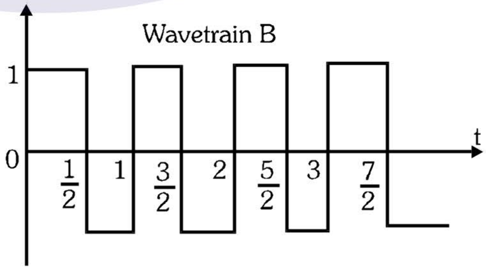
The wave velocity is, $V = f\lambda = \frac{\omega}{k} = 4ms^{-1}$ in $+x$ direction

(b) The particle velocity and acceleration are, $v_p = \frac{\partial y}{\partial t} = -(20\pi)(0.05)\cos\left(\frac{5\pi}{2} - \pi - \frac{\pi}{4}\right) = 2.22m/s$

$$a_p = \frac{\partial^2 y}{\partial t^2} = -(20\pi)^2(0.05)\sin\left(\frac{5\pi}{2} - \pi - \frac{\pi}{4}\right) = 140\ m/s^2$$

Illustration Calculate the ratio of intensity of wavetrain A to wavetrain B.





Solution $\because I \propto a^2n^2 \therefore \frac{I_A}{I_B} = \frac{a_A^2 n_A^2}{a_B^2 n_B^2} = \left(\frac{2}{1}\right)^2 \times \left(\frac{1}{2}\right)^2 = 1$

Illustration Determine the change in volume of 6 liters of alcohol if the pressure is decreased from 200 cm of Hg to 75 cm. [velocity of sound in alcohol is 1280 m/s, density of alcohol = 0.81 gm/cc, density of Hg = 13.6 gm/cc and g = 9.81 m/s^2]

Solution For propagation of sound in liquid $v = \sqrt{(B/\rho)}$ i.e., $B = v^2\rho$

But by definition $B = -V\dfrac{\Delta P}{\Delta V}$ So $-V\dfrac{\Delta P}{\Delta V} = v^2\rho$, i.e. $\Delta V = \dfrac{V(-\Delta P)}{\rho v^2}$

Here $\Delta P = H_2\rho g - H_1\rho g = (75 - 200) \times 13.6 \times 981 = -1.667 \times 10^6$ dynes/cm^2

$$\text{So } \Delta V = \frac{(6\times10^3)(1.667\times10^6)}{0.81\times(1.280\times10^5)^2} = 0.75 \text{ cc}$$

Illustration **(a)** Speed of sound in air is 332 m/s at NTP. What will be the speed of sound in hydrogen at NTP if the density of hydrogen at NTP is (1/16) that of air.

(b) Calculate the ratio of the speed of sound in neon to that in water vapour at any temperature. [Molecular weight of neon = 2.02×10^{-2} kg/mol and for water vapours = 1.8×10^{-2} kg/mol]

Solution The velocity of sound in air is given by $v = \sqrt{\dfrac{E}{\rho}} = \sqrt{\dfrac{\gamma P}{\rho}} = \sqrt{\dfrac{\gamma RT}{M_w}}$

(a) In terms of density and pressure $\dfrac{v_H}{v_{air}} = \sqrt{\dfrac{P_H}{\rho_H}\times\dfrac{\rho_{air}}{P_{air}}} = \sqrt{\dfrac{\rho_{air}}{\rho_H}}$ [as $P_{air} = P_H$]

$$\Rightarrow \quad v_H = v_{air}\times\sqrt{\frac{\rho_{air}}{\rho_H}} = 332\times\sqrt{\frac{16}{1}} = 1328 \text{ m/s}$$

(b) In terms of temperature and molecular weight $\dfrac{v_{Ne}}{v_W} = \sqrt{\dfrac{\gamma_{Ne}}{M_{Ne}}\times\dfrac{M_W}{\gamma_W}}$ [as $T_{Ne} = T_W$]

Now as neon is mono atomic ($\gamma = 5/3$) while water vapours poly atomic ($\gamma = 4/3$) so

$$\frac{v_{Ne}}{v_W} = \sqrt{\frac{(5/3)\times1.8\times10^{-2}}{(4/3)\times2.02\times10^{-2}}} = \sqrt{\frac{5}{4}\times\frac{1.8}{2.02}} = 1.055$$

Illustration In interference phenomena if the degree of interference pattern in interference is 60% then find the ratio of intensity & amplitudes of interferring wave form.

