WORK, POWER AND ENERGY

WORK

Work is said to be done by a force when the force produces a displacement in the body on which it acts in any direction except perpendicular to the direction of the force.

Work Done by Constant Force

Work is the product of the applied force and the displacement of the body in the direction of force.

$$W = F\,s\cos\theta = \vec{F}.\vec{s}$$

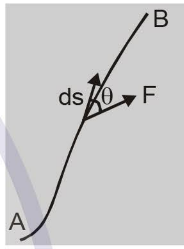
- If $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$ and $\vec{s} = x\hat{i} + y\hat{j} + z\hat{k}$ Then $W = \vec{F}.\vec{s} = F_x x + F_y y + F_z z$

  Dimension : $M^1L^2T^{-2}$ UNIT S I : joule C.G.S. : erg $\boxed{1 \text{ joule} = 10^7 \text{ erg}}$

Work Done by a Variable Force

When magnitude and direction of the force varies with position or time, the work done by force for infinitesimal displacement ds is $dW = \vec{F}.d\vec{s}$

The total work done for displacement from A to B is $W_{AB} = \int_A^B \vec{F}.d\vec{s} = \int_A^B (F\cos\theta)ds$



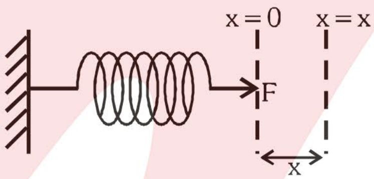
In terms of rectangular components

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}, \qquad d\vec{s} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$W_{AB} = \int_A^B (F_x\hat{i} + F_y\hat{j} + F_z\hat{k}).(dx\hat{i} + dy\hat{j} + dz\hat{k}) = \int_{x_A}^{x_B} F_x dx + \int_{y_A}^{y_B} F_y dy + \int_{z_A}^{z_B} F_z dz$$

(i) Work done by Stretching Force:

$$W = \int_{x_1}^{x_2} F\cos\theta\, dx$$



- Work done to stretch spring by displacement x from normal length

  $x_1 = 0$ $x_2 = x$ and $\theta = 0°$

$$W = \int_0^x F\,dx$$

$\because$ $F = kx$ $\qquad W = \int_0^x kx\,dx$

$$\boxed{W = \frac{1}{2}kx^2}$$

- Work done to stretch spring from displacement x_1 to x_2

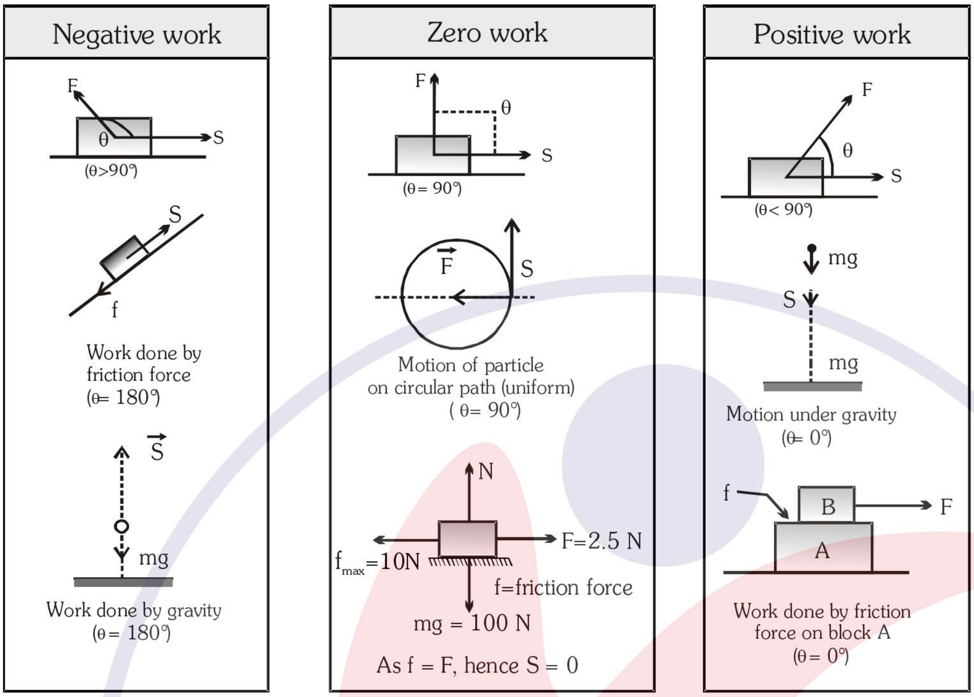
  Then $x_1 = x_1$ $x_2 = x_2$ and $\theta = 0°$

$$W = \int_{x_1}^{x_2} F\,dx$$

$\because$ $F = kx$ $\qquad W = \int_{x_1}^{x_2} kx\,dx$

$$\boxed{W = \frac{1}{2}k\left(x_2^2 - x_1^2\right)}$$

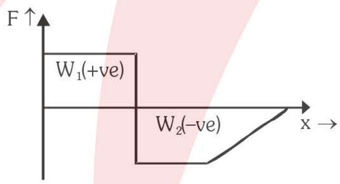
- **Nature of work done** : Although work done is a scalar quantity, yet its value may be positive, negative or even zero



