LAWS OF MOTION AND FRICTION

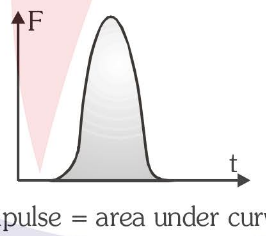
- **Force**
  A push or pull that one object exerts on another.
- **Forces in nature**
  There are four fundamental forces in nature :
  1. Gravitational force 2. Electromagnetic force 3. Strong nuclear force 4. Weak force
- **Types of forces on macroscopic objects**
  (a) Field Forces or Range Forces :
  These are the forces in which contact between two objects is not necessary.
  Ex. (i) Gravitational force between two bodies.
  (ii) Electrostatic force between two charges.
  (b) Contact Forces :
  Contact forces exist only as long as the objects are touching each other.
  Ex. (i) Normal forces. (ii) Frictional force
  (c) Attachment to Another Body :
  Tension (T) in a string and spring force (F = kx) comes in this group.
- **Newton's first law of motion (or Galileo's law of Inertia)**
  Every body continues in its state of rest or uniform motion in a straight line unless compelled by an external force to change that state.
  Inertia : Inertia is the property of the body due to which body opposes the change of it's state. Inertia of a body is measured by mass of the body.

$$\boxed{\text{inertia} \propto \text{mass}}$$

- **Newton's second law**

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt} \quad \text{(Linear momentum } \vec{p} = m\vec{v}\text{)}$$

  - For constant mass system $\vec{F} = m\vec{a}$
- **Momentum :** It is the product of the mass and velocity of a body i.e. momentum $\vec{p} = m\vec{v}$
  SI Unit : kg m s^{-1} **Dimensions** : $[M L T^{-1}]$
- **Impulse** : Impulse = product of force with time.



impulse = area under curve

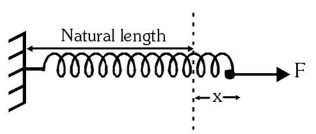
For a finite interval of time from t_1 to t_2 then the impulse = $\int_{t_1}^{t_2} \vec{F}dt$

If constant force acts for an interval Δt then : Impulse = $\vec{F}\Delta t$

Impulse – Momentum theorem

Impulse of a force is equal to the change of momentum $\boxed{\vec{F}\Delta t = \Delta\vec{p}}$

- **Newton's third law of motion** : Whenever a particle A exerts a force on another particle B, B simultaneously exerts a force on A with the same magnitude in the opposite direction.
- **Spring Force (According to Hooke's law) :**
  In equilibrium F=kx (k is spring constant)



Note : Spring force is non impulsive in nature.

- **Motion of bodies in contact**

  When two bodies of masses m_1 and m_2 are kept on the frictionless surface and a force F is applied on one body, then the force with which one body presses the other at the point of contact is called force of contact. These two bodies will move with same acceleration a.

  (i) When the force F acts on the body with mass m_1 as shown in fig.(i)

  $$F = (m_1 + m_2)a$$

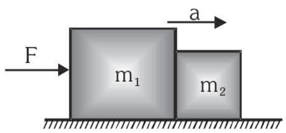


Fig.(1) : When the force F acts on mass m_1

If the force exerted by m_2 on m_1 is f_1 (force of contact) then for body m_1, $(F - f_1) = m_1 a$

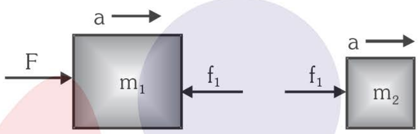


Fig. 1(a) : F.B.D. representation of action and reaction forces.

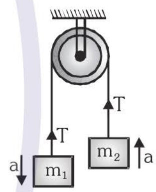
For body m_2, $f_1 = m_2 a \Rightarrow$ action of m_1 on m_2: $f_1 = \dfrac{m_2 F}{m_1 + m_2}$

- **Pulley system**

  A single fixed pulley changes the direction of force only and in general, assumed to be massless and frictionless.

SOME CASES OF PULLEY

Case – I



Let $m_1 > m_2$

now for mass m_1, $m_1 g - T = m_1 a$

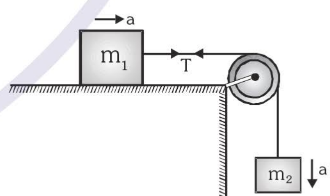
for mass m_2, $T - m_2 g = m_2 a$

$$\text{Acceleration} = a = \frac{(m_1 - m_2)}{(m_1 + m_2)} g = \frac{\text{net pulling force}}{\text{total mass to be pulled}}$$

$$\text{Tension} = T = \frac{2m_1 m_2}{(m_1 + m_2)} g = \frac{2 \times \text{Product of masses}}{\text{Sum of two masses}} g$$

Reaction at the suspension of pulley $R = 2T = \dfrac{4m_1 m_2 g}{(m_1 + m_2)}$

Case – II



For mass m_1 : $T = m_1 a$

For mass m_2 : $m_2 g - T = m_2 a$

Acceleration $a = \dfrac{m_2 g}{(m_1 + m_2)}$ and $T = \dfrac{m_1 m_2}{(m_1 + m_2)} g$ Force on pulley = $T\sqrt{2}$

FRAME OF REFERENCE

- **Inertial frames of reference :** A reference frame which is either at rest or in uniform motion along the straight line with respect to an inertial frame. A non–accelerating frame of reference is called an inertial frame of reference.

