PROPERTIES OF MATTER AND FLUID MECHANICS

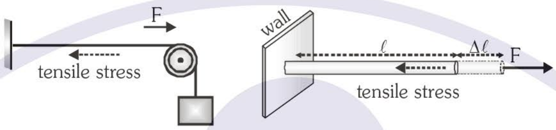
ELASTICITY

STRESS $= \dfrac{\text{Internal restoring force}}{\text{Area of cross} - \text{section}} = \dfrac{F_{Res}}{A}$.

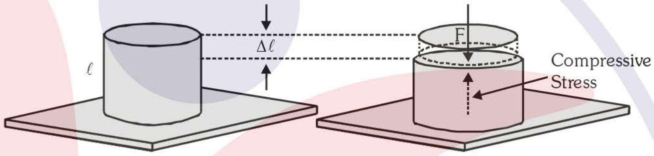
There are three types of stress :-

- **Longitudinal Stress**

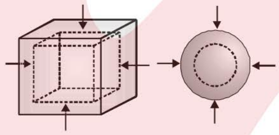
(a) Tensile Stress :



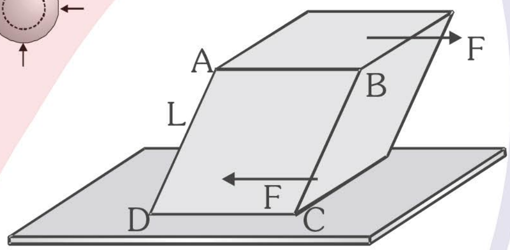
(b) Compressive Stress :



- **Volume Stress / Hydraulic**

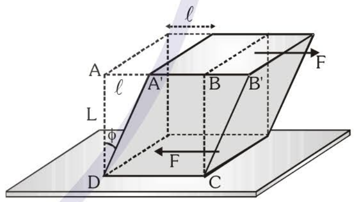


- **Tangential Stress or Shear Stress**



- **Strain** $= \dfrac{\text{Change in dimension of the body}}{\text{Original dimension of the body}}$

- **Longitudinal strain** $= \dfrac{\text{change in length of the body}}{\text{initial length of the body}} = \dfrac{\Delta L}{L}$



- **Volume / Hydraulic strain**$= \dfrac{\text{change in volume of the body}}{\text{original volume of the body}} = \dfrac{\Delta V}{V}$

- **Shear strain** : $\tan\phi = \dfrac{\ell}{L}$ for small angle $\phi = \dfrac{\ell}{L} = \dfrac{\text{displacement of upper face with respect to lower face}}{\text{distance between two faces}}$

Breaking Stress

The stress required to cause the actual fracture of a material is called the **breaking stress** or ultimate strength.

Breaking stress $= \dfrac{F}{A}$; F = force required to break the body.

Dependence of breaking stress : (i) Nature of material (ii) Temperature (iii) Impurities.

Independence of breaking stress : (i) Cross sectional area or thickness (ii) Applied force.

Maximum load (force) which can applied on the wire depends on

(i) Cross sectional area or thickness (ii) Nature of material (iii) Temperature (iv) Impurities.

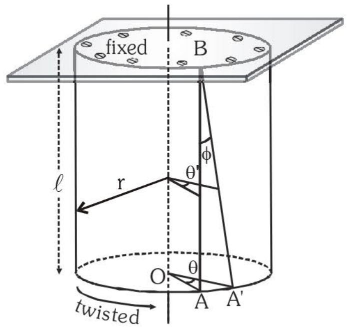
- **Relation between angle of twist (θ) & angle of shear (ϕ)**

AA' = r θ and Arc AA' = $\ell\phi$

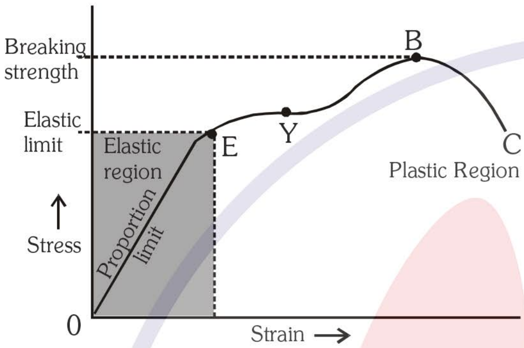
So $r\theta = \ell\phi \Rightarrow \phi = \frac{r\theta}{\ell}$

where θ = angle of twist,

ϕ = angle of shear



- **Stress – Strain Graph**



- **Hooke's Law** within elastic limit $\boxed{\text{Stress} \propto \text{strain}}$

- **Young's modulus of elasticity** $Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}} = \frac{F\ell}{A\,\Delta\ell}$

- If L is the length of wire, r is radius and ℓ is the increase in length of the wire by suspending a weight Mg at its one end then Young's modulus of elasticity of the material of wire $Y = \frac{(Mg/\pi r^2)}{(\ell/L)} = \frac{MgL}{\pi r^2 \ell}$

- **Increment in length due to own weight** $\Delta\ell = \frac{MgL}{2AY} = \frac{\rho g L^2}{2Y}$

- **Bulk modulus of elasticity** $K = \frac{\text{Volume stress}}{\text{Volume strain}} = \frac{F/A}{\left(\frac{-\Delta V}{V}\right)} = \frac{P}{\left(\frac{-\Delta V}{V}\right)}$

- **Compressibility** $C = \frac{1}{\text{Bulk modulus}} = \frac{1}{K}$

- **Modulus of rigidity** $\eta = \frac{\text{shearing stress}}{\text{shearing strain}} = \frac{(F_{\text{tangential}})/A}{\phi}$

- **Poisson's ratio** $(\sigma) = \frac{\text{lateral strain}}{\text{Longitudinal strain}} = \frac{\beta}{\alpha}$ $\left(\text{where } \beta = -\frac{\Delta D}{D},\ \alpha = \frac{\Delta L}{L}\right)$

$-1 \le \sigma \le 0.5$ (theoritical limit), $\sigma \approx 0.2$ to 0.4 (practical limit)

- **Work done in stretching wire**

$W = \frac{1}{2} \times Y \times (\text{strain})^2 \times \text{volume}$; $W = \frac{1}{2}$ (stress) (strain) (volume).