Solution. $\dfrac{I_{max} - I_{min}}{I_{max} + I_{min}} = \dfrac{60}{100} = \dfrac{3}{5}$ By C & D $\dfrac{I_{max}}{I_{min}} = \dfrac{5+3}{5-3} = \dfrac{4}{1}$

Thus $\dfrac{a_1 + a_2}{a_1 - a_2} = \dfrac{2}{1}$ & $\dfrac{a_1}{a_2} = \dfrac{2+1}{2-1} = \dfrac{3}{1}$ thus $\dfrac{I_1}{I_2} = \dfrac{9}{1}$ Ans.

Illustration T.F. having n = 300 Hz produces 5 beats/sec. with another T.F. If impurity is added on the arm of known tuning fork number of beats decreases then find frequency of unknown T.F. ?

Solution. 295 $\underset{\text{wax is added}}{\xleftarrow{300}}$ 305

If it would be 305 Hz, beats would have increased but with 295 Hz beats decreases so answer is 295 Hz.

Illustration A T. F. having n = 158 Hz, produce 3 beats/sec. with another T. F. As we file the arm of unknown, beats become 7 then find frequency of unknown.

Solution. before filling 158 ± 3 so 155 or 161

after filling b = 7

155 158 161 165 (after filling)

165 (after filling)

filling filling

Both T.F. give 7 beat/sec. after filling. So answer is both.

Illustration 41 tuning forks are arranged in a series in such a way that each T.F. produce 3 beats with its neighbouring T.F. If the frequency of last is 3 times of first then find the frequency of 1st 11th 16th 21st & last T.F.

Solution. $n_1 = n$ (let) So $n_{41} = 3n$ (according to Que.)

$n_2 = n + b$

$n_3 = n + 2b$ So $n_{41} = n + 40 \times 3$

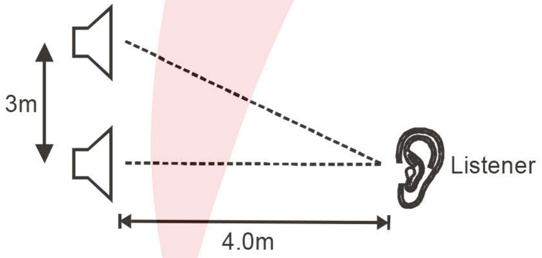
$n_4 = n + 3b$ $3n = n + 120$

$n_{41} = n + 40\ b$ $n = 60$ Hz

$n_{11} = n + 10\ b = 90$ Hz, $n_{16} = n + 15\ b = 105$ Hz

$n_{21} = n + 20b = 120$ Hz

Illustration Two loudspeakers as shown in fig. below separated by a distance 3 m, are in phase. Assume that the amplitudes of the sound from the speakers is approximately same at the position of a listener, Who is at a distance 4.0 m in front of one of the speakers. For what frequencies does the listener hear minimum signal ? Given that the speed of sound in air is 330 ms^{-1}.



Solution The distance of the listener from the second speaker = $\sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$ m

path difference = (5 – 4.0) m = 1 m

For fully destructive interference 1 m = (2m + l)λ/2

Hence λ = 2/(2m + 1) m

The corresponding frequencies are given by

$n = [330 \times (2m + 1)]/2\ s^{-1}$, for m = 0, 1, 2, 3, 4, ...

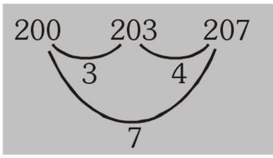
$= 165\ (2m + 1)\ s^{-1}$, for m = 0, 1, 2, 3, 4, ...

Therefore the frequencies for which the listener would hear a minimum intensity 165 Hz. 495 Hz, 825 Hz, ...

Illustration Three tuning forks of frequencies 200, 203 and 207 Hz are sounded together. Find out the beat frequency.

Solution

$$\frac{1}{3} \quad \frac{2}{3} \left(\frac{3}{3}\right)$$



Divide 1 second into 3, 4 or 7 equal divisions

$$\frac{1}{4} \quad \frac{2}{4} \quad \frac{3}{4} \left(\frac{4}{4}\right)$$

$$\frac{1}{7} \quad \frac{2}{7} \quad \frac{3}{7} \quad \frac{4}{7} \quad \frac{5}{7} \quad \frac{6}{7} \left(\frac{7}{7}\right)$$

Eliminate common time instants. Total Maxima in one second 3 + 3 + 6 = 12

$\Rightarrow$ 12 beats per second

Illustration Two tuning forks A and B produce 8 beats/s when sounded together. A gas column 37.5 cm long in a pipe closed at one end resonate to its fundamental mode with fork A whereas a column of length 38.5 cm of the same gas in a similar pipe is required for a similar resonance with fork B. Calculate the frequency of these two tuning forks. **[AIPMT 2006]**