Graphical Method to Calculate Work Done

If force displacement (F – x) curve is given then net area under F – x curve is equal to work done.

$W = W_1 + (-W_2)$



Facts:

- Work is defined for an interval or displacement, there is no term like instantaneous work similar to instantaneous velocity.
- For a particular displacement work is independent of time, work will be same for same displacement whether the time taken is small or large.
- When several forces act, work by a force for a particular displacement is independent of other forces.
- Displacement depends on reference frame so work done by a force is reference frame dependent so work done by a force can be different in different reference frame.
- Work is done by the source or agent that applies the force.
- When $\theta = 0°$: A force does maximum positive work.
- When $\theta = 180°$ A force does maximum negative work.

POWER

Power of a body is defined as the rate at which the body can do the work.

Average Power $\text{Power} = \dfrac{\text{work}}{\text{time}}$ or $P = \dfrac{W}{t}$

W is the amount of the work done in total time t.

Instantaneous Power (P)

Instantaneous power is the power at any given instant. Suppose an agent does an infinitesimally amount of work dW in an infinitesimally time dt then,

$$P = \frac{dW}{dt} \quad \text{or} \quad P = \frac{\vec{F}.d\vec{s}}{dt} \quad (\because dW = \vec{F}.d\vec{s}) \quad \text{or} \quad P = \vec{F}.\frac{d\vec{s}}{dt} \quad \text{or} \quad P = \vec{F}.\vec{v}$$

Dimensions : $[M^1L^2T^{-3}]$ SI UNIT : $\text{watt} = \left(\dfrac{\text{joule}}{\text{second}}\right)$

Some other UNITS of power :

1 kilo watt (kW) = 1000 watt = 10^3 W

1 mega watt = 1000,000 watt or 1MW = 10^3 kW

1 horse power (h.p.) = 746 W = 550 ft. lb/s

FPS UNIT : (foot × pound)/second, 1W = 0.738 ft. lb/s

A convenient rule of thumb approximation is

$$1hp \approx \frac{3}{4}\,kW$$

Efficiency

Machines are designed to convert energy into some of the useful work however, because of frictional effects, the work performed by the machine is always less than the energy put into machine.

The efficiency (η) of a machine $\eta = \dfrac{\text{work done}}{\text{energy input}}$

FACTS:

- When an agent delivers power at a uniform rate, the average power is equal to the instantaneous power.
- Power is the ratio of two scalars (Also' it is the scalar product of two vectors) so power is scalar quantity.
- When the time taken to complete a given amount of work is important we measure the power of agent doing work.
- The slope of work – time graph gives the instantaneous power

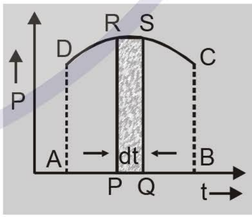
$$\text{slope} = \tan\theta = \frac{dW}{dt} = P\,(\text{instantaneous power})$$

- Area under power – time graph gives the work done.

Area under P–t graph = Area ABCD

$= \Sigma(PR \times PQ) = \Sigma(P \times dt)$

Area under power – time graph $= \int P dt = W$



CONSERVATIVE AND NONCONSERVATIVE FORCE

Conservative Force

A force is said to be conservative if work done by the force on a particle moving between two points does not depends on the path taken by the particle.

Nonconservative Force

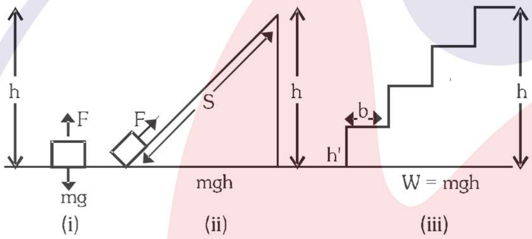
A force is said to be non conservative, if work done by the force or against the force in moving a body from one position to another, depends on the path.