$W = \frac{1}{2} \times \frac{F}{A} \times \frac{\Delta\ell}{\ell} \times A \times \ell = \frac{1}{2} F \times \Delta\ell$

Energy density/energy stored per unit volume = $\frac{(\text{Stress})^2}{2Y}$

Factor Affecting Elasticity

- **Effect of Temperature**

  T ↑ ⇒ Y↓ Due to weekness of intermolecular force.

  When temperature is increased, the elastic properties in general decreases i.e. elastic constants decrease. Plasticity increases with temperature.

  For a special kind of steel, elastic constants do not vary appreciably with temperature. This steel is called INVAR steel.

- **Interatomic Force Constant :**

  k or $k_a = Y \cdot r_0$

  Y = Young's modulus ; r_0 = interatomic distance under normal circumstances

- **Relation between Y, K, η and σ :** To be remembered

  $Y = 3K(1-2\sigma)$, $Y = 2\eta(1+\sigma)$, $\frac{9}{Y} = \frac{3}{\eta} + \frac{1}{K}$.

HYDROSTATICS

- **Density** $= \frac{\text{mass}}{\text{volume}}$

- **Specific weight** $= \frac{\text{weight}}{\text{volume}} = \rho g$

- **Relative density** $= \frac{\text{density of given any body}}{\text{density of pure water at } 4°C}$

- **Density of a Mixture of substance in the proportion of mass**

  the density of the mixture is $\rho = \dfrac{M_1 + M_2 + M_3 ....}{\dfrac{M_1}{\rho_1} + \dfrac{M_2}{\rho_2} + \dfrac{M_3}{\rho_3} + ...}$

- **Density of a mixture of substance in the proportion of volume**

  the density of the mixture is $\rho = \dfrac{\rho_1 V_1 + \rho_2 V_2 + \rho_3 V_3}{V_1 + V_2 + V_3 + ...}$

  If liquids with same masses are mixed i.e. $m_1 = m_2 = m$ then $\rho_{mix.} = \dfrac{2\rho_1\rho_2}{\rho_1 + \rho_2}$ (Harmonic mean of individual densities)

  If liquids with same volumes are mixed i.e. $V_1 = V_2 = V$ then $\rho_{mix.} = \dfrac{\rho_1 + \rho_2}{2}$ (Arithmetic mean of individual densities)

- **Pressure** $= \frac{\text{Normal force}}{\text{Area}} = \frac{\text{Thrust}}{\text{Area}}$

- **Variation of pressure with depth**

  Pressure is same at two points in the same horizontal level $P_1 = P_2$

  The difference of pressure between two points separated by a depth h

  $$(P_2 - P_1) = h\rho g$$

- **Pascal's Law**
  - A liquid exerts equal pressures in all directions.
  - If the pressure in an enclosed fluid is changed at a particular point, the change is transmitted to every point of the fluid and to the walls of the container without being diminished in magnitude. *[for ideal fluids]*

- **Types of Pressure :** Pressure is of three types

  (i) Atmospheric pressure (P_0)

  (ii) Gauge pressure (P_{gauge})

  (iii) Absolute pressure ($P_{abs.}$)

- **Atmospheric pressure :** Force exerted by air column on unit cross–section area at sea level called atmospheric pressure (P_o)

$$P_o = \frac{F}{A} = 101.3 \text{ kN/m}^2 \qquad \therefore P_o = 1.013 \times 10^5 \text{ N/m}^2$$

Barometer is used to measure atmospheric pressure.

Which was discovered by **Torricelli**.

Atmospheric pressure varies from place to place and at a particular place from time to time.

- **Gauge Pressure :**

  Excess Pressure ($P - P_{atm}$) measured with the help of pressure measuring instrument called Gauge pressure.
  $P_{gauge} = h\rho g$ or $P_{gauge} \propto h$

  Gauge pressure is always measured with help of **"manometer"**

- Pressure due to liquid on a vertical wall is different at different depths, so average fluid pressure on side wall a of container filled upto height h = mean pressure = $\frac{h\rho g}{2}$

- **Absolute Pressure :**

  Sum of atmospheric and Gauge pressure is called absolute pressure.

$$P_{abs} = P_{atm} + P_{gauge} \Rightarrow P_{abs} = P_o + h\rho g$$

The pressure which we measure in our automobile tyres is gauge pressure.

- **Buoyant force** = Weight of displaced fluid = $V\sigma g$

- **Apparent weight** = Weight − Upthrust

- **Relative density of body** = $\frac{\text{Density of body}}{\text{Density of water at } 4°C}$

Principle of Floatation

When a body of density (ρ) and volume (V) is completely immersed in a liquid of density (σ), the forces acting on the body are :

(i) Weight of the body $W = Mg = V\rho g$ directed vertically downwards through the Centre of gravity of the body.

