THERMAL PHYSICS

TEMPERATURE, THERMAL EXPANSION

- **Zeroth law of thermodynamics**

  If objects A and B are separately in thermal equilibrium with a third object C (say thermometer), then objects A and B are in thermal equilibrium with each other. Zeroth law of thermodynamics introduce the concept of temperature.

- **Comparison between Different Temperature Scales**

$$\boxed{\frac{K-273}{100}=\frac{C}{100}=\frac{F-32}{180}=\frac{X-LFP}{UFP-LFP}}$$

☞ **THERMAL EXPANSION**

When matter is heated without any change in its state, it usually expands.

- Solids can expand in one dimension (Linear expansion), two dimensions (Superficial expansion) and three dimensions (Volumetric expansion) while liquids and gases usually suffers change in volume only.
- **Linear expansion :** $\ell = \ell_0 (1 + \alpha\Delta\theta)$
- **Superficial (areal) expansion :** $A = A_0 (1 + \beta\Delta\theta)$
- **Volumetric expansion :** $V = V_0 (1 + \gamma\Delta\theta)$ $\boxed{\alpha : \beta : \gamma = 1 : 2 : 3}$

HEAT

Heat required to increase the temperature of a substance is $Q = mcdT$ Here c = Specific heat

Heat required for change of phase is $Q = mL$ Here L = Latent heat

- Value of specific heats of gas can vary from zero (0) to infinity depending on the condition to be heated.
- Generally two types of specific heats are defined for a gas –

  (a) Specific heat at constant volume (C_V) (b) Specific heat at constant pressure (C_P)

  Thermal capacity (Heat capacity)

  The quantity of heat required to raise the temperature of the whole of that substance through 1°C is called thermal capacity. The thermal capacity of substance is = mc

☞ **Effect of change in pressure on M.P. and B.P. for ice/water**

- If P ↑ $\xrightarrow{\text{then result}}$ M.P. ↓ & B.P. ↑ • If P ↓ $\xrightarrow{\text{then result}}$ M.P. ↑ & B.P. ↓

  Law of Mixtures :

  Principle of calorimetry represents the law of conservation of heat energy. $\boxed{\textbf{Heat lost = Heat gained}}$

MODES OF HEAT TRANSFER

- **Conduction**

  The process in which the material takes an active part by molecular action and energy is passed from one particle to another is called conduction. It is predominant in solids.

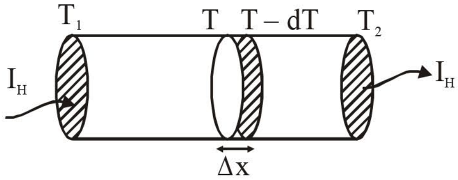
- **Convection**

  The transfer of energy by actual motion of particle of medium from one place to another is called convection. It is predominant in fluids (liquids and gases).

- **Radiation**

  Quickest way of transmission of heat is known as radiation. In this mode of energy transmission, heat is transferred from one place to another without effecting the inter–venning medium.

☞ Thermal conduction



Rate of heat transfer (Heat current) $I_H = \dfrac{Q}{t} = -KA\dfrac{\Delta T}{\Delta x}$

$\boxed{\dfrac{Q}{t} = \dfrac{KA(T_1 - T_2)}{L}}$ K= Thermal conductivity → it is the measure of the ability of material to conduct heat.

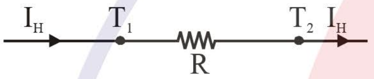
Here K is a constant depending on nature of the material of quantity $\left(\dfrac{dT}{dx}\right)$ is called temperature gradiant.

The (–) sign shows heat flows from high to low temperature (ΔT is a –ve quantity).

Application of Thermal Conduction

- Cooking utensils are made of aluminium and brass whereas their handles are made of wood.
- Ice is covered in gunny bags to prevent melting of ice.
- Two thin blankets are warmer than a single blanket of double the thickness.

Thermal Resistance to conduction



$R = \dfrac{\Delta T}{I_H} = \dfrac{T_1 - T_2}{I_H}$; $R = \dfrac{L}{KA}$

In series combination

$$K_{eq} = \frac{L_1 + L_2}{\dfrac{L_1}{K_1} + \dfrac{L_2}{K_2}} = \frac{\Sigma L_i}{\Sigma \dfrac{L_i}{K_i}}$$

For identical rods

$K_S = \dfrac{2K_1K_2}{K_1 + K_2}$ (Harmonic mean)

In parallel combination

$$K_{eq} = \frac{K_1A_1 + K_2A_2}{A_1 + A_2} = \frac{\Sigma K_iA_i}{\Sigma A_i}$$

For identical rods

$K_P = \dfrac{K_1 + K_2}{2}$ (Arithmatic mean)

☞ Thermal Radiation

The process of the transfer of heat from one place to another place without heating the intervening medium is called radiation.

FUNDAMENTAL DEFINATION

- **Absorptive power or absorptive coefficient 'a' :** $a = \dfrac{Q_a}{Q}$ (unitless)
- **Emissive power (e)** : $e = Q/At$ (watt/m²)
- **Spectral Emmisive power (e_λ)** : Emissive power or total emissive power $e = \int_0^\infty e_\lambda \, d\lambda$ **SI UNIT :** W/m² Å
- **Emissivity (e_r) :** $e_r = \dfrac{Q_{GB}}{Q_{IBB}} = \dfrac{e_{GB}}{E_{IBB}} = \dfrac{\text{emitted radiation by gray body}}{\text{emitted radiation by ideal black body}}$

  GB = gray or general body, IBB = Ideal black body

  (i) No unit (ii) For ideal black body $e_r = 1$ (iii) range $0 < e_r < 1$
- **Prevost's theory of heat energy exchange**

  According to Prevost at every possible temperature (except zero kelvin temperature) there is a continuous heat energy exchange between a body and its surrounding and this exchange carry on for infinite time.
- **KIRCHHOFF'S LAW** :

  At constant temperature

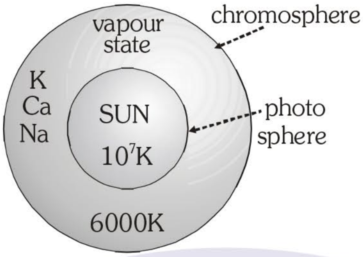
  $\dfrac{e_\lambda}{a_\lambda} = E_\lambda$ = constant $\left[\dfrac{e_\lambda}{a_\lambda}\right]_1 = \left[\dfrac{e_\lambda}{a_\lambda}\right]_2$ = constant hence $e_\lambda \propto a_\lambda$

  Good absorbers are good emitters and bad absorbers are bad emitters

Applications of Kirchoff Law

- **Fraunhoffer's lines**

Fraunhoffer lines are dark lines in the spectrum of the Sun. When white light emitted from the central core of the Sun (Photosphere) passes through its atmosphere (chromosphere) radiations of those wavelengths will be absorbed by the gases present, resulting in dark lines in the spectrum of Sun.