  All the fundamental laws of physics have been formulated in respect of inertial frame of reference.

- **Non–inertial frame of reference :** An accelerating frame of reference is called a non–inertial frame of reference. Newton's laws of motion are not directly applicable in such frames, before application we must add pseudo force.

- **Pseudo force**: The force on a body due to acceleration of non–inertial frame is called fictitious or apparent or pseudo force and is given by $\vec{F} = -m\vec{a}_0$, where $\vec{a}_0$ is acceleration of non–inertial frame with respect to an inertial frame and m is mass of the particle or body. The direction of pseudo force must be opposite to the direction of acceleration of the non–inertial frame.

- When we draw the free body diagram of a mass, with respect to an **inertial frame of reference** we apply only the real forces (forces which are actually acting on the mass). But when the free body diagram is drawn from a non–inertial frame of reference a pseudo force (in addition to all real forces) has to be applied to make the equation $\vec{F} = m\vec{a}$ to be valid in this frame also.

- **Man in a Lift**

(a) If the lift moving with constant velocity v upwards or downwards. In this case there is no accelerated motion hence no pseudo force experienced by observer inside the lift.

So apparent weight $W'=Mg$=Actual weight.

(b) If the lift is accelerated upward with constant acceleration a. Then forces acting on the man w.r.t. observer inside the lift are

(i) Weight $W=Mg$ downward

(ii) Fictitious force $F_0=Ma$ downward.

So apparent weight $W'=W+F_0=Mg+Ma=M(g+a)$

(c) If the lift is accelerated downward with acceleration $a<g$.

Then w.r.t. observer inside the lift fictitious force $F_0=Ma$ acts upward while weight of man $W=Mg$ always acts downward.

So apparent weight $W'=W+F_0 = Mg - Ma = M(g-a)$

Special Case :

If $a=g$ then $W'=0$ (condition of weightlessness).

Thus, in a freely falling lift the man will experience weightlessness.

(d) If lift accelerates downward with acceleration $a > g$. Then as in Case (c). Apparent weight $W'=M(g-a)$ is negative, i.e., the man will be accelerated upward and will stay at the ceiling of the lift.

FRICTION

Friction is the force between two surfaces in contact which opposes relative motion. or the force of a medium acting on a moving object. (i.e. air on aircraft.)

Frictional forces arise due to molecular interactions. In some cases friction acts as a supporting force and in some cases it acts as opposing force.

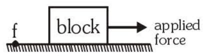
- **Cause of Friction:** Friction is arises on account of strong atomic or molecular forces of attraction between the two surfaces at the point of actual contact.

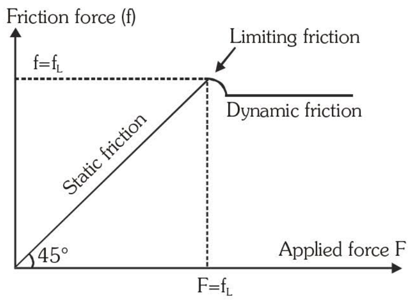
- **Types of friction**

Friction

- Static friction (No relative motion between objects)
- Dynamic friction (Kinetic friction) (There is relative motion between objects)

- **Graph between applied force and force of friction**



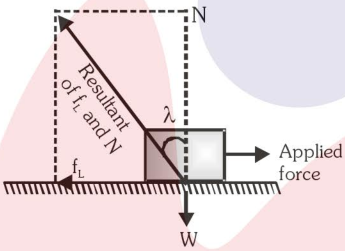


- **Static friction coefficient** $\mu_s = \frac{f_L}{N}$, $0 \le f_s \le \mu_s N$, $\vec{f}_s = -\vec{F}_{applied}$

  $(f_s)_{max} = \mu_s N =$ limiting friction

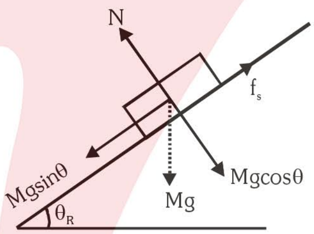
- **Sliding friction coefficient** $\mu_k = \frac{f_k}{N}$, $\vec{f}_k = -(\mu_k N)\hat{v}_{relative}$

- **Angle of Friction (λ)**



$\tan\lambda = \frac{f_L}{N} = \frac{\mu_s N}{N} = \mu_s$

- **Angle of repose** : The maximum angle of an inclined plane for which a block remains stationary on the plane.