(ii) Buoyant force or Upthrust $Th = V\sigma g$ directed vertically upwards through Centre of buoyancy.

The following three cases are possible :

Case I Density of the body is greater than that of liquid ($\rho > \sigma$)

In this case $W > Th$

So the body will sink to the bottom of the liquid.

$$W_{App} = W - Th = V\rho g - V\sigma g = V\rho g\,(1 - \sigma/\rho) = W\,(1 - \sigma/\rho).$$

Case II Density of the body is equal to the density of liquid ($\rho = \sigma$)

In this case $W = Th$

So the body will float fully submerged in the liquid. It will be in neutral equilibrium.

$$W_{App} = W - Th = 0$$

Case III Density of the body is lesser than that of liquid ($\rho < \sigma$)

In this case $W < Th$

So the body will float partially submerged in the liquid. In this case the body will move up and will rise partially the volume of liquid displaced by the body (V_{in}) will be less than the volume of body (V). This ensures that Th equally to W

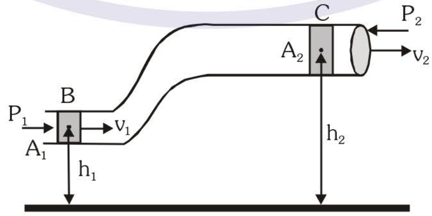
$$\therefore \quad W_{App} = W - Th = 0 \Rightarrow w = Th \quad \Rightarrow V\rho g = V_{in}\sigma g$$

The above three cases constitute the ***laws of floatation*** which states that a body will float in a liquid if weight of the liquid displaced by the immersed part of the body is at least equal to the weight of the body.

HYDRODYNAMICS

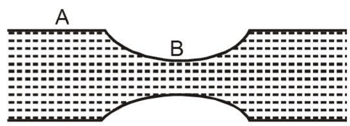
- **Steady and Unsteady Flow :** *Steady flow* is defined as that type of flow in which the fluid characteristics like velocity, pressure and density at a point do not change with time.
- **Streamline Flow :** In steady flow all the particles passing through a given point follow the same path and hence a unique line of flow. This line or path is called a *streamline*.
- **Laminar and Turbulent Flow :** *Laminar flow* is the flow in which the fluid particles move along well–defined streamlines which are straight and parallel.
- **Equation of continuity** $A_1v_1 = A_2v_2$ Based on conservation of mass
- **Bernoulli's theorem** : $\boxed{P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}}$

  Based on energy conservation



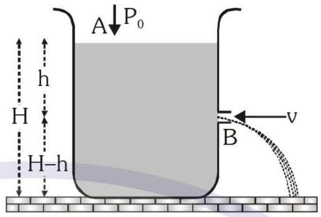
- **For horizontal flow in venturimeter**

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$



or $\frac{P}{\rho g} + \frac{v^2}{2g} + h = \text{constant}$ (Energy per unit weight)

In the above equation $\frac{P}{\rho g}$ is called *the pressure head*, $\frac{v^2}{2g}$ is called *the velocity head* and h is called *the gravitational/potential head.*



- **Velocity of efflux** $v = \sqrt{2gh}$
- **Horizontal range** $R = 2\sqrt{h(H-h)}$

Time taken to empty the container

$$t = \frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{H_1} - \sqrt{H_2}\right]$$

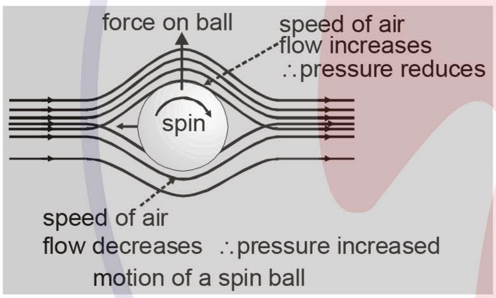
A – Area of container

a – Area of orifice

H_1 – Initial level from Orifice t = 0

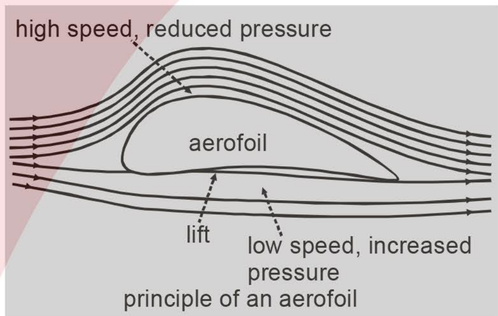
H_2 _ Final level from Orifice t

- **Magnus Effect (Observed in a Spinning Ball)**



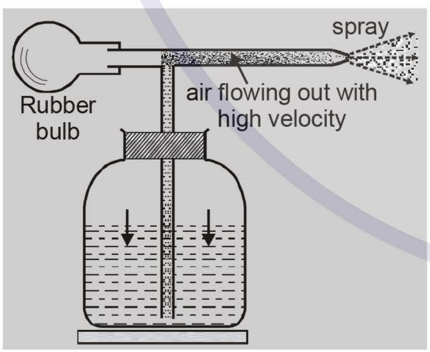
motion of a spin ball

- **Aerofoil**

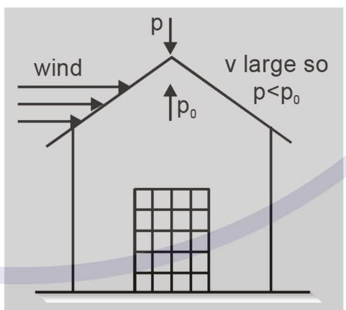


principle of an aerofoil

- **Sprayer or Atomizer**



- **Blowing-off of Tin Roof Tops in Wind Storm**



VISCOSITY

- **Newton's law of viscosity** $F = -\eta A \frac{dv}{dy}$

• **SI UNITS :** $\frac{N \times s}{m^2}$ or deca poise or 1 pa-s or 1 poiseuille