Difference Between Conservative and Non-conservative Forces

Conservative force	Non-conservative force
(i) Work done does not depend upon path	(i) Work done depends upon path
(ii) Work done in a round trip is zero	(ii) Work done in a round trip is not zero
(iii) When only a conservative force acts within a system, the kinetic energy and potential energy can change. However their sum, the mechanical energy of the system does not change.	(iii) Work done against a non-conservative force may be dissipated as heat energy.
(iv) Work done is completely recoverable.	(iv) Work done is not completely recoverable.

Example of Conservative Force and Non-conservative Force

Conservative force	Non-conservative force
(i) Gravitational force.	(i) Force of friction.
(ii) Electrostatic force.	(ii) Viscous force.
(iii) Spring or restoring force.	(iii) Force due to air resistance.
(iv) Intermolecular force.	(iv) Magnetic force due to current carring element.
(v) Magnetic force produced by bar magnet.	



In all 3 cases work done by gravity is same irrespective of their different paths. (W = –mgh)

Central Force

The forces acting along the line joining the particles are called as central force.

$$\vec{F}_{central} = C.F(r).\hat{r} \quad [C = \text{constant},\ F(r) = \text{function of position},\ \hat{r} = \text{direction}]$$

eg. Gravitational force, electrostatic force

FACTS:

- All central forces are conservative but all conservative forces are not central forces.
- The concept of potential energy exists only in the case of conservative force.

ENERGY

Energy is defined as the internal capacity of doing work.
When we say that a body has energy it mean that it can do work.

(i) Different forms of energy :

Mechanical energy, electrical energy, optical (light) energy, acoustical (sound)energy, molecular energy, atomic and nuclear energy.
These forms of energy can change from one form to the other.

(ii) Mass energy relation

According to Einstein mass energy equivalence principle, mass and energy are inter convertible i.e. they can be changed into each other.
Equivalent energy corresponding to mass m is $E = mc^2$
where, m : mass of the particle   c : speed of light

- Energy is a scalar quantity
- Dimensions : $[M^1L^2T^{-2}]$
- SI UNIT : joule

Other units 1 erg = 10^{-7} joule   1kWh = 36×10^5 joule
1 eV = 1.6×10^{-19} joule   1 cal = 4.2 joule

In mechanics we only concerned with mechanical energy which is of two type.
(a) kinetic energy   (b) Potential energy

Kinetic Energy

Kinetic energy is the internal capacity of doing work of the object by virtue of its motion.

or

K.E. of a body can be calculated by the amount of work done in stopping the moving body. or from the amount of work done in giving the present velocity to the body from the state of rest.

If a particle of mass m is moving with velocity 'v' much less than the velocity of the light then the kinetic energy K.E. is given by $K.E. = \frac{1}{2}mv^2$

Relation between K.E. (K) and linear momentum (p) :

$p = mv$ and $K = \frac{1}{2}mv^2 = \frac{1}{2m}(m^2v^2) = \frac{p^2}{2m}$

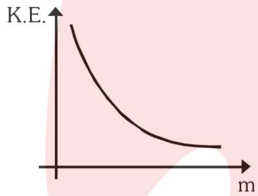
$K = \frac{p^2}{2m}$ or $p = \sqrt{2mK}$

Graph

(i) $K.E. = \frac{p^2}{2m}$ if p = constant.

$K.E. \propto \frac{1}{m}$

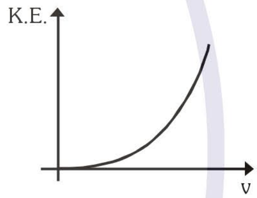
$y \propto \frac{1}{x}$



(ii) $K.E. = \frac{1}{2}mv^2$

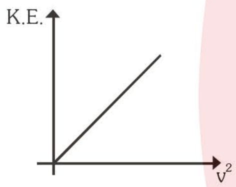
$K.E. \propto v^2$

$y \propto x^2$



(iii) $K.E. \propto v^2$

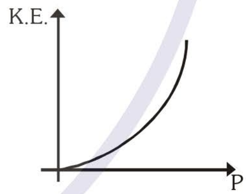
$y \propto x$



(iv) $K.E. = \frac{p^2}{2m}$

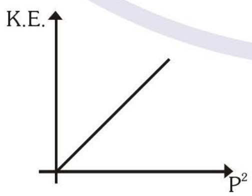
$K.E. \propto p^2$

$y \propto x^2$



(v) $K.E. \propto p^2$

$y \propto x$



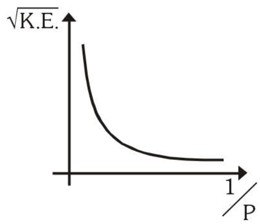
(vi) $2\,m\,K.E. = p^2$

$\sqrt{2mK.E.} = p$

$\sqrt{K.E.} \propto p$

$\sqrt{K.E.} \propto \frac{1}{1/p}$

$y \propto \frac{1}{x}$



Questions based on % change:-

Case-I: change $\leq 5\%$

(i) $K.E. = \frac{P^2}{2m} \Rightarrow K.E. \propto P^2 \Rightarrow \boxed{\frac{\Delta K}{K} = \frac{2\Delta P}{P}}$