$\tan\theta_R = \mu_s$

- For smooth surface $\theta_R = 0$

KEY POINTS

- Aeroplanes always fly at low altitudes because according to Newton's III law of motion as aeroplane displaces air & at low altitude density of air is high.
- Rockets move by pushing the exhaust gases out so they can fly at low & high altitude.
- Pulling a lawn roller is easier than pushing it because pushing increases the apparent weight and hence friction.
- A moongphaliwala sells his moongphali using a weighing machine in an elevator. He gain more profit if the elevator is accelerating up because the apparent weight of an object increases in an elevator while accelerating upward.
- Pulling (figure I) is easier than pushing (figure II) on a rough horizontal surface because normal reaction is less in pulling than in pushing.

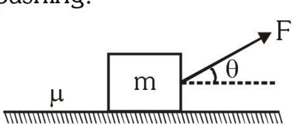


Fig. I

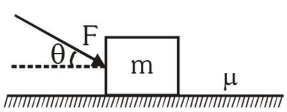
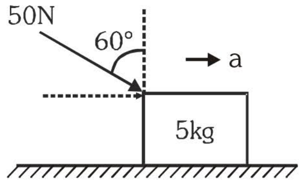


Fig. II

- While walking on ice, one should take small steps to avoid slipping. This is because smaller step ensure smaller friction.
- A man in a closed cabin (lift) falling freely does not experience gravity as inertial and gravitational mass have equivalence.

Illustration A force of 50 N acts in the direction as shown in figure. The block is of mass 5kg, resting on a smooth horizontal surface. Find out the acceleration of the block.

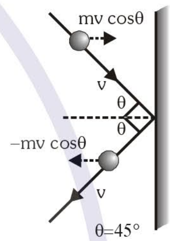


Solution Horizontal component of the force = 50 sin 60° = $\frac{50\sqrt{3}}{2}$

acceleration of the block, a = $\frac{\text{component of force in the direction of acceleration}}{\text{mass}}$

$= \frac{50\sqrt{3}}{2} \times \frac{1}{5} = 5\sqrt{3}\ \text{m/s}^2$

Illustration A ball of 0.20 kg hits a wall with a velocity of 25 m/s at an angle of 45°. If the ball rebounds at 90° to the direction of incidence, calculate the magnitude of change in momentum of the ball.



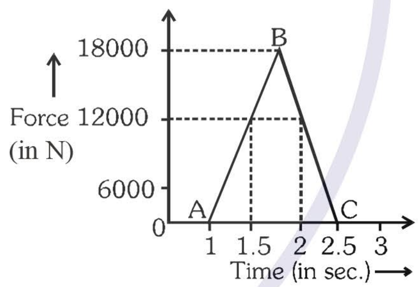
Solution Change in momentum = (– mv cos 45°) – (mv cos 45°) = – 2mv cos 45°

$|\Delta\vec{p}| = 2mv\cos45° = 2 \times 0.2 \times 25 \times \frac{1}{\sqrt{2}} = 5\sqrt{2}$ N-s

Illustration Figure shows an estimated force–time graph for a base ball struck by a bat. From this curve, determine

(a) Impulse delivered to the ball

(b) Average force exerted on the ball.



Solution

(a) Impulse = Area under F-t curve

= Area of ΔABC = $\frac{1}{2} \times 18000 \times (2.5 - 1)$

= 1.35×10^4 kg–m/s

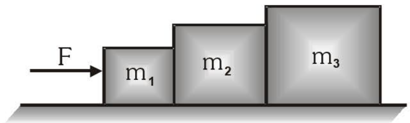
(b) Average force = $\frac{\text{Impulse}}{\text{Time}} = \frac{1.35 \times 10^4}{(2.5-1)} = 9000$ N

Illustration A 600 kg rocket is set for a vertical firing. If the exhaust speed of gases is 1000 m/s, then calculate the mass of gas ejected per second to supply the thrust needed to overcome the weight of rocket.

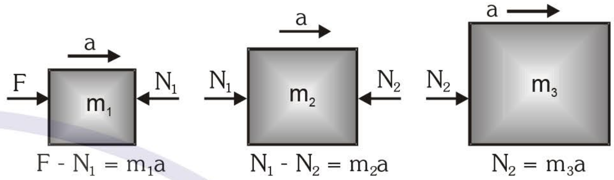
Solution Force required to overcome the weight of rocket F = mg and thrust needed = $v_{rel}\frac{dm}{dt}$

so $v_{rel}\frac{dm}{dt} = mg \Rightarrow \frac{dm}{dt} = \frac{mg}{v_{rel}} = \frac{600 \times 9.8}{1000} = 5.88$ kg/s

Illustration Three blocks of masses $m_1 = 1$ kg, $m_2 = 1.5$ kg and $m_3 = 2$ kg are in contact with each other on a frictionless surface as shown in fig. Find the (a) horizontal force F needed to push the blocks as a single unit with an acceleration of 4 m/s^2 (b) resultant force on each block and (c) magnitude of contact forces between the blocks.