• **CGS UNITS :** dyne–s/cm² or poise (1 decapoise = 10 poise) $\eta = \frac{\text{Shear stress}}{\text{Shear strain rate}}$

- **Dependency of viscosity of fluids**

  On Temperature of Fluid

  (a) Since cohesive forces decrease with increase in temperature as increase in K.E.. Therefore with the rise in temperature, the viscosity of liquids decreases.

  (b) The viscosity of gases is the result of diffusion of gas molecules from one moving layer to other moving layer. Now with increase in temperature, the rate of diffusion increases. So, the viscosity also increases. Thus, the viscosity of gases increases with the rise of temperature.

  On Pressure of Fluid

  (a) The viscosity of liquids increases with the increase of pressure.

  (b) The viscosity of gases is practically independent of pressure.

  On Nature of Fluid

- **Poiseuille's formula** $Q = \frac{dV}{dt} = \frac{\pi pr^4}{8\eta L}$

- **Viscous force** $\vec{F} = -6\pi\eta r\vec{v}$ (Stokes law)

- **Terminal velocity** $v_T = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta} \Rightarrow v_T \propto r^2$ ρ - density of body, σ - density of medium

- **Reynolds number** $R_e = \frac{\text{Inertial force}}{\text{Viscous force}}$

R_e	< 1000	>2000	between 1000 to 2000
Type of flow	streamline	often turbulent	unsteady

SURFACE TENSION

Surface tension is basically a property of liquid. The liquid surface behaves like a stretched elastic membrane which has a natural tendency to contract and tends to have a minimum possible area. This property of liquid is called *surface tension*.

Intermolecular forces

(a) Cohesive force

The force acting between the molecules of one type of molecules of same substance is called cohesive force.

(b) Adhesive force

The force acting between different types of molecules or molecules of different substance is called adhesive force.

- ❒ Intermolecular forces are different from the gravitational forces not obey the inverse–square law
- ❒ The distance upto which these forces effective, is called molecular range. This distance is nearly 10^{-9} m. Within this limit this increases very rapidly as the distance decreases.
- ❒ Molecular range depends on the nature of the substance

Properties of surface tension

- Surface tension is a scalar quantity.
- Force due to surface tension is acts tangential to liquid surface.
- Surface tension is always produced due to cohesive force.
- More is the cohesive force, more is the surface tension.
- When surface area of liquid is increased, molecules from the interior of the liquid rise to the surface. For this, work is done against the downward cohesive force.

Dependency of Surface Tension

- **On Cohesive Force :** Those factors which increase the cohesive force between molecules increase the surface tension and those which decrease the cohesive force between molecules decrease the surface tension.
- **On Impurities :** If the impurity is completely soluble then on mixing it in the liquid, its surface tension increases. e.g., on dissolving ionic salts in small quantities in a liquid, its surface tension increases. If the impurity is partially soluble in a liquid then its surface tension decreases because adhesive force between insoluble impurity molecules and liquid molecules decreases cohesive force effectively, e.g.

  (a) On mixing detergent in water its surface tension decreases.

  (b) Surface tension of water is more than (alcohol + water) mixture.
- **On Temperature**

  On increasing temperature surface tension decreases. At critical temperature and boiling point it becomes zero.

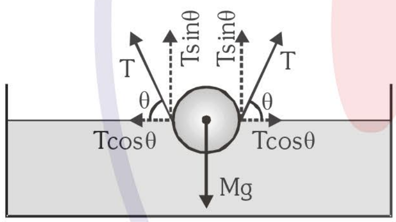
  Note : Surface tension of water is maximum at 4°C
- **On Contamination**

  The dust particles or lubricating materials on the liquid surface decreases its surface tension.

Definition of surface tension

The force acting per unit length of an imaginary line drawn on the free liquid surface at right angles to the line and in the plane of liquid surface, is defined as surface tension.

❒ For floating needle $2T\ell \sin\theta = mg$



- **Required excess force for lift**

❒ Wire $F_{ex} = 2T\ell$

❒ Hollow disc $F_{ex} = 2\pi T (r_1 + r_2)$

❒ For ring $F_{ex} = 4\pi rT$

❒ Circular disc $F_{ex} = 2\pi rT$

❒ Square frame $F_{ex} = 8aT$

❒ Square plate $F_{ex} = 4aT$

- **Work** = surface energy = $T\Delta A$

❒ Liquid drop $W = 4\pi r^2 T$

❒ Soap bubble $W = 8\pi r^2 T$

- **Splitting of bigger drop into smaller droples** $R = n^{1/3} r$

$(SE)_{Big} = T(4\pi R^2)$ $(SE)_{small} = T(4\pi R^2) \cdot n^{1/3}$

$$W = \Delta SE = 4\pi R^3 T\left(\frac{1}{r} - \frac{1}{R}\right) = 4\pi R^2 T\,(n^{1/3} - 1)$$

- **Excess pressure** $P_{ex} = P_{in} - P_{out}$

❒ In liquid drop $P_{ex} = \dfrac{2T}{R}$

❒ In soap bubble $P_{ex} = \dfrac{4T}{R}$

ANGLE OF CONTACT (θ_c)

The angle enclosed between the tangent plane at the liquid surface and the tangent plane at the solid surface at the point of contact inside the liquid is defined as the *angle of contact*.