At the time of total solar eclipse direct light rays emitted from photosphere cannot reach on the Earth and only rays from chromosphere are able to reach on the Earth surface. At that time we observe bright fraunhoffer lines.

- **In deserts days are hot and nights are cold**
- **STEFAN'S LAW**

Amount of radiation emitted **$E \propto T^4$**

$E = \sigma T^4$ (This law is true for only ideal black body)

SI Unit : E = watt/m² σ = Stefen's constant = 5.67×10^{-8} watt /m² K⁴ (universal constant)

Dimensions of σ : $M^1 L^0 T^{-3} \theta^{-4}$

Total radiation energy emitted out by surface of area A in time t :

Ideal black body $Q_{IBB} = \sigma A T^4 t$ and for any other body $Q_{GB} = e_r \sigma A T^4 t$

Including the temp of sorrounding (T_0 = temp of surrounding)

$R_H = \dfrac{Q}{t} = \sigma A e_r (\theta^4 - \theta_0^4)$; R_H = Rate of heat loss

$R_F = \dfrac{d\theta}{dt} = \dfrac{\sigma A e_r}{mcJ}(\theta^4 - \theta_0^4)$; R_F = Rate of fall in temp.

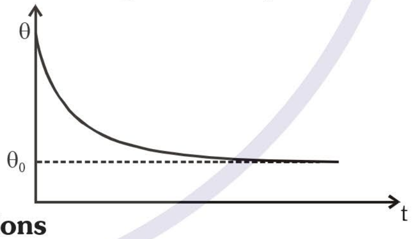
$\dfrac{d\theta}{dt} \propto (\theta - \theta_0)$ [(when $(\theta - \theta_0) \ngtr 35$°C]

Eg. Two identical hollow and solid spheres are cooled in the same sorrounding the solid sphere will cool late.

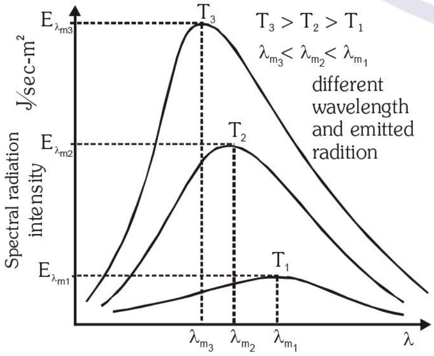
- **NEWTON'S LAW OF COOLING**

For Numerical Problems, Newton's Law of cooling

$$\boxed{\left[\frac{\theta_1 - \theta_2}{t}\right] = +K\left[\frac{\theta_1 + \theta_2}{2} - \theta_0\right]}$$



Spectral Energy distribution curve of Black Body radiations



(i) $\lambda_m \propto \dfrac{1}{T}$

(ii) $E_{\lambda_m} \propto T^5$

(iii) Area $= \displaystyle\int_0^\infty E_\lambda \, d\lambda = E = \sigma T^4$

Wein's Displacement Law $\lambda_m = \dfrac{b}{T}$ where $b = 2.89 \times 10^{-3}$ m–k

- **Solar constant 'S'**

  The Sun emits radiant energy continuously in space of which an in significant part reaches the Earth. The solar radiant energy received per unit area per unit time by a black surface held at right angles to the Sun's rays and placed at the mean distance of the Earth (in the absence of atmosphere) is called solar constant. The value of solar constant is 1340 watt/m^2 or 1.937 cal/cm^2–min.

  $$S = \sigma T^4 \left(\frac{R}{d}\right)^2$$

KINETIC THEORY OF GASES

Intermolecular force Solid > liquid > real gas > ideal gas (zero)

Potential energy Solid < liquid < real gas < ideal gas (zero)

At a given temperature for solid, liquid and gas:

(i) Internal kinetic energy : Same for all

(ii) Internal potential Energy : Maximum for ideal gas (PE = 0) and Minimum for solids (PE = –ve)

(iii) Internal Energy : Maximum for Ideal gas and Minimum for solid

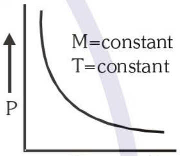
Ideal Gas Concept

- Volume of gas molecules is negligible as compared to volume of container so volume of gas = volume of container (Except 0 K)
- No intermolecular force act between gas molecules.

Equation of state for Ideal gas

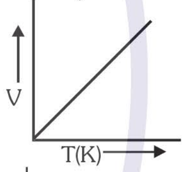
$$PV = \mu RT \Rightarrow PV = \frac{M}{M_w}RT = \left[\frac{R}{N_0}\right] N\,T = NkT \Rightarrow \boxed{\frac{P}{\rho} = \frac{RT}{M_w} = \frac{kT}{m}}$$

Gas Laws



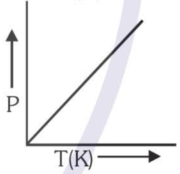
- **Boyle's Law** $V \propto \frac{1}{P}$ if M and T = Constant $\Rightarrow \boxed{P_1V_1 = P_2V_2}$
- **Charle's Law**

  $V \propto T$ if M and P = Constant $\Rightarrow \boxed{\frac{V_1}{V_2} = \frac{T_1}{T_2}}$ Slope of V–T curve = $\frac{\mu R}{P}$



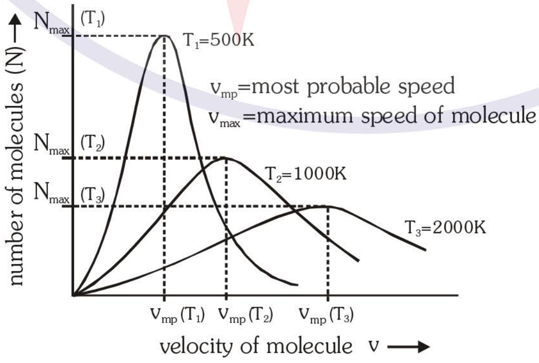
- **Gay–Lussac's Law**

  $P \propto T$ if M and V = constant $\Rightarrow \boxed{\frac{P_1}{P_2} = \frac{T_1}{T_2}}$ Slope of P–T curve = $\frac{\mu R}{V}$



- **Avogadro's Law** $N_1 = N_2$ if P,V and T are same.
- **Dalton's Partial Pressure Mixture Law :** $P = P_1 + P_2 + \ldots$ If V, T → Same
- **Maxwell's law of distribution of velocities**

  Each velocity increases with increase in temp. and no. of molecules corresponding to most probable speed decreases. Area under the curve gives the total number of molecules which is constant at each temperature. i.e. for each curve.



$T_3 > T_2 > T_1$

Degree of freedom (f)

- The number of independent ways in which a molecule or an atom can exhibit motion or have energy is called it's degrees of freedom.
- The degrees of freedom are of three types :

(a) **Translational Degree of freedom :** Maximum three degree of freedom are there corresponding to translational motion.

(b) **Rotational Degree of freedom** : The number of degrees of freedom in this case depends on the structure of the molecule.