Solution



(a) $F = (m_1 + m_2 + m_3)\,a$

$= (1 + 1.5 + 2) \times 4$

$= 4.5 \times 4 = 18$ N

(b) For m_1

$F - N_1 = m_1a = 1 \times 4$

$\Rightarrow$ $F - N_1 = 4$ N ......(i)

for m_2,

$N_1 - N_2 = m_2\,a = 1.5 \times 4 = 6$

$\Rightarrow$ $N_1 - N_2 = 6N$ ......(ii)

for m_3,

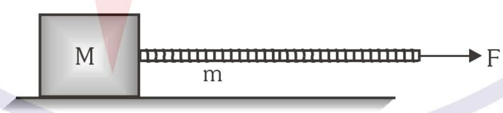
$N_2 = m_3a = 2 \times 4$

$\Rightarrow$ $N_2 = 8N$ ......(iii)

(c) Contact force between m_2 and m_3 is $N_2 = 8$ N

and contact force between m_1 and m_2 is $N_1 = N_2 + 6 = 8 + 6 = 14N$.

Illustration A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m as shown in fig. A horizontal force F is applied to one end of the rope. Find the (i) Acceleration of the rope and the block (ii) Force that the rope exerts on the block. (iii) Tension in the rope at its mid point.

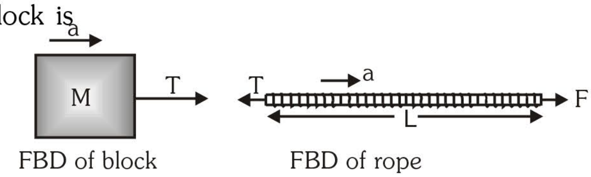


Solution

(i) Acceleration $a = \dfrac{F}{(m+M)}$

(ii) Force exerted by the rope on the block is

$$T = Ma = \frac{M.F}{(m+M)}$$



(iii) $T_1 = \left(\dfrac{m}{2} + M\right)a = \left(\dfrac{m+2M}{2}\right)\left(\dfrac{F}{m+M}\right)$

Tension in rope at midpoint is $T_1 = \dfrac{(m+2M)F}{2(m+M)}$

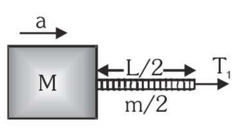
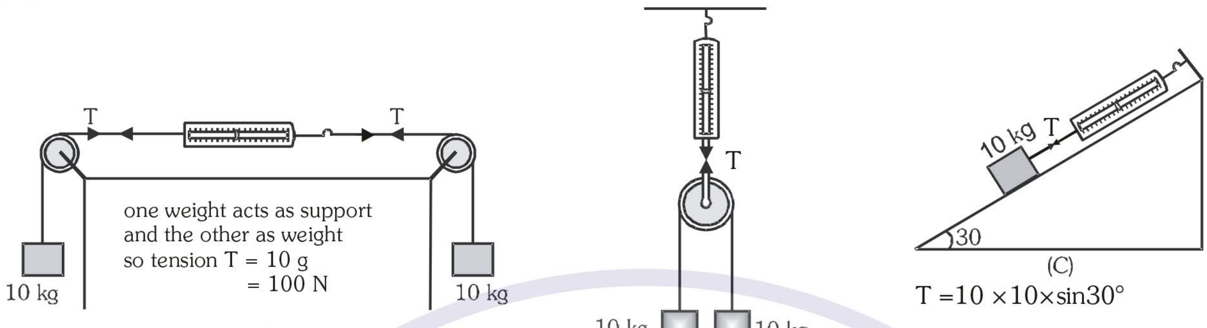


Illustration The system shown in fig. is in equilibrium. If the spring balance is calibrated in newtons, what does it record in each case? ($g = 10\ m/s^2$)

Solution



(A) (B) (C)

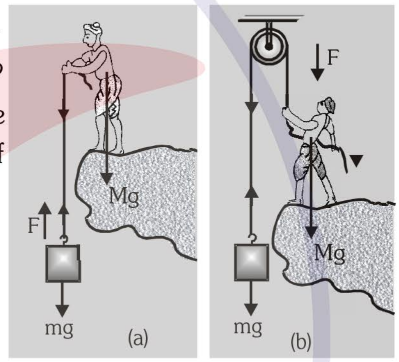
$T = 2\times10\times g = 2\times10\times10$

$= 200\ N$

$T = 10\times10\times\sin30°$

$= 10\times10\times\frac{1}{2} = 50\ N$

Illustration A block of mass 25 kg is raised in two different ways by a 50 kg man as shown in fig. What is the action in the two cases? If the floor yields to a normal force of 700 N, which mode should the man adopt to lift the block without the yielding of the floor?