Solution

$$\left.\begin{aligned} &\text{For tuning fork 'A'} \quad \frac{\lambda_1}{4} = 37.5 \quad \text{so } n_1 = \frac{v}{\lambda_1} = \frac{v}{4\times 37.5} \\ &\text{For tuning fork 'B'} \quad \frac{\lambda_2}{4} = 38.5 \quad \therefore n_2 = \frac{v}{\lambda_2} = \frac{v}{4\times 38.5} \end{aligned}\right\}$$

$$\therefore n_1 - n_2 = 8 \Rightarrow \frac{v}{4\times 37.5} - \frac{v}{4\times 38.5} = 8 \quad \therefore \quad v = (8\times 4\times 37.5\times 38.5)$$

$$n_1 = \frac{8\times 4\times 37.5\times 38.5}{4\times 37.5} = 308 \text{ Hz} \quad \text{and} \quad n_2 = 308 - 8 = 300 \text{ Hz}$$

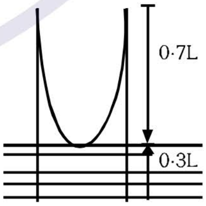
Illustration A transverse wave, travelling along the positive x-axis, given by $y=A\sin(kx-\omega t)$ is superposed with another wave travelling along the negative x-axis given by $y=-A\sin(kx+\omega t)$. The point x=0 is

Solution At $x=0$, $y_1 = A\sin(-\omega t)$ and $y_2 = -A\sin\omega t$; $y_1 + y_2 = -2A\sin\omega t$ (antinode)

Illustration If an OOP (open organ pipe) of fundamental frequency 1400 Hz dipped 30% in water then calculate produced frequency.

Solution

$$\frac{v}{2L} = 1400 \text{ Hz} \qquad \Rightarrow \qquad \frac{v}{L} = 2800 \text{ Hz}$$



$$n' = \frac{v}{4L'} = \frac{v}{4[0\cdot 7L]} = \frac{v}{2\cdot 8L} = \frac{2800}{2\cdot 8} = 1000 \text{ Hz}$$

Illustration A string with a mass density of 4×10^{-3} kg/m is under tension of 360 N and is fixed at both ends. One of its resonance frequencies is 375 Hz. The next higher resonance frequency is 450 Hz. Find the mass of the string. **[AIPMT 2007]**

Solution

$$n_1 = 375 = \frac{p}{2\ell}\sqrt{\frac{T}{m}} \text{ and } n_2 = 450 = \frac{p+1}{2\ell}\sqrt{\frac{T}{m}} \text{ where p is number of loops}$$

$$\Rightarrow \quad \frac{450}{375} = \frac{p+1}{p} \Rightarrow p = 5$$

$$\text{so} \quad \ell = \frac{p}{2 \times n_1}\sqrt{\frac{T}{m}} = \frac{5}{2 \times 375}\sqrt{\frac{360}{4 \times 10^{-3}}} = 2\text{m}$$

$\Rightarrow$ Mass of wire = (m) (ℓ) = (4×10^{-3}) (2) = 8×10^{-3} kg

OR

Difference between two consecutive resonating frequency $n_2 - n_1 = \frac{1}{2\ell}\sqrt{\frac{T}{m}}$

$$\Rightarrow \quad 450 - 375 = \frac{1}{2\ell}\sqrt{\frac{360}{4 \times 10^{-3}}} \Rightarrow \quad \ell = \frac{1}{2 \times 75}\sqrt{\frac{360}{4 \times 10^{-3}}} = \frac{1}{150} \times \frac{6 \times 10^2}{2} = 2 \text{ m}$$

$\Rightarrow$ Mass of wire = (m) (ℓ) = (4×10^{-3}) (2) = 8×10^{-3} kg

Illustration For given C.O.P. (closed organ pipe) if 9th O.T. (over tone) has frequency 1900 Hz. then fundamental frequency of same length O.O.P. is ?

Solution 19n = 1900 ; n = 100

OOP = 2n = 200 Hz

For same length OOP have double freq. than COP.

Illustration Two C.O.P. having length 20 cm & 20.5 cm produce 5 beat/sec determine the freg of both C.O.P.