The angle of contact depends the nature of the solid and liquid in contact.

- **Angle of contact** $\theta < 90° \Rightarrow$ concave shape, Liquid rise up

  Angle of contact $\theta > 90° \Rightarrow$ convex shape, Liquid falls

  Angle of contact $\theta = 90° \Rightarrow$ plane shape, Liquid neither rise nor falls

- **Effect of Temperature on angle of contact**

  On increasing temperature surface tension decreases, thus $\cos\theta_c$ increases $\left[\because \cos\theta_c \propto \frac{1}{T}\right]$ and θ_c decrease. So on increasing temperature, θ_c decreases.

- **Effect of Impurities on angle of contact**

  (a) Solute impurities increase surface tension, so $\cos\theta_c$ decreases and angle of contact θ_c increases.

  (b) Partially solute impurities decrease surface tension, so angle of contact θ_c decreases.

- **Effect of Water Proofing Agent**

  Angle of contact increases due to water proofing agent. It gets converted acute to obtuse angle.

- **Capillary rise** (i) Pressure Balance Method

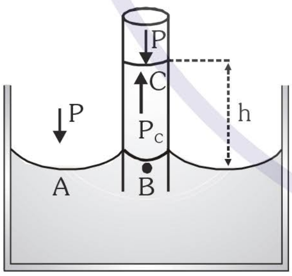
$$\rho g h = \frac{2T}{R}$$

$$h = \frac{2T}{R\rho g}$$

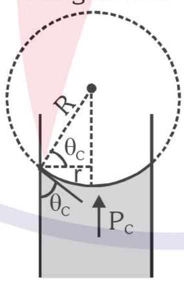
$$h = \frac{2T\cos\theta}{r\rho g}$$

(ii) Force Balance Method

$$(2\pi r)T\cos\theta = mg$$



R= Radius of the meniscus



$R = \frac{r}{\cos\theta}$

- Zurin's law $h \propto \frac{1}{r}$

- When two soap bubbles are in contact then radius of curvature of the common surface $r = \frac{r_1 r_2}{r_1 - r_2}$ $(r_1 > r_2)$

- When two soap bubbles are combining to form a new bubble then radius of new bubble $r = \sqrt{r_1^2 + r_2^2}$

- Force required to separate two plates $F = \frac{2AT}{d}$

Illustration The stress versus strain graphs for two materials A and B are shown below.

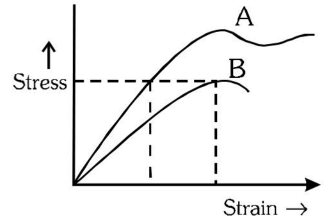
Explain the following

(a) Which material has greater Young's modulus ?

(b) Which material is more ductile ?

(c) Which material is more brittle ?

(d) Which of the two is more stronger material ?



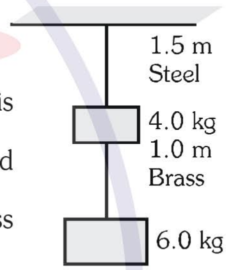
Solution (a) Material A has greater value of Young's modulus, because slope of A is greater than that of B.

(b) Material A is more ductile because there is a large plastic deformation range between the elastic limit and the breaking point.

(c) Material B is more brittle because the plastic region between the elastic limit and breaking point is small.

(d) Strength of a material is determined by the stress required to cause fracture. Material A is stronger than material B.

Illustration Two wires of diameter 0.25 cm, one made of steel and the other made of brass are loaded as shown in Fig. The unloaded length of steel wire is 1.5 m and that of brass wire is 1.0 m. Young's modulus of steel is 2.0×10^{11} Pa and that of brass is 0.91×10^{11} Pa. Calculate the elongations of the steel and brass wires. (1 Pa = 1 Nm^{-2})



Solution $\because$ The elongation in steel wire $\Delta\ell_S = \dfrac{Mg\ell_S}{\pi r^2 Y_S} = \dfrac{(4+6)\times 9.8\times 1.5}{3.14\times\left(0.125\times 10^{-2}\right)^2\times 2\times 10^{11}} = 1.50\times 10^{-4}$ m

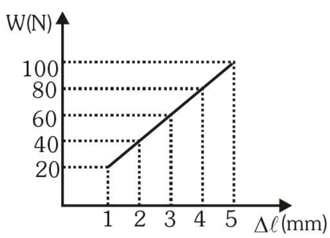
The elongation in brass wire $\Delta\ell_B = \dfrac{Mg\ell_B}{\pi r^2 Y_B} = \dfrac{6\times 9.8\times 1.0}{3.14\times\left(0.125\times 10^{-2}\right)^2\times 0.91\times 10^{11}} = 1.32\times 10^{-4}$ m

Illustration Calculate the force required to increase the length of a steel wire of cross-sectional area 10^{-6} m^2 by 0.5%. given : $Y_{(for\ steel)} = 2\times 10^{11}$ $N\text{-}m^{-2}$.

Solution $\dfrac{\ell}{L}\times 100 = 0.5\% \quad \Rightarrow \quad \dfrac{\ell}{L} = 5\times 10^{-3}$

so $F = YA\dfrac{\ell}{L} = 2\times 10^{11}\times 10^{-6}\times 5\times 10^{-3} = 10^3$ N.