(c) **Vibrational Degree of freedom :** It is exhibited at high temperatures.

- For monoatomic gas : f = 3 (T = 3)
  For diatomic gas : f = 5 (T = 3, R = 2)
  For triatomic/polyatomic gas (Linear) : f = 5 (T = 3, R = 2)
  For triatomic/polyatomic gas (non-linear) : f = 6 (T = 3, R = 3)

Maxwell's law of equipartition of energy

The total kinetic energy of a gas molecules is equally distributed among its all degree of freedom and the energy associated with each degree of freedom at absolute temperature T is $\frac{1}{2}kT$

Different K.E. of gas (Internal Energy)

- **Translatory kinetic energy (E_T)** $E_T = \frac{1}{2}Mv_{rms}^2 = \frac{3}{2}PV = \frac{3}{2}\mu RT = \frac{3}{2}\frac{M}{Mw}RT = \frac{3}{2}\frac{N}{N_0}RT$

- **Energy per unit volume or energy density (E_V)** $E_V = \frac{\text{Total energy}}{\text{Volume}} = \frac{E}{V}$; $E_V = \frac{1}{2}\left[\frac{M}{V}\right]v_{rms}^2 = \frac{1}{2}\rho v_{rms}^2$

$$E_{molar} = \frac{3}{2}RT$$

$$E_{gram} = \frac{3}{2}\frac{RT}{Mw}$$

$$E_{molecule} = \frac{3}{2}\frac{RT}{N_0} = \frac{3}{2}kT$$

$$\boxed{\begin{aligned} v_{rms} &= \sqrt{\frac{3RT}{M_w}} = \sqrt{\frac{3kT}{m}} \\ v_{mp} &= \sqrt{\frac{2RT}{M_w}} = \sqrt{\frac{2kT}{m}} \\ v_{av} &= \sqrt{\frac{8RT}{M_w}} = \sqrt{\frac{8kT}{\pi m}} \end{aligned}}$$

for 'f' degree of freedom $E = \frac{f}{2}RT$

Mean freepath → Average distance travelled between two successive collisions

n = no. of molecules per unit volume

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 n} = \frac{V}{\sqrt{2}\pi d^2 N} = \frac{kT}{\sqrt{2}\pi d^2 P}$$

THERMODYNAMIC

- **Work done by thermodynamic system** $W = \int_0^W dW = \int_{V_i}^{V_f} PdV$

☞ **First law of thermodynamics** **$dU = \delta Q - \delta W$ or $\Delta U = Q - W$ Here $\Delta U = \mu C_V \Delta T$ & $Q = \mu C \Delta T$**

Heat supplied to the system and work done by the system are path dependent. Change in internal energy $\Delta U = U_f - U_i$ does not depends on path it depends only on initial and final positions of the system.

☞ **DIFFERENT PROCESSES**

- **Isometric or Isochoric Process** (V = constant or P ∝ T)

  $W = 0$ and $Q = \Delta U = \mu\ C_v \Delta T$

- **Isobaric Process** (P = constant or V ∝ T)

  $Q = \mu C_P \Delta T$; $W = \mu R \Delta T = P(V_f - V_i)$; $\Delta U = \mu C_V \Delta T$

sign convesion

Q = +ve heat is absorbed by the gas

Q = –ve heat is rejected by the gas

W = +ve (expansion)

W = –ve (compression)

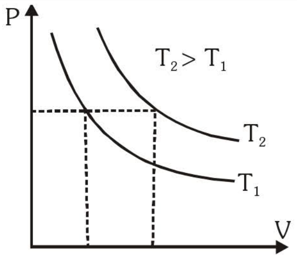
Isothermal Process (T = constant or PV = constant)

Work Done

∴ Work done : $W = \int_{V_1}^{V_2} \frac{\mu RT}{V} dV = \mu RT \log_e\left[\frac{V_2}{V_1}\right] = 2.303\mu RT\ \log_{10}\left[\frac{P_1}{P_2}\right]$ [∵ $P_1V_1 = P_2V_2$]

- Two isotherms for a given gas at two different temperatures T_1 and T_2 are shown in figure

  The curves drawn for the same gas at different temperatures are mutually parallel and do not cut each other.

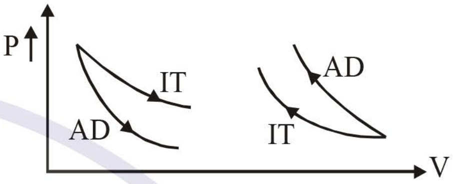


- If changes are executed in a vessel of infinite conductivity then they will be isothermal.
- **Adiabatic Process** ($Q = 0 \Rightarrow W = -\Delta U$)

  Eq^n of state PV^{γ} = const. ; $TV^{\gamma-1}$ = const. ; $P^{1-\gamma}T^{\gamma}$ = const.

$$\left|\frac{dP}{dV}\right|_{adia} = \gamma\frac{P}{V} = \gamma\left|\frac{dP}{dV}\right|_{iso} \Rightarrow \frac{\text{slope of adiabatic changes}}{\text{slope of isothermal changes}} = \gamma$$

Work done $W = \dfrac{\mu R}{(\gamma-1)}(T_1 - T_2)$



Examples of adiabatic process

- If a gas is suddenly expanded by moving the piston outwards, there will be a decrease in the temperature of the gas.
- Propagation of sound waves in a gas.

Relation between degrees of freedom and specific heat of gas

$$C_V = \frac{dU}{dT} = \frac{f}{2}R = \frac{R}{\gamma-1} \cdot C_P = C_V + R = \left[\frac{f}{2}+1\right]R = \frac{\gamma R}{\gamma-1} \text{ and } \gamma = \frac{C_P}{C_V} = 1 + \frac{2}{f}$$

General expression for C (C_P or C_V) in the process PV^x = constant $C = \dfrac{R}{\gamma-1} + \dfrac{R}{1-x}$

For isobaric process P = constant so $x = 0$ $\therefore$ $C = C_P = \dfrac{R}{\gamma-1} + R = C_V + R$

For isothermal process, PV = constant so $x = 1$ $\therefore$ $C = \infty$

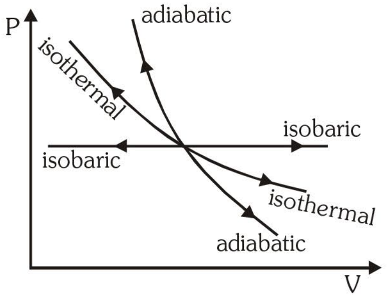
For adiabatic process PV^{γ} = constant so $x = \gamma$ $\therefore$ $C = 0$