Solution Mass of the block, m = 25 kg ;

mass of the man, M = 50 kg

Force applied to lift the block

$F = mg = 25\times9.8 = 245\ N$

Weight of the man,

$Mg = 50\times9.8 = 490\ N$

(a) When the block is raised by the man by applying a force F in the upward direction, reaction being equal and opposite to F will act on the floor in addition to the weight of the man.

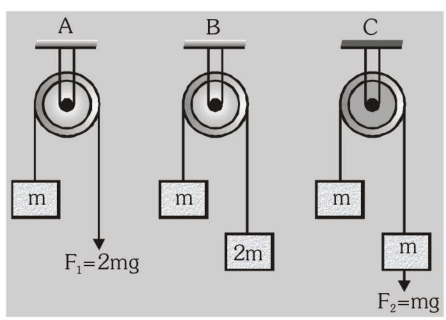
$\therefore$ Action on the floor $Mg + F = 490 + 245 = 735\ N$

(b) When the block is raised by the man applying force F over the rope (passing over the pulley) in the downward direction, reaction being equal and opposite to F will act on the floor against the weight of the man.

$\therefore$ Action on the floor $Mg - F = 490 - 245 = 245\ N$

since floor yields to a normal force of 700 N, mode (b) should be adopted by the man to lift the block.

Illustration In the figure blocks A, B and C have accelerations a_1, a_2 and a_3 respectively. F_1 and F_2 are external forces of magnitudes 2mg and mg respectively. Find the value of a_1, a_2 and a_3.

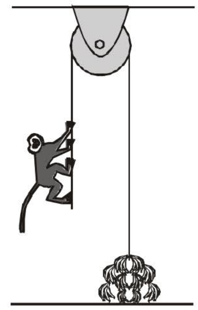


Solution

$$a_1 = \frac{2mg - mg}{m} = g \quad ; \quad a_2 = \frac{2m - m}{2m + m}g = \frac{g}{3}$$

$$a_3 = \frac{mg + mg - mg}{2m} = \frac{g}{2}\text{; Clearly } a_1 > a_3 > a_2.$$

Illustration A 12 kg monkey climbs a light rope as shown in fig. The rope passes over a pulley and is attached to a 16 kg bunch of bananas. Mass and friction in the pulley are negligible so that the effect of pulley is only to reverse the direction of force of the rope. What maximum acceleration can the monkey have without lifting the bananas? (Take g = 10 m/s²)



Solution For Monkey

$T - 120 = 12 \times a$ ....(i)

For Bananas

$160 - T = N$

Condition for just loosing the contact is N = 0

$160 - T = 0 \Rightarrow T = 160$ ....(ii)

from equation (i) & (ii)

$160 - 120 = 12 \times a \Rightarrow a = 3.33\ m/s^2$

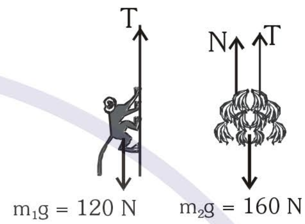
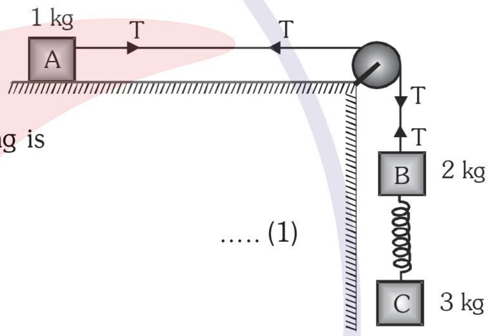


Illustration In the system shown in figure all surfaces are smooth, string is massless and inextensible. (in steady state) Find the

(a) acceleration of the system

(b) tension in the string and

(c) extension in the spring if force constant of spring is $k = 50$ N/m (Take g=10 m/s²)

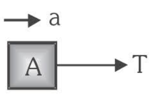


Solution (a) $3g - kx = 3a$ ..... (1)

$2g + kx - T = 2a$ ..... (2)

$T = a$ ..... (3)

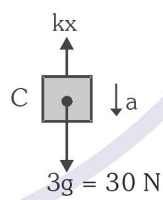
∴ Acceleration of the system is $a = \frac{50}{6}\ ms^{-2}$

(b) 

Free body diagram of 1 kg block gives $T = ma = (1)\left(\frac{50}{6}\right)N = \frac{50}{6}N$

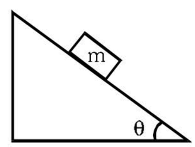
(c) Free body diagram of 3 kg block gives

$30 - kx = ma$ but $ma = 3 \times \frac{50}{6} = 25$ N



$x = \frac{30-25}{k} = \frac{5}{50} = 0.1$ m = 10 cm

Illustration What horizontal acceleration should be provided to the wedge so that the block of mass m placed on the wedge falls freely?