(ii) $P = \sqrt{2mK.E.} \Rightarrow P \propto \sqrt{K.E.} \Rightarrow P \propto (K.E.)^{1/2} \Rightarrow \boxed{\frac{\Delta P}{P} = \frac{1}{2}\frac{\Delta K}{K}}$

Case-II: More than 5% ($\geq 5\%$): Here E = Kinetic Energy

$$E \propto P^2 \Rightarrow \frac{\Delta E}{E} = \left[\frac{P_2^2 - P_1^2}{P_1^2}\right] \times 100 \Rightarrow \boxed{\frac{\Delta E}{E} = \left[\left(\frac{P_2}{P_1}\right)^2 - 1\right] \times 100}$$

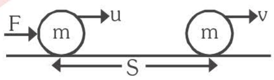
$$P \propto \sqrt{E} \Rightarrow \frac{\Delta P}{P} = \left[\frac{\sqrt{E_2} - \sqrt{E_1}}{\sqrt{E_1}}\right] \times 100 \Rightarrow \boxed{\frac{\Delta P}{P} = \left[\sqrt{\frac{E_2}{E_1}} - 1\right] \times 100}$$

Work Energy Theorem

Work done by all forces (Net force) = change in K.E. ($W = \Delta K.E.$)

Proof:

(i) For constant force



$v^2 = u^2 + 2as$

$v^2 - u^2 = 2\frac{F}{m}s$ $\qquad$ $F.S = (v^2 - u^2)m$

$F.S = \Delta K.E.$ $\qquad$ $\boxed{W = \Delta K.E.}$

(ii) For variable force

$$W = \int F\,dx = \int ma\,dx = \int m\left(\frac{vdv}{dx}\right)dx$$

$$W = \int_u^v mvdv \qquad W = m\left[\frac{v^2}{2}\right]_u^v$$

$$W = \frac{m}{2}\left[v^2 - u^2\right] \qquad \boxed{W = \Delta K.E.}$$

FACTS:

- If there is no change in the speed of a particle, there is no change in kinetic energy. So work done by the resultant force is zero.
- If K.E. of the body decreases then work done is negative i.e. the force opposes the motion of the body.
- If K.E. of the body increases then work done is positive.
- In above discussion, we have assumed that the work done by the force is effective only in changing the kinetic energy of the body. It should however be remembered that work done on a body may also be stored in the body in the form of potential energy.
- As mass m and v^2 or $\vec{v}.\vec{v}$ are always positive so K.E. can never be negative.
- The kinetic energy depends on the frame of reference.
- Work done on the body is the measure of K.E. of the body.
- The expression $K.E. = \frac{1}{2}mv^2$ holds even when the force applied varies in magnitude or in direction or in both.

Potential Energy

The energy possesed by any body or a system is due to change in its position, shape or configuration is called as potential energy.

or

Work done by external force against conservative force is equal to potential energy of that system.

Potential energy gradient : $\frac{dU}{dr} = \frac{d}{dr}w_{ex} = \frac{d}{dr}Fr\cos\theta$

For conservative force $\theta = 180°$

so $\frac{dU}{dr} = -F$ $\boxed{F_C = -\frac{dU}{dr}}$

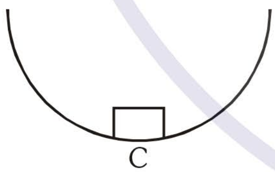
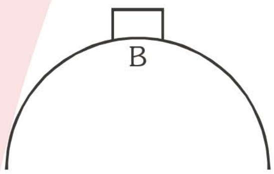
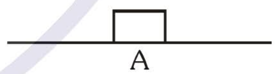
$\int_{U_1}^{U_2} dU = -\int_{r_1}^{r_2} \vec{F}_C.d\vec{r}$ $U_2 - U_1 = -W_c$ $\boxed{\Delta U = -W_c}$ or $\boxed{\Delta U = W_{ex}}$

FACTS:

- It is a scalar quantity.
- It can be positive or negative or zero.
- It is a relative quantity & it depends on frame of reference but change in potential energy is independent of frame of reference.
- Potential energy is always defined only for conservative force fields.
- Whenever work is done by the conservative forces potential energy decreses & whenever work is done against conservative forces, potential energy increases.
- It is function of position doesn't depend on path.