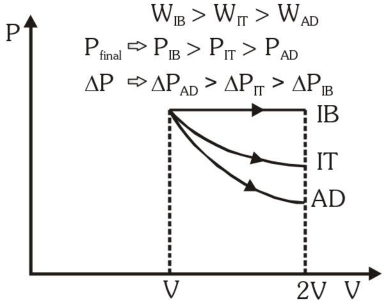
Atomicity of gas	**f**	C_V	C_P	γ
Monoatomic	3	$\frac{3}{2}R$	$\frac{5}{2}R$	$\frac{5}{3} = 1.67$
Diatomic, Triatomic and Triatomic linear (at normal temperature)	5	$\frac{5}{2}R$	$\frac{7}{2}R$	$\frac{7}{5} = 1.4$
Poly atomic Triangular Non-linear	6	$\frac{6}{2}R = 3R$	$\frac{8}{2}R = 4R$	$\frac{4}{3} = 1.33$

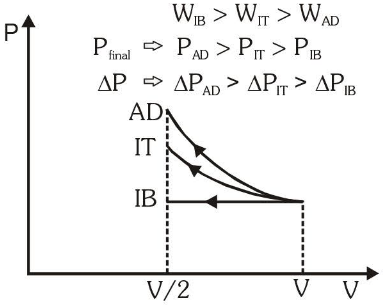
☞ **Why C_P is greater than C_V ?**

In comparision to constant volume, at constant pressure additional heat is required to do the work So, C_P is more than C_V

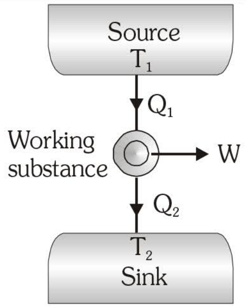
- When a gas expands its volume increases, then final pressure is less for adiabatic expansion. But, when a gas compresses its volume decreases, then the final pressure is more in case of adiabatic compression.







HEAT ENGINE

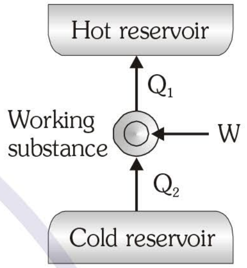


$$[\%\ \text{efficiency} = \frac{\text{work done}}{Q_{\text{input}}} \times 100]$$

Heat engine is a device which converts heat into work. In a cycle of heat engine the working substance extracts heat Q_1 from source, does some work W and rejects remaining heat Q_2 to the sink.

Efficiency of heat engine $\eta = \dfrac{\text{work done (W)}}{\text{heat taken from source } (Q_1)} \Rightarrow \dfrac{T_1 - T_2}{T_1} = \dfrac{Q_1 - Q_2}{Q_1}$

REFRIGERATOR



It is reverse of heat engine. It extracts heat (Q_2) from a cold reservoir, same external work W is done on it and rejects heat (Q_1) to hot reservoir.

$$\beta = \frac{\text{Heat extracted from cold reservoir}}{\text{Work done on refrigerator}} = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{1}{\frac{Q_1}{Q_2} - 1}$$

For Carnot reversible refrigerator $\dfrac{Q_1}{Q_2} = \dfrac{T_1}{T_2}$ $\quad \therefore\ \beta = \dfrac{Q_2}{W} = \dfrac{1}{\left[\dfrac{Q_1}{Q_2} - 1\right]} = \dfrac{1}{\left[\dfrac{T_1}{T_2} - 1\right]} \Rightarrow \beta = \dfrac{T_2}{T_1 - T_2}$

Relation between η & β → $\beta = \dfrac{1}{\eta} - 1$

Illustration A thermometer with an arbitrary scale has the ice point at –20° and the steam point at 180°. When the thermometer reads 5°, a centigrade thermometer will read

(1) 7.5°C (2) 12.5°C (3) 16.5°C (4) –9.37°C

Solution : $\dfrac{C - 0}{100 - 0} = \dfrac{t - (-20)}{180 - (-20)}$ (Here t = 5°) $\Rightarrow \dfrac{C}{100} = \dfrac{5 + 20}{200} \Rightarrow C = 12.5°C$

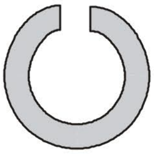
Illustration The temperature of an iron piece is raised from 30°C to 90°C. What is the change in its temperature on the Fahrenheit scale and on the Kelvin scale?

Solution $\Delta C = 90° - 30° = 60°C$

Temperature difference on Fahrenheit Scale $\Delta F = \dfrac{9}{5}\Delta C = \dfrac{9}{5}(60°C) = 108°F$

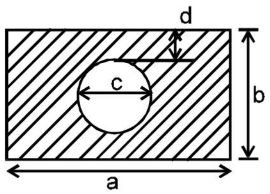
Temperature difference on Kelvin Scale $\Delta K = \Delta C = 60K$

Illustration A small ring having small gap is shown in figure on heating what will happen to the size of gap.



Solution : Gap will also increase due to thermal expansion.

Illustration A rectangular plate has a circular cavity as shown in the figure. If we increase its temperature then which dimension will increase in following figure.



Solution : Distance between any two point on an object increases with increase in temperature.

So, all dimension a, b, c and d will increase

Illustration There are two spheres of same radius and material at same temperature but one being solid while the other hollow. Which sphere will expand more if they are heated to the same temperature,

Solution As thermal expansion of isotropic solids is similar to true photographic enlargement, expansion of a cavity is same as if it had been a solid body of the same material i.e. $\Delta V = V\gamma\,\Delta\theta$

As here V, γ and $\Delta\theta$ are same for both solid and hollow spheres treated (cavity) ; so the expansion of both will be equal.

Illustration What is the percentage change in length of 1m iron rod if its temperature changes by 100ºC. α for iron is 2×10^{-5}/ºC.

Solution : percentage change in length due to temperature change

$$\frac{\Delta\ell}{\ell} \times 100 = \alpha\Delta\theta \times 100 = 2 \times 10^{-5} \times 100 \times 100 = 0.2\%$$

Illustration 5 kg of steam at 100℃ is mixed with 10 kg of ice at 0℃. Choose incorrect alternative (Given s_{water} = 1 cal/g℃, L_F = 80 cal/g, L_V = 540 cal/g)

(A) Equilibrium temperature of mixture is 160℃

(B) Equilibrium temperature of mixture is 100℃

(C) At equilibrium, mixture contains $13\frac{1}{3}$ kg of water

(D) At equilibrium, mixture contains $1\frac{2}{3}$ kg of steam

Solution **Ans. (A)**

Required heat	Available heat
10 kg ice (0℃)	5 kg steam (100℃)
↓ 800 kcal	↓ 2700 Kcal
10 g water (0℃)	5 g water (100℃)
↓ 1000 kcal	
10 g water (100℃)	

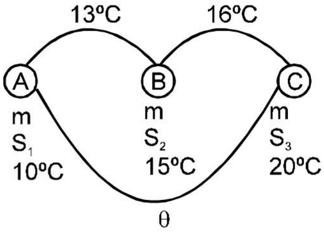
So available heat is more than required heat therefore final temperature will be 100℃.