Solution For free fall, normal reaction should be zero

N = 0

For equilibrium perpendicular to the wedge

$0 + ma\sin\theta = mg\cos\theta$

$\Rightarrow \boxed{a = \frac{g}{\tan\theta}}$

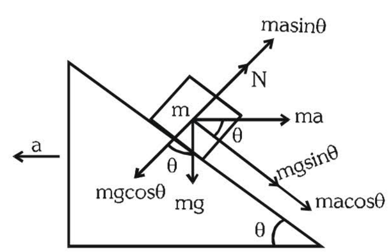
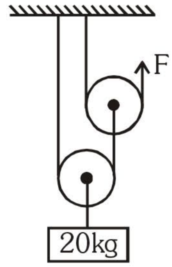


Illustration A student is able to lift a bag containing books of 20kg-wt by applying a force of 5kg-wt. Find the mechanical advantage.

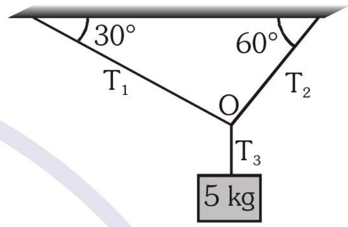


Solution W = 20kg-wt

F = 5kg-wt

$$M.A. = \frac{20}{5} = 4$$

Illustration Calculate the tensions T_1, T_2 and T_3 in the massless strings shown in figure ($g = 10\ ms^{-2}$)

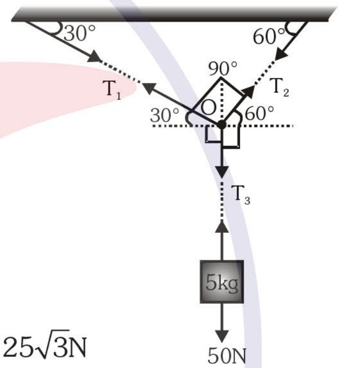


Solution Considering the adjoining figure

T_3 = wt. of the 5 kg block (mg)

$T_3 = 5 \times 10 = 50$ N

Now applying Lami's theorem at point O.

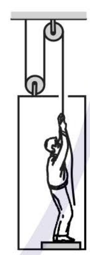


$$\frac{T_1}{\sin(90° + 60°)} = \frac{T_2}{\sin(90° + 30°)} = \frac{T_3}{\sin(180° - 60° - 30°)}$$

$$\Rightarrow \quad \frac{T_1}{\cos 60°} = \frac{T_2}{\cos 30°} = \frac{50}{\sin 90°}$$

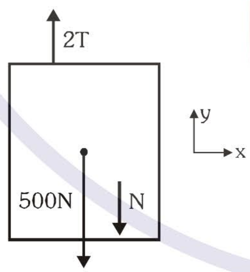
$$T_1 = 50\frac{\cos 60°}{\sin 90°} = 25\text{ N} \quad \text{and} \quad T_2 = 50\frac{\cos 30°}{\sin 90°} = 25\sqrt{3}\text{N}$$

Illustration A 70 kg man standing on a weighing machine in a 50 kg lift pulls on the rope, which supports the lift as shown in the figure. Find the force with which the man should pull the rope to keep the lift stationary. Also, find the weight of the man as shown by the weighing machine.



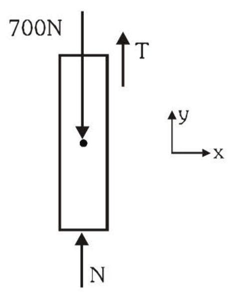
Solution Magnitude of tension everywhere in the string is same.

For equilibrium of the lift.



$$\sum F_y = 0 \quad \Rightarrow \quad 500 + N = 2T \quad \ldots(i)$$

To analyse the equilibrium of the man let us assume him as a block



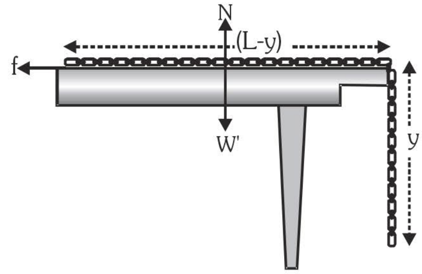
$$\sum F_y = 0 \quad \Rightarrow \quad N + T = 700 \quad \ldots(ii)$$

From equations (i) & (ii), we have T = 400 N and N = 300 N

Here, T is the pull of mass and N is the reading of the weighing machine.

Illustration Length of a chain is L and coefficient of static friction is μ. Calculate the maximum length of the chain which can hang from the table without sliding.

Solution



Let y be the maximum length of the chain that can be hang outside the table without sliding.