TRANSLATIONAL EQUILIBRIUM

A system is said to be in translational equilibrium, if net force on that system is zero.

	Types of equilibrium	
Stable equilibrium	***Unstable equilibrium***	***Neutral equilibrium***
		
$(F = 0, \frac{dF}{dx} < 0)$	$(F = 0, \frac{dF}{dx} > 0)$	$(F = 0, \frac{dF}{dx} = 0)$
$\frac{dU}{dx} = 0$ & $\frac{d^2U}{dx^2} > 0$	$\frac{dU}{dx} = 0$ & $\frac{d^2U}{dx^2} < 0$	$\frac{dU}{dx} = 0$ & $\frac{d^2U}{dx^2} = 0$
U_{min} (point C)	U_{max} (point B)	$U_{constant}$ (point A)

LAW OF CONSERVATION OF ENERGY

Energy neither be created nor be destroyed, it only can be converted from one form to other form

Law of Conservation of Mechanical Energy

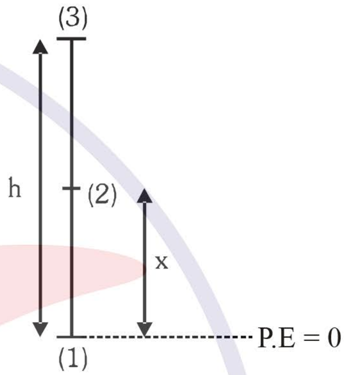
Under the effect of "conservative force" mechanical energy of the system always remains constant. This is called law of conservation of mechanical energy.

M.E. = K.E. + U = constant

$\Delta K + \Delta U = 0$ $\boxed{\Delta K = -\Delta U}$

(i) For gravitational field

Let a ball of mass m is dropped from position (3) (as shown in figure)



At point 1: PE = 0

$KE = \frac{1}{2}mv^2$ $\because v = \sqrt{2gh}$ So, KE = mgh

At point 2:

PE = mgx $KE = \frac{1}{2}mv^2$

$\because v = \sqrt{2g(h-x)}$ KE = mg(h – x)

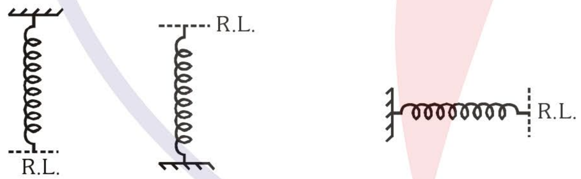
At point 3: PE = mgh KE = 0

so during motion of ball at any position

$\boxed{PE = mgx}$ $\boxed{KE = mg(h-x)}$

$\boxed{ME_{(1)} = ME_{(2)} = ME_{(3)}}$

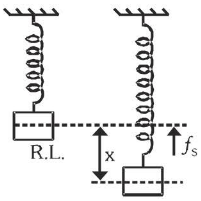
Spring Section



[R.L. = Reference level]

Step (1)- Reference level at the free end of natural length of spring.

Step (2)- If x is extension/compression in spring then spring force is in the opposite direction of extension/compression and proportional to x.



$f_s \propto -x$

$f_s = -Kx$

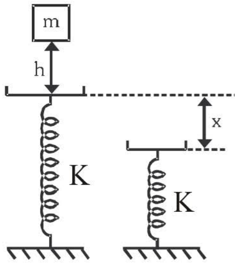
where K = spring force constant or stiffness constant.

Step(3)- Potential energy of spring. $PE = \frac{1}{2}Kx^2$

Special cases:

(i) A particle of mass m is freely released from height h on a spring, spring is compressed by x then

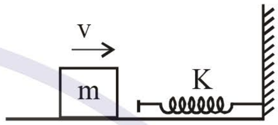
$$\boxed{mg(h+x)=\frac{1}{2}Kx^2}$$



(ii) If body of mass m moving with speed v collides with a spring and spring is compressed by x.

$$\frac{1}{2}mv^2=\frac{1}{2}Kx_{max}^2$$

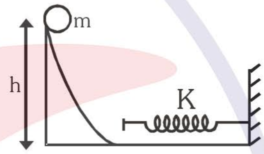
$$\boxed{x_{max}=v\sqrt{\frac{m}{k}}}$$



(iii) If body is released from hight h and collide with spring .

$$mgh=\frac{1}{2}Kx_{max}^2$$

$$\Rightarrow \quad \sqrt{\frac{2mgh}{K}}=x_{max}$$

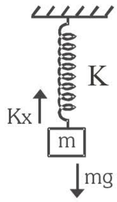


(iv) If block is attached with lower end of spring fixed at upper end in vertical position.