Illustration The graph shows the extension of a wire of length 1m suspended from a roof at one end and with a load W connected to the other end. If the cross sectional area of the wire is 1 mm^2, then the Young's modulus of the material of the wire is



Solution $Y = \dfrac{F/A}{\Delta\ell/\ell} = \dfrac{W\ell}{A\Delta\ell} \Rightarrow \dfrac{W}{\Delta\ell} = \dfrac{YA}{\ell} = \text{slope} \Rightarrow Y = \dfrac{\ell}{A}(\text{slope}) = \dfrac{1}{10^{-6}}\left(\dfrac{40-20}{(2-1)\times 10^{-3}}\right) = 2\times 10^{10}\ Nm^{-2}.$

Illustration A sphere contracts in volume by 0.01% when taken to the bottom of sea 1 km deep. Find the bulk's modulus of the material of the sphere. Given : density of sea water is 1 gcm^{-3}, g = 980 cms^{-2}.

Solution $\frac{|\Delta V|}{V} = \frac{0.01}{100}$, h = 1 km = 10^5 cm, ρ = 1 gcm^{-3} ; $\Delta P = 10^5 \times 1 \times 980$ dyne-cm^{-2}, K = ?

$$K = \frac{\Delta P}{|\Delta V|/V} = \frac{\Delta P \times V}{|\Delta V|} = \frac{10^5 \times 980 \times 100}{0.01} \text{dyne-cm}^{-2} = 9.8 \times 10^{11} \text{ dyne-cm}^{-2}.$$

Illustration Young modulus of elasticity of steel is 2 x 10^{11} N/m^2. If interatomic distance for steel is 3.2 A°, then find the interatomic force constant.

Solution $k = Y \times r_0 = 2 \times 10^{11} \times 3.2 \times 10^{-10} = 64 \text{ Nm}^{-1}$.

Illustration Two immiscible liquids of densities 2.5 g/cm^3 and 0.8 g/cm^3 are taken in the ratio of their masses as 2:3 respectively. Find the average density of the liquid combination.

Solution Let masses be 2M & 3M then $V = V_1 + V_2 = \left(\frac{2M}{2.5} + \frac{3M}{0.8}\right)$ cm^3

Total mass = 2M+3M = 5M

Therefore, the average density $\rho_{av} = \frac{5M}{V} = \frac{5M}{\frac{2M}{2.5} + \frac{3M}{0.8}} = \frac{5}{\frac{2}{2.5} + \frac{3}{0.8}} = \frac{10}{9.1}$ g/cm^3 = 1.09 g/cm^3

Illustration Calculate the depth of a well if the pressure at its bottom is 15 times that at a depth of 3 metres. Atmospheric pressure is 10 m column of water.

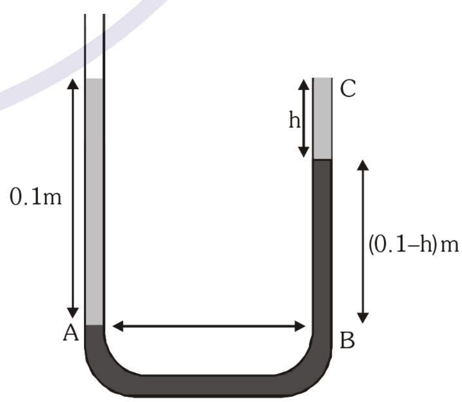
Solution Let the depth of the well be h then according to the question,

$P_{atm} + h\rho_w g = 15 (P_{atm} + 3\rho_w g)$

$h\rho_w g = 14 P_{atm} + 45 \rho_w g = 14 (10 \times \rho_w g) + 45 \rho_w g$

h = 185 m.

Illustration A vertical U–tube of uniform cross–section contains mercury in both arms. A glycerine (relative density = 1.3) column of length 10 cm is introduced into one of the arms. Oil of density 800 kg m^{-3} is poured into the other arm until the upper surface of the oil and glycerine are at the same horizontal level. Find the length of the oil column. Density of mercury is 13.6×10^3 kgm^{-3}.



Solution Pressure at A and B must be same

Pressure at A = $P_0 + 0.1 \times (1.3 \times 1000) \times g$

Pressure at B = $P_0 + h \times 800 \times g + (0.1 - h) \times 13.6 \times 1000$ g

$\Rightarrow 0.1 \times 1300 = 800\,h + (0.1 - h) \times 13600$

$\Rightarrow$ h = 0.096 m = 9.6 cm

Illustration An iceberg is floating partially immersed in sea-water. The density of sea-water is 1.03 g/cm^3 and that of ice is 0.92 g/cm^3. What is the fraction of the total volume of the iceberg above the level of sea-water ?

Solution In case of floatation weight = upthrust i.e.