Mass of heat condensed $= \frac{800+1000}{540} = \frac{10}{3}$ kg. Total mass of water $= 10 + \frac{10}{3} = \frac{40}{3} = 13\frac{1}{3}$ kg

Total mass of steam $= 5 - \frac{10}{3} = \frac{5}{3} = 1\frac{2}{3}$ kg

Illustration The temperature of equal masses of three different liquids A, B, and C are 10℃ 15℃ and 20℃ respectively. The temperature when A and B are mixed is 13℃ and when B and C are mixed, it is 16℃. What will be the temperature when A and C are mixed?

Solution : when A and B are mixed



$$mS_1 \times (13 - 10) = m \times S_2 \times (15 - 13)$$

$$\Rightarrow 3S_1 = 2S_2 \qquad .....(1)$$

when B and C are mixed $S_2 \times 1 = S_3 \times 4$ ....(2)

when C and A are mixed $S_1(\theta - 10) = S_3 \times (20 - \theta)$ ....(3)

by using equation (1), (2) and (3) we get $\theta = \frac{140}{11}$℃

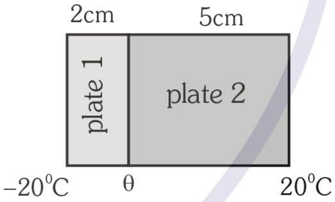
Illustration Two vessels of different materials are identical in size and wall–thickness. They are filled with equal quantities of ice at 0℃. If the ice melts completely, in 10 and 25 minutes respectively then compare the coefficients of thermal conductivity of the materials of the vessels.

Solution Let K_1 and K_2 be the coefficients of thermal conductivity of the materials, and t_1 and t_2 be the time in which ice melts in the two vessels. Since both the vessels are identical, so A and L in both the cases is same.

$$\text{Now, } Q = \frac{K_1A(\theta_1 - \theta_2)t_1}{L} = \frac{K_2A(\theta_1 - \theta_2)t_2}{L} \qquad \Rightarrow \frac{K_1}{K_2} = \frac{t_2}{t_1} = \frac{25\text{ min}}{10\text{ min}} = \frac{5}{2}$$

Illustration Two plates of equal areas are placed in contact with each other. Their thickness are 2.0 cm and 5.0 cm respectively. The temperature of the external surface of the first plate is –20℃ and that of the external surface of the second plate is 20℃. What will be the temperature of the contact surface if the plate (i) are of the same material, (ii) have thermal conductivities in the ratio 2 : 5.

Solution Rate of flow of heat in the plates is



$$\frac{Q}{t} = \frac{K_1A(\theta_1 - \theta)}{L_1} = \frac{K_2A(\theta - \theta_2)}{L_2} \quad ...(i)$$

(i) Here $\theta_1 = -20$℃, $\theta_2 = 20$℃,

$L_1 = 2\text{ cm} = 0.02\text{ m}$, $L_2 = 5\text{ cm} = 0.05\text{ m}$ and $K_1 = K_2 = K$

$\therefore$ equation (i) becomes $\frac{KA(-20-\theta)}{0.02} = \frac{KA(\theta - 20)}{0.05}$

$\therefore 5(-20-\theta) = 2(\theta - 20) \Rightarrow -100 - 5\theta = 2\theta - 40 \Rightarrow 7\theta = -60 \Rightarrow \theta = -8.6$℃

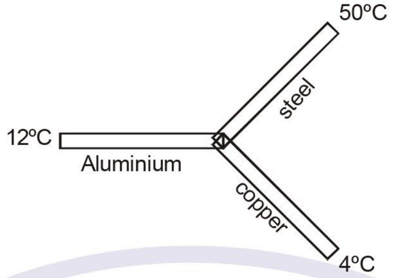
(ii) $\frac{K_1}{K_2} = \frac{2}{5}$ or $K_1 = \frac{2}{5}K_2$

$\therefore$ from equation (i) $\frac{2/5\,K_2A(-20-\theta)}{0.02} = \frac{K_2A(\theta-20)}{0.05} \Rightarrow -20 - \theta = \theta - 20$

$\therefore \theta = 0$℃

Illustration Three identical rods of length 1m each, having cross-section area of 1cm^2 each and made of Aluminium, copper and steel respectively are maintained at temperatures of 12℃, 4℃ and 50℃ respectively at their separate ends. Find the temperature of their common junction.

$[\,K_{Cu}=400\ \text{W/m-K}\ ,\ K_{Al} = 200\ \text{W/m-K},\ K_{steel} = 50\ \text{W/m-K}\,]$



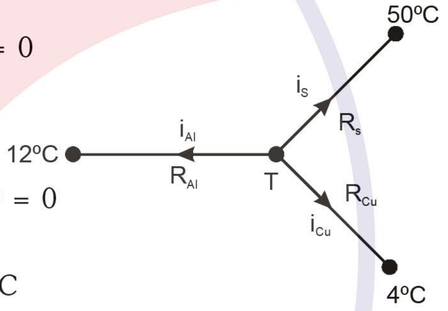
Solution

$$R_{Al} = \frac{L}{KA} = \frac{1}{200\times 10^{-4}} = \frac{10^4}{200}$$

Similarly $R_{steel} = \dfrac{10^4}{50}$ and $R_{copper} = \dfrac{10^4}{400}$

Let temperature of common junction = T

then from Kirchoff;s current laws, $i_{Al} + i_{steel} + i_{Cu} = 0$



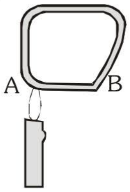
$$\Rightarrow \quad \frac{T-12}{R_{Al}} + \frac{T-50}{R_{steel}} + \frac{T-4}{R_{Cu}} = 0$$

$$\Rightarrow \quad (T - 12)\,200 + (T - 50)\,50 + (T - 4)\,400 = 0$$

$$\Rightarrow \quad 4(T - 12) + (T - 50) + 8\,(T - 4) = 0$$

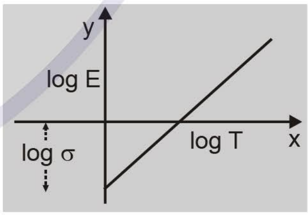
$$\Rightarrow \quad 13T = 48 + 50 + 32 = 130 \ \Rightarrow\ T = 10℃$$

Illustration Water in a closed tube is heated with one arm vertically placed above the lamp. In what direction water will begin the circulate along the tube ?



Solution On heating the liquid at A will become lighter and will rise up. This will push the liquid in the tube upwards and so the liquid in the tube will move clockwise i.e. from B to A.

Illustration Draw a graph between $\log E$ and $\log T$



Solution $E = \sigma T^4$ (taking log)

$$\log E = \log(\sigma T^4)$$

$$\log E = 4\log T + \log\sigma$$

This is equilant to $y = mx - C$ ($\sigma < 1$ so its log is a negative quantity)

Illustration If temperature of ideal black body is increased by 50%, what will be percentage increase in quantity of radiations emitted from its surface.