- Equilibrium condition: $F_{net}=0$

  $mg-Kx=0$

  $$\boxed{\frac{mg}{K}=x}$$



- ***Maximum extension:***

  Decrease in G.P.E. of block = increase in P.E. of spring

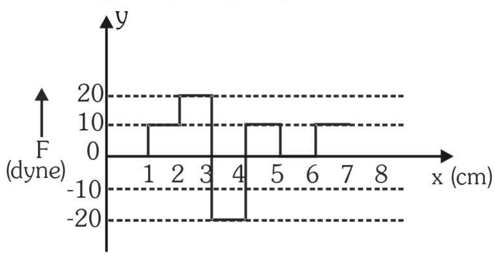
  $$mgx_{max}=\frac{1}{2}Kx_{max}^2 \qquad \boxed{\frac{2mg}{K}=x_{max}}$$

Illustration A position dependent force $F=7-2x+3x^2$ acts on a small body of mass 2 kg and displaces it from x = 0 to x = 5 m. Calculate the work done in joules.

Solution

$$W=\int_{x_1}^{x_2}Fdx=\int_0^5(7-2x+3x^2)dx=\left[7x-\frac{2x^2}{2}+\frac{3x^3}{3}\right]_0^5=135\,J.$$

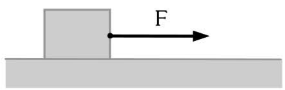
Illustration Corresponding to the force-displacement diagram shown in adjoining diagram, calculate the work done by the force in displacing the body from x=1 cm to x=5 cm.



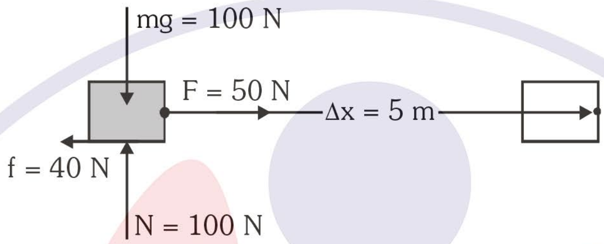
Solution Work = Area between the curve and displacement axis

= 10 + 20 − 20 + 10 = 20 ergs.

Illustration A 10 kg block placed on a rough horizontal floor is being pulled by a constant force of 50 N. Coefficient of kinetic friction between the block and the floor is 0.4. Find the work done by each individual force acting on the block over a displacement of 5 m.



Solution Forces acting on the block are (i) its weight (mg = 100 N),



(ii) normal reaction (N = 100 N) by the ground, (iii) force of kinetic friction (f = 40 N) and

(iv) the applied force (F = 50 N). All these force and the displacement of the block are shown in the figure.

All these forces are constant forces, therefore we use equation $W_{i \to f} = \vec{F} \cdot \Delta\vec{r}$.

Work done W_g by the gravity i.e. weight of the block $W_g = 0$ $\left(\because m\vec{g} \perp \Delta\vec{x}\right)$

Work done W_N by the normal reaction $W_N = 0$ $\left(\because \vec{N} \perp \Delta\vec{x}\right)$

Work done W_F by the applied force $W_F = 250$ J $\left(\because \vec{F} \parallel \Delta\vec{x}\right)$

Work done W_f by the force of kinetic friction $W_f = -200$ J. $\left(\because \vec{f} \uparrow\downarrow \Delta\vec{x}\right)$

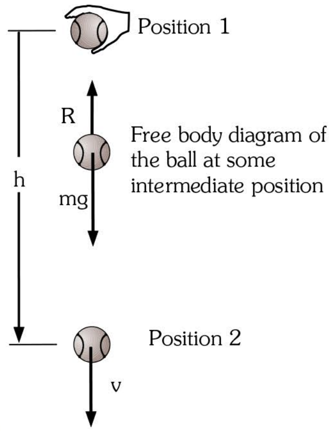
Illustration Calculate the work done by the force $\vec{F} = \left(3\hat{i} - 2\hat{j} + 4\hat{k}\right)$ N in carrying a particle from point (–2 m,1 m,3 m) to point (3 m,6 m,–2 m).

Solution The force $\vec{F}$ is a constant force, therefore we can use equation $W_{i \to f} = \vec{F} \cdot \Delta\vec{r}$.

$$W = \vec{F} . \Delta\vec{r} = (3\hat{i} - 2\hat{j} + 4\hat{k}).(5\hat{i} + 5\hat{j} - 5\hat{k}) = -15 \text{ J}.$$

Illustration A 5 kg ball when falls through a height of 20 m acquires a speed of 10 m/s. Find the work done by air resistance.