$mg = V_{in}\sigma g$ or $V\rho g = V_{in}\sigma g$

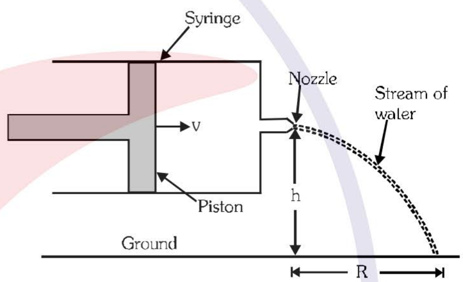
or $V_{in} = \frac{\rho}{\sigma}V$

so $V_{out} = V - V_{in} = V\left[1-\frac{\rho}{\sigma}\right]$

$\therefore$ The required fraction is,

or $f_{out} = \frac{V_{out}}{V} = \left[1-\frac{\rho}{\sigma}\right] = \left[1-\frac{0.92}{1.03}\right] = \frac{0.11}{1.03} = 0.106$

Illustration A syringe containing water is held horizontally with its nozzle at a height h above the ground as shown in fig. The cross-sectional areas of the piston and the nozzle are A and a respectively. The piston is pushed with a constant speed v. Find the horizontal range R of the stream of water on the ground



Solution let v' be the horizontal speed of water when it emerges from the nozzle then from equation of continuity

$$Av = av' \Rightarrow v' = \frac{Av}{a}$$

Let t be the time taken by the stream of water to strike the ground then $h = \frac{1}{2}gt^2$

$$\Rightarrow t = \sqrt{\frac{2h}{g}} \Rightarrow \text{horizontal distance } R = v'\sqrt{\frac{2h}{g}} = \frac{Av}{a}\sqrt{\frac{2h}{g}}.$$

Illustration Water is flowing through two horizontal pipes of different diameters which are connected together. In the first pipe the speed of water is 4 m/s. and the pressure is 2×10^4 N/m^2. Calculate the speed and pressure of water in the second pipe. The diameters of the pipes are 3 cm and 6 cm respectively ?

Solution If A is the area of cross–section of a pipe at a point and v is the velocity of flow of water at that point, then by the principle of continuity Av = constant $\Rightarrow A_1v_1 = A_2v_2$

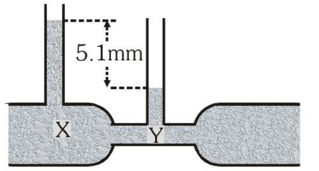
$$\Rightarrow \pi r_1^2 v_1 = \pi r_2^2 v_2$$

$$\Rightarrow v_2 = \left(\frac{r_1}{r_2}\right)^2 v_1 = \left(\frac{1.5\times10^{-2}}{3\times10^{-2}}\right)^2 \times 4 = 1 \text{ m/s.}$$

From Bernoulli's theorem : $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 \Rightarrow P_2 = P_1 + \frac{1}{2}\rho(v_1^2 - v_2^2)$

$$\therefore P_2 = 2 \times 10^4 + \frac{1}{2}\times(10^3)\times(16-1) = 2\times10^4 + 7.5\times10^3 = 2.75\times10^4 \text{ N/m}^2$$

Illustration The diagram (fig.) shows venturimeter through which water is flowing. The speed of water at X is 2 cm/s. Find the speed of water at Y (taking g = 1000 cm/s²).



Solution By using Bernoulli's principle –

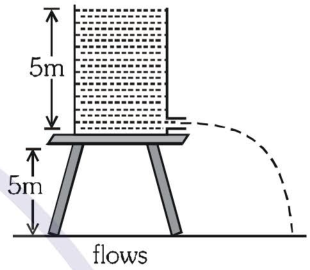
$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 \Rightarrow P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) \Rightarrow \rho gh = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

putting the values in equation $1000 \times 0.51 = \frac{1}{2}(v_2^2 - 2^2) \Rightarrow v_2 = 32$ cm/s

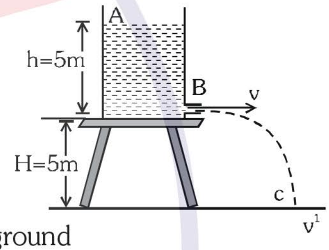
Illustration A cylindrical tank 1m in radius rests on a platform 5 m high. Initially, the tank is filled with water to a height of 5 m. A small plug whose area is 10^{-4} m² is removed from an orifice located on the side of the tank at the bottom. Calculate the :

(i) initial speed with which water flows out from the orifice

(ii) initial speed with which the water strikes the ground.



Solution (i) Applying Bernoulli's theorem between the water surface and the orifice,



$$P_0 + \frac{1}{2}\rho(0)^2 + \rho gh = P_0 + \frac{1}{2}\rho v^2 + \rho g(0)$$

$$\rho gh = \frac{1}{2}\rho v^2 \; ; \; v = \sqrt{2gh} = \sqrt{2 \times 10 \times 5} = 10 \text{ m/s}.$$

(ii) Let v' be the initial velocity with which the water strikes the ground

Then, applying Bernoulli's theorem between the top of the tank and the ground level, we get

$$v' = \sqrt{2g(H+h)} = \sqrt{2 \times 10 \times 10} = 10\sqrt{2} = 14.1 \text{ m/s}.$$

Illustration The velocity of water in a river is 18 km/h at the surface. If the river is 5 m deep and the flow is streamlined, find the shearing stress between the horizontal layers of water assuming uniform veloicty gradient. Viscosity of water is 10^{-3} poiseuille.

Solution As velocity at the bottom of the river will be zero,

Velocity gradient $\frac{dv}{dy} = \frac{18 \times 10^3}{60 \times 60 \times 5} = 1\,s^{-1}$

Shear stress $= \frac{F}{A} = \eta\frac{dv}{dy} = 10^{-3} \times 1 = 1 \times 10^{-3}$ N/m².

Illustration A spherical ball of radius 1×10^{-4} m and density 10^4 kg/m³ falls freely under gravity through a distance h before entering a tank of water. If the velocity of the ball does not change, after entering the water find h. Viscosity of water is 9.8×10^{-6} N-s/m².