Solution $E \propto T^4$ and $\therefore$ $E' \propto (1.5)^4 T^4 \propto \left[\dfrac{15}{10}\right]^4 T^4 \propto \left[\dfrac{3}{2}\right]^4 T^4 \propto \dfrac{81}{16}T^4$

$$\frac{E'-E}{E}\times 100\% = \left[\frac{\frac{81}{16}T^4 - T^4}{T^4}\right]\times 100\% = 406\ \% \approx 400\ \%$$

Illustration If λ_m for the moon is 14.5 micron, then find its temperature. (Assume balck body)

Solution Wien's displacement law $\lambda_m T = b \quad \therefore T = \frac{b}{\lambda_m} = \frac{2.89 \times 10^{-3}}{14.5 \times 10^{-6}} = 199.3\text{K}$

Illustration Calculate the temperature at which a perfect black body radiates at the rate of 5.67 W cm^{-2}. Stefan's constant is 5.67×10^{-8} J s^{-1} m^{-2} K^{-4}.

Solution Given E = 5.67 W cm^{-2} = $5.67 \times 10^{+4}$ W m^{-2} , $\sigma = 5.67 \times 10^{-8}$ J s^{-1} m^{-2} K^{-4}

$$\text{Using, } E = \sigma T^4; \quad T^4 = \frac{E}{\sigma} \qquad \text{or } T = \left[\frac{E}{\sigma}\right]^{\frac{1}{4}} = \left[\frac{5.67 \times 10^{+4}}{5.67 \times 10^{-8}}\right]^{\frac{1}{4}} = (10^{12})^{1/4} = 1000 \text{ K}$$

Illustration Two bodies A and B have thermal emissiviities of 0.01 and 0.81 respectively. The outer surface areas of the two bodies are same, the two bodies emit total radiant power at the same rate. The wavelength λ_B corresponding to maximum spectral radiancy of B is shifted from the wavelength corresponding to maximum spectral radiancy in the radiation of A by 1.0 μm. If the temperature of A is 5802K, Calculate :-

(a) The temperature of B (b) Wavelength λ_B

Solution (a) As both bodies A and B having same radiant power

$$\therefore P_A = P_B \qquad \Rightarrow e_A \sigma A_A T_A^4 = e_B \sigma A_B T_B^4 \Rightarrow (0.01)\sigma A T_A^4 = (0.81)\sigma A T_B^4$$

$$T_B = \left(\frac{0.01}{0.81}\right)^{1/4} T_A = \frac{T_A}{3} = \frac{5802}{3} = \boxed{1934 \text{ K}}$$

(b) According to wein's displacement law

$$\lambda_A T_A = \lambda_B T_B \Rightarrow \lambda_B = \left(\frac{5802}{1934}\right)\lambda_A = 3\lambda_A$$

$$\text{As } \lambda_B - \lambda_A = 1\ \mu\text{m} \quad \Rightarrow \lambda_B - \frac{\lambda_B}{3} = 1\ \mu\text{m} \Rightarrow \frac{2\lambda_B}{3} = 1\ \mu\text{m} \Rightarrow \lambda_B = 1.5\ \mu\text{m}.$$

Illustration Assuming Newton's law of cooling to be valid. The temperature of body changes from 60°C to 40°C in 7 minutes. Temperature of surroundings being 10°C, Find its temperature after next 7 minutes.

Solution According to Newton's law of cooling $\frac{\theta_2 - \theta_1}{t} = K\left(\frac{\theta_1 + \theta_2}{2} - \theta_0\right)$

Since the temperature decreases from 60°C to 40°C in 7 minutes

$$\frac{60-40}{7} = K\left(\frac{60+40}{2} - 10\right) \Rightarrow \frac{20}{7} = K(50 - 10) \Rightarrow K = \frac{1}{14}$$

$$\frac{40 - \theta'}{7} = \frac{1}{14}\left(\frac{40 + \theta'}{2} - 10\right)$$

$$\Rightarrow 40 - \theta' = \frac{1}{4}(40 + \theta' - 20) \Rightarrow 160 - 4\theta' = 20 + \theta' \Rightarrow 5\theta' = 140 \Rightarrow \theta' = 28^\circ\text{C}$$

Illustration Define (i) Steady state and (ii) Temperature gradient in conduction of heat through a conducting rod.

Solution (i) When one end of a rod is heated, the temperature of various points of the rod changes continuously but after some time a state is reached, when the temperature of each cross–section becomes steady which is called steady state. In this state the heat received by any section will be totally transfered to the next section so no heat is absorbed by any cross section.

(ii) Temperature gradient is defined as the rate of change of temperature with distance in the direction of flow of heat.

Illustration By increasing temperature of gas by 5° C its pressure increases by 0.5% from its initial value at constant volume then what is initial temperature of gas ?

Solution $\because$ At constant volume $T \propto P$ $\quad \therefore \dfrac{\Delta T}{T}\times 100 = \dfrac{\Delta P}{P}\times 100 = 0.5 \Rightarrow T = \dfrac{5\times 100}{0.5} = 1000\text{K}$

Illustration A closed container of volume 0.02 m^3 contains a mixture of neon and argon gases at a temperature of 27°C and pressure of 1×10^5 N/m^2. The total mass of the mixture is 28 g. If the gram molecular weights of neon and argon are 20 and 40 respectively, find the mass of the individual gases in the container, assuming them to be ideal. Given : R = 8.314 J/mol-K.

Solution Let m gram be the mass of neon. Then, the mass of argon is (28 – m)g.

Total number of moles of the mixture, $\mu = \dfrac{m}{20} + \dfrac{28-m}{40} = \dfrac{28+m}{40}$ ...(i)

Now, $\mu = \dfrac{PV}{RT} = \dfrac{1\times 10^5 \times 0.02}{8.314\times 300} = 0.8$ ...(ii)

By (i) and (ii), $\dfrac{28+m}{40} = 0.8 \Rightarrow 28 + m = 32 \Rightarrow m = 4$ gram

or mass of argon = (28 – 4)g = 24 g

Illustration At the top of a mountain a thermometer reads 7°C and barometer reads 70 cm of Hg. At the bottom of the mountain they read 27°C and 76 cm of Hg respectively. Compare the density of the air at the top with that at the bottom.

Solution By gas equation $PV = \dfrac{M}{M_w}RT \Rightarrow \dfrac{P}{\rho T} = \dfrac{R}{M_w}\left[\because \mu = \dfrac{M}{M_w} \text{ and } \dfrac{M}{V} = \rho\right]$

Now as M_W and R are same for top and bottom $\left[\dfrac{P}{\rho T}\right]_T = \left[\dfrac{P}{\rho T}\right]_B$

So $\dfrac{\rho_T}{\rho_B} = \dfrac{P_T}{P_B}\times\dfrac{T_B}{T_T} = \dfrac{70}{76}\times\dfrac{300}{280} = \dfrac{75}{76} = 0.9868$

Illustration A vessel of volume 8.0×10^{-3} m^3 contains an ideal gas at 300 K and 200 k Pa. The gas is allowed to leak till the pressure falls to 125 kPa. Calculate the amount of the gas leaked assuming that the temperature remains constant.