Solution The ball starts falling from position 1, where its speed is zero; hence, kinetic energy is also zero.



$K_1 = 0$ J ...(i)

During the downward motion of the ball, constant gravitational force mg acts downwards and air resistance R of unknown magnitude acts upwards as shown in the free body diagram. The ball reaches position 20 m below the position-1 with a speed v = 10 m/s, so the kinetic energy of the ball at position 2 is

$K_2 = \frac{1}{2}mv^2 = 250$ J ...(ii)

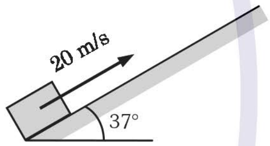
Work done by gravity

$W_{g,1\to2} = mgh = 1000$ J ...(iii)

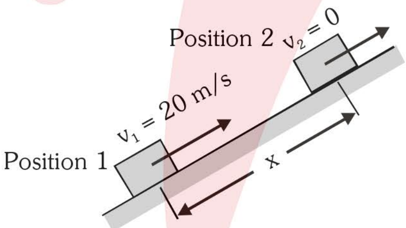
Denoting the work done by the air resistance as $W_{R,1\to2}$ and making use of eq. (i), (ii) and (iii) in work-kinetic energy theorem, we have

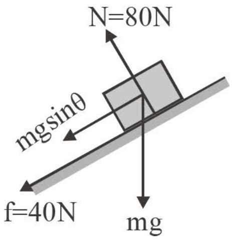
$W_{1\to2} = K_2 - K_1 \Rightarrow W_{g,1\to2} + W_{R,1\to2} = K_2 - K_1 \Rightarrow W_{R,1\to2} = -750\,J$.

Illustration A box of mass m = 10 kg is projected up an inclined plane from its foot with a speed of 20 m/s as shown in the figure. The coefficient of friction μ between the box and the plane is 0.5. Find the distance travelled by the box on the plane before it stops for the first time.



Solution The box starts from position 1 with speed v_1 = 20 m/s and stops at position 2.





Kinetic energy at position 1: $K_1 = \frac{1}{2}mv_1^2 = 2000$ J

Kinetic energy at position 2: $K_2 = 0$

Work done by external forces as the box moves from position 1 to position 2 is,

$W_{1\to2} = W_{g,1\to2} + W_{f,1\to2} = -60x - 40x = -100x$ J

Applying work energy theorem for the motion of the box from position 1 to position 2, we have $W_{1\to2} = K_2 - K_1 \Rightarrow -100\,x = 0 - 2000 \Rightarrow x = 20$ m.

Illustration A particle of mass m moves with velocity $v = a\sqrt{x}$ where a is a constant. Find the total work done by all the forces during a displacement from x = 0 to x = d.

Solution Work done by all forces $= W = \Delta KE = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

Here $v_1 = a\sqrt{0} = 0$, $v_2 = a\sqrt{d}$, So $W = \frac{1}{2}ma^2d - 0 = \frac{1}{2}ma^2d$.

Illustration The potential energy for a conservative force system is given by $U = ax^2 - bx$, where a and b are constants. Find out the (a) expression for force, (b) equilibrium position and (c) potential energy at equilibrium.

Solution (a) For conservative force $F = -\frac{dU}{dx} = -(2ax - b) = -2ax + b$

(b) At equilibrium $F = 0 \Rightarrow -2ax + b = 0 \Rightarrow x = \frac{b}{2a}$

(c) $U = a\left(\frac{b}{2a}\right)^2 - b\left(\frac{b}{2a}\right) = \frac{b^2}{4a} - \frac{b^2}{2a} = -\frac{b^2}{4a}$.

Illustration The potential energy of a particle of mass 1 kg free to move along the x-axis is given by $U(x) = \left(\frac{x^2}{2} - x\right)$ joules. If total mechanical energy of the particle is 2 J, then find its maximum speed.

Solution Potential energy $U = \left(\frac{x^2}{2} - x\right) = \frac{x^2}{2} - x$

For minimum U, $\frac{dU}{dx} = \frac{2x}{2} - 1 = 0$ and $\frac{d^2U}{dx^2} = 1 =$ positive

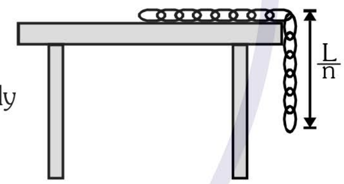
so at x = 1, U is minimum. Hence $U_{min} = -\frac{1}{2}$ J.