Length of chain on the table = $(L - y)$

Weight of the part of the chain on table $W' = \frac{M}{L}(L - y)g$

Weight of hanging part of the chain $W = \frac{M}{L}yg$

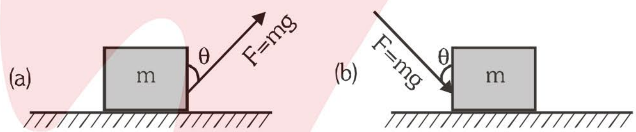
For equilibrium :

limiting force of friction on (L–y) length = weight of hanging part of the chain of y length

$\mu N = W \Rightarrow \mu W' = W$

$$\Rightarrow \mu\frac{M}{L}(L - y)g = \frac{M}{L}yg \Rightarrow \mu L - \mu y = y \Rightarrow y = \frac{\mu L}{1+\mu}$$

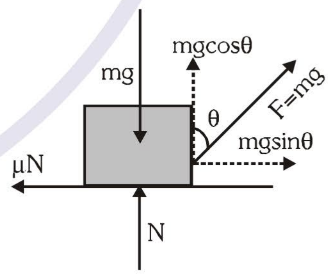
Illustration A block of mass m rests on a rough horizontal surface as shown in figure (a) and (b). Coefficient of friction between the block and surface is μ. A force F = mg act at an angle θ with the vertical side of the block. Find the condition for which the block will move along the surface.



Solution **For (a) :** normal reaction $N = mg - mg\cos\theta$, limiting frictional force $= \mu N = \mu(mg - mg\cos\theta)$

Now, block can be pulled when : Horizontal component of force $\geq$ limiting frictional force

i.e. $mg\sin\theta \geq \mu(mg - mg\cos\theta)$



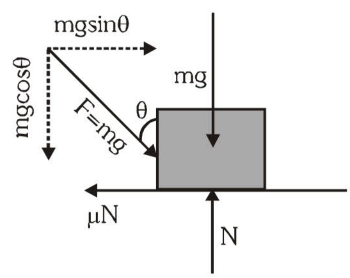
$$\Rightarrow \quad 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} \geq \mu(1 - \cos\theta)$$

$$\Rightarrow \quad 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} \geq 2\mu\sin^2\frac{\theta}{2} \Rightarrow \cot\frac{\theta}{2} \geq \mu$$

For (b) : Normal reaction $N = mg + mg\cos\theta = mg(1 + \cos\theta)$

Hence, block can be pushed along the horizontal surface when horizontal component of force $\geq$ limiting frictional force

i.e. $mg\sin\theta \geq \mu\, mg(1 + \cos\theta)$



$$\Rightarrow \quad 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} \geq \mu \times 2\cos^2\frac{\theta}{2} \Rightarrow \tan\frac{\theta}{2} \geq \mu$$

Illustration A body of mass m rests on a horizontal floor with which it has a coefficient of static friction μ. It is desired to make the body move by applying the minimum possible force F. Find the magnitude of F and the direction in which it has to be applied.

Solution Let the force F be applied at an angle θ with the horizontal as shown in figure.

For vertical equilibrium,

$N + F\sin\theta = mg$ i.e $N = mg - F\sin\theta$ ......(i)

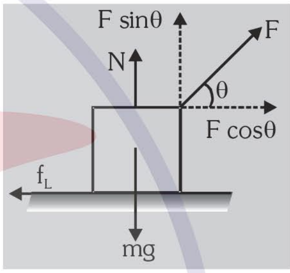
for horizontal motion

$F\cos\theta \geq f_L$ i.e. $F\cos\theta \geq \mu N$ [as $f_L = \mu N$] ......(ii)

substituting expression for N from equation (i) in (ii),

$F\cos\theta \geq \mu(mg - F\sin\theta) \Rightarrow F \geq \dfrac{\mu mg}{(\cos\theta + \mu\sin\theta)}$ ......(iii)

For the force F to be minimum $(\cos\theta + \mu\sin\theta)$ must be maximum,



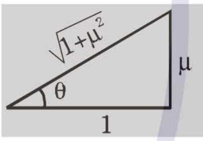
$$\frac{d}{d\theta}(\cos\theta + \mu\sin\theta) = 0$$

or $-\sin\theta + \mu\cos\theta = 0$ i.e., $\tan\theta = \mu$ ..... (iv)

$\therefore$ $\sin\theta = \dfrac{\mu}{\sqrt{1+\mu^2}}$ and $\cos\theta = \dfrac{1}{\sqrt{1+\mu^2}}$

substituting these in equation (iii)

$$F \geq \frac{\mu mg}{\dfrac{1}{\sqrt{1+\mu^2}} + \dfrac{\mu^2}{\sqrt{1+\mu^2}}} \quad \text{i.e.} \quad F \geq \frac{\mu mg}{\sqrt{1+\mu^2}}$$