Solution As the gas leaks out, the volume and the temperature of the remaining gas do not change. The number of moles of the gas in the vessel in given by $\mu = \dfrac{PV}{RT}$.

The number of moles in the vessel before the leakage is $\mu_1 = \dfrac{P_1V}{RT}$ and that after the leakage is $\mu_2 = \dfrac{P_2V}{RT}$.

The amount leaked is $\mu_1 - \mu_2 = \dfrac{(P_1 - P_2)V}{RT} = \dfrac{(200-125)\times 10^3 \times 8.0 \times 10^{-3}}{8.3\times 300} = 0.24$ mole

Illustration 1500 ml of a gas at a room temperature of 23°C is inhaled by a person whose body temperature is 37°C, if the pressure and mass stay constant, what will be the volume of the gas in the lungs of the person ?

Solution $T_1 = 273 + 23 = 296$ K; $T_2 = 273 + 37 = 310$ K. Pressure and amount of the gas are kept constant,

$$\text{So } \frac{V_1}{T_1} = \frac{V_2}{T_2} \therefore V_2 = V_1 \times \frac{T_2}{T_1} = 1500 \times \frac{310}{296} = 1570.95 \text{ ml}$$

Illustration The velocities of ten particles in ms^{-1} are 0, 2, 3, 4, 4, 4, 5, 5, 6, 9. Calculate (i) average speed and (ii) rms speed (iii) most probable speed.

Solution (i) average speed, $v_{av} = \frac{0+2+3+4+4+4+5+5+6+9}{10}$

$$= \frac{42}{10} = 4{\cdot}2 \text{ ms}^{-1}$$

(ii) rms speed, $v_{rms} = \left[\frac{(0)^2+(2)^2+(3)^2+(4)^2+(4)^2+(4)^2+(5)^2+(5)^2+(6)^2+(9)^2}{10}\right]^{1/2}$

$$= \left[\frac{228}{10}\right]^{1/2} = 4.77 \text{ ms}^{-1}$$

(iii) most probable speed $v_{mp} = 4$ m/s

Illustration At what temperature root mean square velocity of hydrogen becomes double of its value at S.T.P., pressure remaining constant ?

Solution Let v_1 be the r.m.s. velocity at S.T.P. and v_2 be the r.m.s. velocity at unknown temperature T_2.

$$\therefore \quad \frac{v_1^2}{v_2^2} = \frac{T_1}{T_2}$$

$$\text{or} \quad T_2 = T_1\left[\frac{v_2}{v_1}\right]^2 = 273 \times (2)^2 = 273 \times 4 = 1092 \text{ K} = (1092 - 273) = 819°\text{C}$$

Illustration The pressure of one mole monoatomic gas increases linearly from 4×10^5 Nm^{-2} to $8 \times 10^{+5}$ Nm^{-2} when its volume increases from $0.2 m^3$ to $0.5 m^3$. Calculate.

(i) Work done by the gas, (ii) Increase in the internal energy,

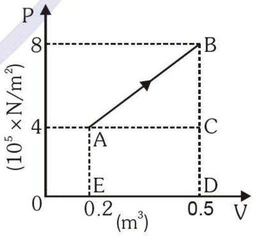
(iii) Amount of heat supplied, (iv) Molar heat capacity of the gas $R = 8.31$ J mol^{-1} K^{-1}

Solution $P_1 = 4 \times 10^5$ Nm^{-2} $P_2 = 8 \times 10^{+5}$ Nm^{-2}, $V_1 = 0.2$ m^3, $V_2 = 0.5$ m^3

(i) Work done by the gas = Area under P–V graph (Area ABCDEA)

$$= \frac{1}{2}(AE + BD) \times AC = \frac{1}{2}(4 \times 10^5 + 8 \times 10^5) \times (0.5–0.2)$$

$$= \frac{1}{2} \times 12 \times 10^5 \times 0.3 = 1.8 \times 10^5 \text{ J}$$



(ii) Increase in internal energy

$$\Delta U = C_V(T_2 - T_1) = \frac{C_V}{R}R(T_2–T_1) = \frac{C_V}{R}(P_2V_2 - P_1V_1)$$

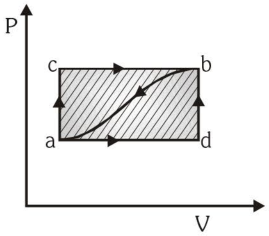
For monoatomic gas $C_V = \frac{3}{2}R$

$$\therefore \Delta U = \frac{3}{2}[(8 \times 10^5 \times 0.5) - (4 \times 10^5 \times 0.2)] = \frac{3}{2}[4 \times 10^5 - 0.8 \times 10^5] = 4.8 \times 10^5 \text{ J}$$

(iii) $Q = \Delta U + W = 4.8 \times 10^5 + 1.8 \times 10^5 = 6.6 \times 10^5$ J

(iv) $C = \frac{Q}{n\Delta T} = \frac{QR}{nR\Delta T} = \frac{QR}{n(P_2V_2 - P_1V_1)} = \frac{6.6 \times 10^5 \times 8.31}{1 \times 3.2 \times 10^5} = 17.14$ J/mole K

Illustration As shown in figure when a system is taken from state a to state b, along the path



a → c → b, 60 J of heat flow into the system, and 30 J of work is done :

(i) How much heat flows into the system along the path a → d → b if the work is 10 J.

(ii) When the system is returned from b to a along the curved path, the work done by the system is –20 J. Does the system absorb or liberate heat, and how much?

(iii) If, $U_a = 0$ and $U_d = 22$ J, find the heat absorbed in the process a → d and d → b.

Solution For the path acb $\Delta U = Q - W = 60 - 30 = 30$ J or $U_b - U_a = 30$ J

(i) Along the path adb $Q = \Delta U + W = 30 + 10 = 40$ J

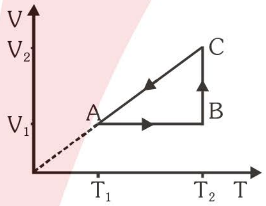
(ii) Along the curved path ba $Q = (U_a - U_b) + W = (-30) + (-20) = -50$ J, heat liberates from system

(iii) $Q_{ad} = U_d - U_a + W_{ad}$

but $W_{ad} = W_{adb} - W_{db} = 10 - 0 = 10$ Hence $Q_{ad} = 22 - 0 + 10 = 32$ J

and $Q_{db} = U_b - U_d + W_{db} = 30 - 22 + 0 = 8$ J

Illustration Figure shows a process ABCA performed on an ideal gas. Find the net heat given to the system during the process.



Solution Since the process is cyclic, hence the change in internal energy is zero.

The heat given to the system is then equal to the work done by it.