Solution $\frac{n}{n+5} = \frac{20}{20.5}$; n = 200 For 20.5 cm

n + 5 = 205 For 20 cm

Illustration Two tuning forks A and B lying on opposite sides of observer 'O' and of natural frequency 85 Hz move with velocity 10 m/s relative to stationary observer O. Fork A moves away from the observer while the fork B moves towards him. A wind with a speed 10 m/s is blowing in the direction of motion of fork A. Find the beat frequency measured by the observer in Hz. [Take speed of sound in air as 340 m/s]

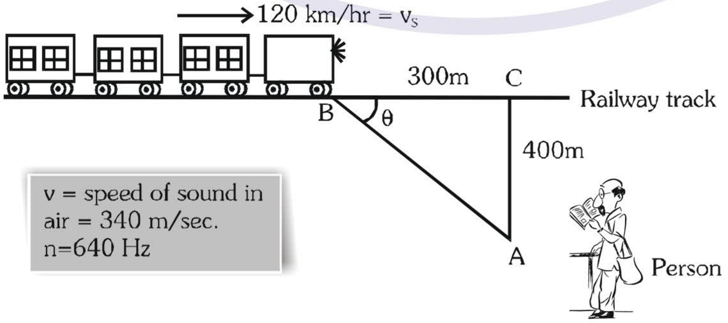
Solution

$$f_{\text{observer for source 'A'}} = f_0\left[\frac{v_{\text{sound}} - v_{\text{medium}}}{v_{\text{sound}} - v_{\text{medium}} + v_{\text{source}}}\right] = \frac{33}{34} f_0 \,;$$

$$f_{\text{observer for source 'B'}} = f_0\left[\frac{v_{\text{sound}} + v_{\text{medium}}}{v_{\text{sound}} + v_{\text{medium}} - v_{\text{source}}}\right] = \frac{35}{34} f_0$$

$$\therefore \text{ Beat frequency} = f_1 - f_2 = \left(\frac{35-33}{34}\right) f_0 = 5$$

Illustration



If engine of train produce horn at B point then find apparent frequency observed by observer at A point.

Solution $n' = \left(\dfrac{v}{v - v_s \cos\theta}\right) n$; Direction in AB velocity

$$= v_s \cos\theta = \left[120 \times \frac{5}{18}\right] \times \cos\theta = 120 \times \frac{5}{18} \times \frac{3}{5} = 20 \text{ m/sec.}$$

$$\Rightarrow \quad n' = \left(\frac{340}{340 - 20}\right) \times 640 = 680 \text{ Hz.}$$

Illustration A SONAR system fixed in a submarine operates at a frequency 40.0 kHz. An enemy submarine moves towards the SONAR with a speed of 360 km h^{-1}. What is the frequency of sound reflected by the submarine ? Take the speed of sound in water to be 1450 ms^{-1}.

Solution As the sound is observed by enemy subsmarine.

Here observer (enemy submarine) is moving towards the source (SONAR)

$\therefore$ Apparent frequency $n' = \left(\dfrac{v + v_0}{v - v_s}\right) n = \left(\dfrac{1450 + 100}{1450}\right) \times 40 \times 10^3 = \dfrac{1550}{1450} \times 40 \times 10^3$ Hz

After the sound is reflected, enemy submarine acts. as a source of frequency n'. This source moves with a speed of 100 ms^{-1} towards the observer (SONAR)

$\therefore$ Apparent frequency of sound reflected by the enemy submarine

$$n'' = \left(\frac{v - v_0}{v - v_s}\right) n' = \left(\frac{1450 - 0}{1450 - 100}\right) \times \left(\frac{1550}{1450} \times 40 \times 10^3\right) = 45.93 \text{ kHz}$$

Illustration Two trains travelling in opposite directions at 126 km/hr each, cross each other while one of them is whistling. If the frequency of the note is 2.22 kHz find the apparent frequency as heard by an observer in the other train :

(a) Before the trains cross each other, (b) After the trains have crossed each other. (v_{sound} = 335 m/sec)

Solution Here $v_1 = 126 \times \dfrac{5}{18} = 35$ m/s

(i) In this situation $\circ\!\longrightarrow v_1 \qquad v_1 \longleftarrow\!\circ$

Observed freq $n' = \left(\dfrac{v + v_1}{v - v_1}\right) \times n = \left(\dfrac{335 + 35}{335 - 35}\right) \times 2220 = 2738$ Hz

(ii) In this situation $v_1 \longleftarrow\!\circ \qquad \circ\!\longrightarrow v_1$

Observed freq $n' = \left(\dfrac{v - v_1}{v + v_1}\right) \times n = \left(\dfrac{335 - 35}{335 + 35}\right) \times 2220 = 1800$ Hz

Illustration A star which is emitting radiation at a wavelength of 5000 Å, is approaching the earth with a velocity of $1{\cdot}5 \times 10^3$ m/s. Calculate the change in wavelength of the radiation as received by the earth.

Solution $\Delta\lambda = \dfrac{v}{c}\lambda = \dfrac{1.5 \times 10^3}{3 \times 10^8} \times 5000 = 0.025$Å