Total mechanical energy = Max KE + Min PE

$\Rightarrow$ Max KE $= \frac{1}{2}mv^2_{max} = 2 - \left(-\frac{1}{2}\right) = \frac{5}{2} \Rightarrow v_{max} = \sqrt{\frac{2}{1} \times \frac{5}{2}} = \sqrt{5}\ ms^{-1}$.

Illustration A chain of mass m and length L is held on a frictionless table in such a way that $\frac{1}{n}$th part is hanging below the edge of table. Calculate the work done to pull the hanging part of the chain.

Solution Required work done = change in potential energy of chain



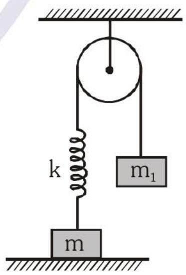
Now, let Potential energy (U) = 0 at table level

so potential energies of chain initially and finally are respectively

$U_i = -mg\left(\frac{L}{2n}\right) = -\left(\frac{M}{L}\right)\frac{L}{n}g\left(\frac{L}{2n}\right) = -\frac{MgL}{2n^2}$, $U_f = 0$

$\therefore$ required work done $= U_f - U_i = \frac{MgL}{2n^2}$

Illustration For what minimum value of m_1 will the block of mass m just leave contact with the surface ?



Solution Let extension in the spring be x_0 due to m_1

then $m_1gx_0 = \frac{1}{2}kx_0^2 \Rightarrow kx_0 = 2m_1g$

but $kx_0 \geq mg$ so $2m_1g \geq mg \Rightarrow m_1 \geq \frac{m}{2}$; therefore minimum value of $m_1 = \frac{m}{2}$.

Illustration A spring is initially compressed by x and then, it is further compressed by y. Find out the work done during the latter compression. (spring constant is k.)

Solution $W_1 = \frac{1}{2}kx^2$; $W_2 = \frac{1}{2}k(x+y)^2$

$W.D. = W_2 - W_1 = \frac{1}{2}k(x^2 + y^2 + 2xy) - \frac{1}{2}kx^2$

$\frac{1}{2}k(y^2 + 2xy) \Rightarrow \frac{1}{2}ky(y + 2x)$

Illustration A body of mass m starting from rest from the origin moves along the x-axis with a constant power (P). Calculate the :

(i) relation between velocity and time. (ii) relation between distance and time.

(iii) relation between velocity and distance.

Solution (i) $P = Fv = mav = m\frac{dv}{dt}v \Rightarrow \int_0^t \frac{P}{m}dt = \int_0^v vdv \Rightarrow \frac{P}{m}t = \frac{v^2}{2}$

$$\Rightarrow \quad v = \sqrt{\frac{2P}{m}t} \quad \Rightarrow \quad \boxed{v \propto t^{1/2}} \quad .....(1)$$

(ii) $\frac{dx}{dt} = \sqrt{\frac{2P}{m}}t^{1/2} \Rightarrow \int_0^x dx = \sqrt{\frac{2P}{m}}\int_0^t t^{1/2}dt$

$$\Rightarrow \quad x = \sqrt{\frac{2P}{m}}\frac{2}{3}t^{3/2} \quad \Rightarrow \quad \boxed{x \propto t^{3/2}} \quad ....(2)$$

(iii) From (1) & (2), $x \propto \left(t^{1/2}\right)^3 \Rightarrow \boxed{x \propto v^3}$ ....(3)

Illustration A truck of mass 10,000 kg moves up an inclined plane rising 1 in 50 with a speed of 36 km/h. Find the power of the engine ($g = 10\ m/s^2$).

Solution Force against which work is done $F = mg\sin\theta = 10{,}000 \times 10 \times \frac{1}{50} = 2000\ N$

speed $v = \frac{36 \times 5}{18} = 10\ m/s$ so $P = 2000 \times 10 = 20\ kW$.

Illustration An engine pumps water of density ρ, through a hose pipe. Water leaves the hose pipe with a velocity v. Find the

(i) rate at which kinetic energy is imparted to water

(ii) power of the engine.

Solution (i) Rate of change of kinetic energy $= \frac{dE_k}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = \frac{1}{2}v^2\frac{dm}{dt} = \frac{1}{2}v^2\frac{d}{dt}(\rho Ax)$

$$= \frac{1}{2}\rho Av^2\frac{dx}{dt} = \frac{1}{2}\rho Av^3$$

(ii) Power $= Fv = \left(v\frac{dm}{dt}\right)v = v^2\frac{dm}{dt} = v^2(\rho Av) = \rho Av^3$