The work done in part AB is $W_1 = 0$ (the volume remains constant). The part BC represents an isothermal process so that the work done by the gas during this part is $W_2 = nRT_2 \ln \frac{V_2}{V_1}$

During the part CA : $V \propto T$ So, V/T is constant and hence, $P = \frac{nRT}{V}$ is constant

The work done by the gas during the part CA is

$$W_3 = P(V_1 - V_2) = nRT_1 - nRT_2 = -nR(T_2 - T_1).$$

The net work done by the gas in the process ABCA is

$$W = W_1 + W_2 + W_3 = nR\left[T_2 \ln\frac{V_2}{V_1} - (T_2 - T_1)\right]$$

The same amount of heat is given to the gas.

Illustration Two samples of a gas initially at same temperature and pressure are compressed from a volume V to $\frac{V}{2}$. One sample is compressed isothermally and the other adiabatically. In which sample is the pressure greater?

Solution Let initial volume, $V_1 = V$ and pressure, $P_1 = P$, final volume, $V_2 = \frac{V}{2}$ and final pressure, $P_2 = ?$

For isothermal compression $P_2V_2 = P_1V_1$ or $P_2 = \frac{P_1V_1}{V_2} = \frac{PV}{\frac{V}{2}} = 2P$

For adiabatic compression $P_2' = P_1\left[\frac{V_1}{V_2}\right]^{\gamma} \Rightarrow P_2' = P\left[\frac{V}{V/2}\right]^{\gamma} = 2^{\gamma}P$

$\Rightarrow \quad P_2' = 2^{\gamma}P \quad \gamma > 1 \quad \therefore \quad 2^{\gamma} > 2$ and $P_2' > P_2$

Pressure during adiabatic compression is greater than the pressure during isothermal compression.

Illustration A sample of oxygen with volume of 500 cc at a pressure of 2 atm is compressed to a volume of 400 cc. What pressure is needed to do this if the temperature is kept constant ?

Solution Temperature is constant, so $P_1V_1 = P_2V_2 \therefore P_2 = P_1\frac{V_1}{V_2} = 2\left[\frac{500}{400}\right] = 2.5$ atm

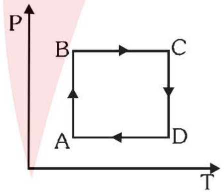
Illustration During an experiment it is found that an ideal gas follows and additional law VP^2 = costant. Initial temperature and volume of gas is T and V then what will be final temperature of the gas it is expanded 2V volume ?

Solutions According to question VP^2 = constant. Using gase equation $PV = \mu RT$

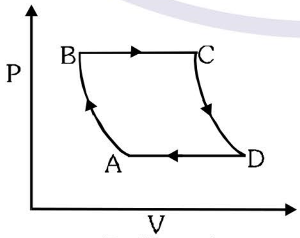
$$\left(\frac{K}{\sqrt{V}}\right)V = \mu RT, \quad \text{i.e.,} \quad \sqrt{V} = \frac{\mu R}{K}T$$

$$\therefore \quad \frac{\sqrt{V_1}}{\sqrt{V_2}} = \left(\frac{T_1}{T_2}\right) \quad \text{i.e.,} \quad \frac{\sqrt{V}}{\sqrt{2V}} = \frac{T}{T'} \quad \text{or } T' = (\sqrt{2})T$$

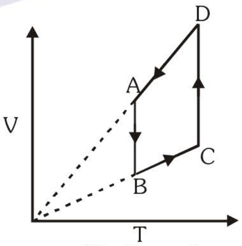
Illustration Plot P – V , V – T graph corresponding to the P–T graph for an ideal gas shown in figure. Explain your answers.



Solution



(P - V curve)



(V - T curve)

For process AB T = constant so $P \propto \frac{1}{V}$ For process BC P = constant so $V \propto T$

For process CD T = constant so $V \propto \frac{1}{P}$ For process DA P = constant so $V \propto T$

Illustration A carnot engine working between 400 K and 800 K has a work output of 1200 J per cycle. What is the amount of heat energy supplied to the engine from source per cycle?

Solution W = 1200J, T_1 = 800K, T_2 = 400K $\therefore \eta = 1 - \dfrac{T_2}{T_1} = \dfrac{W}{Q_1} \Rightarrow 1 - \dfrac{400}{800} = \dfrac{1200}{Q_1} \Rightarrow 0.5 = \dfrac{1200}{Q_1}$

Heat energy supplied by source $Q_1 = \dfrac{1200}{0.5}$ = 2400 joule per cycle

Illustration The temperatures T_1 and T_2 of the two heat reservoirs in an ideal carnot engine are 1500°C and 500°C respectively. Which of the following : increasing T_1 by 100°C or decreasing T_2 by 100°C would result in a greater improvement in the efficiency of the engine?

Solution T_1 = 1500°C = 1500 + 273 = 1773 K and T_2 = 500°C = 500 + 273 = 773 K.

The efficiency of a carnot's engine $\eta = 1 - \dfrac{T_2}{T_1} = 1 - \dfrac{773}{1773} = 0.56$

When the temperature of the source is increased by 100°C, keeping T_2 unchanged, the new temperature of the source is T'_1 = 1500 + 100 = 1600°C = 1873 K.

The efficiency becomes $\eta' = 1 - \dfrac{T_2}{T_1'} = 1 - \dfrac{773}{1873} = 0.59$

On the other hand, if the temperature of the sink is decreased by 100°C, keeping T_1 unchanged, the new temperature of the sink is T'_2 = 500 − 100 = 400°C = 673 K. The efficiency now becomes

$$\eta'' = 1 - \frac{T'_2}{T_1} = 1 - \frac{673}{1773} = 0.62$$

Since η′′ is greater than η′, decreasing the temperature of the sink by 100°C results in a greater efficiency than increasing the temperature of the source by 100°C.

Illustration A heat engine operates between a cold reservoir at temperature T_2 = 300 K and a hot reservoir at temperature T_1. It takes 200 J of heat from the hot reservoir and delivers 120 J of heat to the cold reservoir in a cycle. What could be the minimum temperature of hot reservoir?

Solution Work done by the engine in a cycle is W = 200 − 120 = 80 J. $\eta = \dfrac{W}{Q} = \dfrac{80}{200} = 0.4$

From carnot's Theorem $0.4 \le 1 - \dfrac{T_2}{T_1} = 1 - \dfrac{300}{T_1} \Rightarrow \dfrac{300}{T_1} \le 0.6 \Rightarrow T_1 \ge \dfrac{300}{0.6} \Rightarrow T_1 \ge 500$

Illustration A refrigerator takes heat from water at 0°C and transfer it to room at 27°C. If 100 kg of water is converted in ice at 0°C then calculate the work done. (Latent heat of ice is 3.4×10^5 J/kg)

Solution Coefficient of performance (COP) $= \dfrac{T_2}{T_1 - T_2} = \dfrac{273}{300 - 273} = \dfrac{273}{27}$

$$W = \frac{Q_2}{COP} = \frac{mL}{COP} = \frac{100 \times 3.4 \times 10^5}{273/27} = \frac{100 \times 3.4 \times 10^5 \times 27}{273} = 3.36 \times 10^6 \text{ J}$$