Solution After falling a height h velocity of the ball will become $v = \sqrt{2gh}$. After entering into the water as this velocity does not change, this velocity is equal to the terminal velocity,

$$\sqrt{2gh} = \frac{2}{9}r^2\left[\frac{\rho - \sigma}{\eta}\right]g$$

$$2gh = \left[\frac{2}{9} \times (10^{-4})^2 \times \frac{(10^4 - 10^3) \times 9.8}{9.8 \times 10^{-6}}\right]^2 \Rightarrow h = \frac{20 \times 20}{2 \times 9.8} = 20.41\text{m}$$

Illustration Calculate the work done against surface tension in blowing a soap bubble from a radius 10 cm to 20 cm if the surface tension of soap solution is 25×10^{-3} N/m. Then compare it with a liquid drop for same radii.

Solution (i) For soap bubble : Extension in area $= 2 \times (4\pi r_2^2 - 4\pi r_1^2) = 8\pi [(0.2)^2 - (0.1)^2] = 0.24\pi \text{ m}^2$

Work done W_1 = surface tension × extension in area $= 25 \times 10^{-3} \times 0.24\pi = 6\pi \times 10^{-3}$ J.

(ii) For Liquid Drop : in case of liquid drop there is only one free surface, so extension in area will be half that of soap bubble

$$\therefore W_2 = \frac{W_1}{2} = 3\pi \times 10^{-3} \text{ J}$$

Illustration If W is the amount of work done in forming a soap bubble of volume V, then calculate the amount of work done in forming a bubble of volume 2V from the same solution.

Solution. Volume of bubble $V = \frac{4}{3}\pi r^3 \Rightarrow V \propto r^3$

$$\frac{2V}{V} = \left(\frac{r_2}{r_1}\right)^3 \Rightarrow \frac{r_2}{r_1} = 2^{1/3}$$

Work done in forming the bubble $W = 8\pi r^2 T \Rightarrow W \propto r^2$

$$\frac{W_2}{W_1} = \left(\frac{r_2}{r_1}\right)^2 = (2^{1/3})^2 = 2^{2/3}$$

$$W_2 = 2^{2/3} W$$

Illustration A water drop of radius 1mm is split into 10^6 identical drops. Surface tension of water is 72 dynes/cm. Find the energy spent in this process.

Solution As volume of water remains constant, so $\frac{4}{3}\pi R^3 = n\frac{4}{3}\pi r^3 \Rightarrow r = \frac{R}{n^{1/3}}$

Increase in surface area $\Delta A = n(4\pi r^2) - 4\pi R^2 = 4\pi(n^{1/3} - 1)R^2 = 4\pi(100 - 1)10^{-6}$

$\therefore$ Energy spent $= T\Delta A = 4\pi \times 99 \times 10^{-6} \times 72 \times 10^{-3} = 89.5 \times 10^{-6}$ J

Illustration Prove that If two bubbles of radii r_1 and r_2 $(r_1 < r_2)$ come in contact with each other then the radius of curvature of the common surface $r = \frac{r_1 r_2}{r_2 - r_1}$.

Solution $\because r_1 < r_2 \therefore P_1 > P_2$ Small portion of bubbles is in contact and in equilibrium

$$\Rightarrow P_1 - P_2 = \frac{4T}{r} \Rightarrow \frac{4T}{r_1} - \frac{4T}{r_2} = \frac{4T}{r} \Rightarrow r = \frac{r_1 r_2}{r_2 - r_1}$$

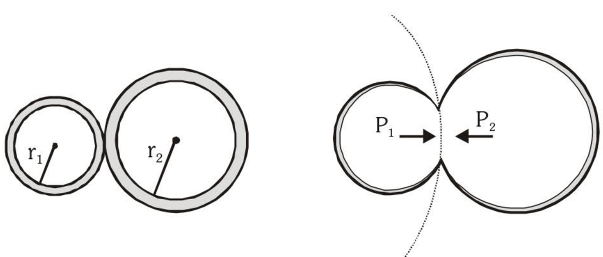


Illustration A hollow sphere which has a small hole in its bottom is immersed in water to a depth of 30 cm before any water enters in it. If the surface tension of water is 75 dynes/cm then find the radius of the hole in metres (taking g=10 m/s^2)

Solution Radius of the hole $r = \frac{2T}{hdg} = \frac{2\times75\times10^{-3}}{30\times10^{-2}\times10^{3}\times10} = 5\times10^{-5}$ m.

Illustration A U - tube is supported with its limbs vertical and is partly filled with water. If the internal diameters of the limbs are 1×10^{-2} m and 1×10^{-4} m respectively. What will be the difference in heights of water in the two limbs? (Surface tension of water is 0.07 N/m.)

Solution Let h_1 and h_2 be the heights of water columns in the limbs of radis r_1 and r_2.

Then $$h_1 = \frac{2T\cos\theta}{r_1dg} = \frac{2\times0.07\times\cos0^\circ}{0.5\times10^{-4}\times1000\times9.8} = 2.8\times10^{-3}\text{ m} = 0.028\times10^{-1}\text{ m}$$

similarly $$h_2 = \frac{2T\cos\theta}{r_2dg} = \frac{2\times0.07\times\cos0^\circ}{0.5\times10^{-2}\times1000\times9.8} = 2.8\times10^{-1}\text{ m}$$

Therefore difference in heights $= h_2 - h_1 = (2.8 - 0.028)\times10^{-1}\text{ m} = 2.772\times10^{-1}\text{ m}$

$= 0.277$ m.