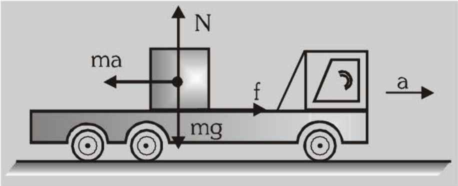
so that $F_{min} = \dfrac{\mu mg}{\sqrt{1+\mu^2}}$ with $\theta = \tan^{-1}(\mu)$

Note : As $-\sqrt{A^2+B^2} \leq A\sin\theta + B\cos\theta \leq \sqrt{A^2+B^2}$

So $(\cos\theta + \mu\sin\theta)_{max} = \sqrt{1+\mu^2}$ Therefore $F_{min} = \dfrac{\mu mg}{\sqrt{1+\mu^2}}$

Illustration A block of mass 1kg lies on a horizontal surface of a truck; the coefficient of static friction between the block and the surface is 0.6, What is the force of friction on the block, if the acceleration of the truck is 5 m/s^2.

Solution Fictitious force (pseudo force) on the block opposite to the acceleration of the block $F = ma = 1 \times 5 = 5N$



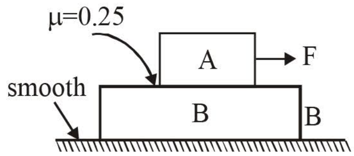
While the limiting friction force

$f_L = \mu_s N = \mu_S mg$

$= 0.6 \times 1 \times 9.8 = 5.88$ newton

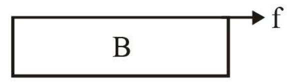
As applied force F is less than the limiting friction force, the block will remain at rest in the truck and the force of friction will be equal to 5 N and in the direction of acceleration of the truck.

Illustration In the figure shown m_A = 10 kg, m_B = 15 kg. Find the maximum value of F, below which the blocks move together.



Solution Assuming that both blocks move together, their common acceleration is $a_C = \frac{F}{10+15} = \frac{F}{25}$

FBD of block B (on which no externally applied force acts).

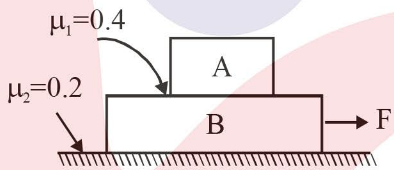


The required friction force f is equal to $f = m_B a_C = \frac{15F}{25} = \frac{3F}{5}$

Now, maximum static friction available is $f_L = \mu N_1 = 0.25(100) = 25$ N (here $N_1 = m_A g = 100N$)

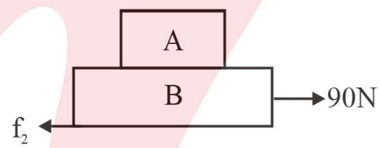
$\therefore f \le f_L \Rightarrow \frac{3F}{5} \le 25 \Rightarrow F \le \frac{125}{3} N \Rightarrow F_{max} = \frac{125}{3} N$

Illustration For the figure shown m_A = 10 kg , m_B = 15 kg and F = 90 N. Find the accelerations of the blocks and the frictional forces acting.



Solution **Step1 :** Draw the FBD of the combined blocks system.

$(f_2)_L = \mu_2 N_2 = 0.2(25g) = 50N \ (\because N_2 = 25g)$



Since 90 N > 50N, net unbalanced forces appear and hence movement begins.

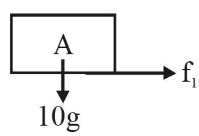
Step 2 : Assuming that both the blocks move together, their combined acceleration is

$$a_C = \frac{90-50}{25} = 1.6 \text{ m/s}^2$$

Draw the FBD of block A (on which externally applied force does not act).

The force required is $f_1 = m_A a_C = 10 \times 1.6 = 16$ N

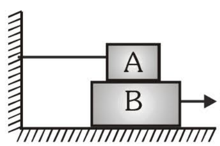
Now, $(f_1)_L = \mu_1 N_1 = 0.4\ (10g) = 40$ N. Clearly, $f_1 < (f_1)_L$



$\therefore$ The frictional force is strong enough to support the combined motion.

$\therefore$ Common acceleration is $a_C = 1.6$ m/s² and $f_1 = 16$ N and $f_2 = 50$ N

Illustration A is a 100 kg block and B is a 200 kg block. As shown in figure, block A is attached to a string tied to a wall. The coefficient of friction between A and B is 0.2 and the coefficient of friction between B and floor is 0.3. Then calculate the minimum force required to move the block B. (take g =10 m/s²).



Solution When B is made to move, by applying a force F, the frictional forces acting on it are f_1 and f_2 with limiting values, $f_1 = (\mu_S)_A m_A g$ and $f_2 = (\mu_S)_B (m_A + m_B)g$

Then minimum value of F should be (such as to overcome these limiting values),

$$F_{min} = f_1 + f_2 = 0.2 \times 100\ g + 0.3 \times 300\ g = 110\ g = 1100\ N$$