DIFFERENTIAL EQUATION- PYQ

1. The differential equation whose solution is $Ax^2 + By^2 = 1$, where A and B are arbitrary constants is of- **[AIEEE-2006]**
(1) first order and second degree
(2) first order and first degree
(3) second order and first degree
(4) second order and second degree

2. The differential equation of all circles passing through the origin and having their centres on the x-axis is- **[AIEEE-2007]**
(1) $x^2 = y^2 + xy\frac{dy}{dx}$ (2) $x^2 + y^2 + 3xy\frac{dy}{dx} = 0$
(3) $y^2 + x^2 + 2xy\frac{dy}{dx} = 0$ (4) $y^2 = x^2 + 2xy\frac{dy}{dx}$

3. The solution of the differential equation $\frac{dy}{dx} = \frac{x+y}{x}$ satisfying the condition $y(1) = 1$ is- **[AIEEE-2008]**
(1) $y = \ell nx + x$ (2) $y = x\ell nx + x^2$
(3) $y = xe^{(x-1)}$ (4) $y = x\ell nx + x$

4. The differential equation of the family of circles with fixed radius 5 units and centre on the line $y = 2$ is- **[AIEEE-2008]**
(1) $(x - 2)y'^2 = 25 - (y - 2)^2$
(2) $(y - 2)y'^2 = 25 - (y - 2)^2$
(3) $(y - 2)^2y'^2 = 25 - (y - 2)^2$
(4) $(x - 2)^2y'^2 = 25 - (y - 2)^2$

5. The differential equation which represents the family of curves $y = c_1e^{c_2x}$, where c_1 and c_2 are arbitrary constants, is :- **[AIEEE-2009]**
(1) $yy'' = y'$ (2) $yy'' = (y')^2$
(3) $y' = y^2$ (4) $y'' = y'y$

6. Solution of the differential equation
$\cos x\, dy = y(\sin x - y)dx$, $0 < x < \frac{\pi}{2}$ is : **[AIEEE-2010]**
(1) $\sec x = (\tan x + c)\, y$
(2) $y \sec x = \tan x + c$
(3) $y \tan x = \sec x + c$
(4) $\tan x = (\sec x + c)\, y$

7. Consider the differential equation **[AIEEE-2011]**
$y^2dx + \left(x - \frac{1}{y}\right)dy = 0$. It $y(1) = 1$,
then x is given by :
(1) $1 - \frac{1}{y} + \frac{e^{\frac{1}{y}}}{e}$ (2) $4 - \frac{2}{y} - \frac{e^{\frac{1}{y}}}{e}$
(3) $3 - \frac{1}{y} + \frac{e^{\frac{1}{y}}}{e}$ (4) $1 + \frac{1}{y} - \frac{e^{\frac{1}{y}}}{e}$

8. Let I be the purchase value of an equipment and V(t) be the value after it has been used for t years. The value V(t) depreciates at a rate given by differential equation $\frac{dV(t)}{dt} = -k(T - t)$, where $k > 0$ is a constant and T is the total life in years of the equipment. Then the scrap value V(T) of the equipment is :- **[AIEEE-2011]**
(1) $I - \frac{k(T-t)^2}{2}$ (2) e^{-kT}
(3) $T^2 - \frac{I}{k}$ (4) $I - \frac{kT^2}{2}$

9. The population p(t) at time t of a certain mouse species satisfies the differential equation $\frac{dp(t)}{dt} = 0.5\,p(t) - 450$. If $p(0) = 850$, then the time at which the population becomes zero is: **[AIEEE-2012]**
(1) ln18 (2) 2 ln18
(3) ln9 (4) $\frac{1}{2}$ ln18

10. At present a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w.r.t. additional number of workers x is given by $\frac{dP}{dx} = 100 - 12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is : **[JEE(Main)-2013]**
(1) 2500 (2) 3000
(3) 3500 (4) 4500

11. Let the population of rabbits surviving at a time t be governed by the differential equation $\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 200$. If $p(0) = 100$, then p(t) equals: **[JEE(Main)-2014]**
(1) $400 - 300\, e^{t/2}$ (2) $300 - 200\, e^{-t/2}$
(3) $600 - 500\, e^{t/2}$ (4) $400 - 300\, e^{-t/2}$

12. Let y(x) be the solution of the differential equation
$(x \log x)\frac{dy}{dx} + y = 2x \log x$, $(x \geq 1)$.
Then y(e) is equal to : **[JEE (Main) 2015]**
(1) 2 (2) 2e
(3) e (4) 0

13. If a curve $y = f(x)$ passes through the point $(1, -1)$ and satisfies the differential equation, $y(1 + xy)\,dx = x\,dy$, then $f\left(-\frac{1}{2}\right)$ is equal to : **[JEE(Main)-2016]**

(1) $\frac{4}{5}$ (2) $-\frac{2}{5}$ (3) $-\frac{4}{5}$ (4) $\frac{2}{5}$

14. If $(2+\sin x)\frac{dy}{dx}+(y+1)\cos x=0$ and $y(0) = 1$, then $y\left(\frac{\pi}{2}\right)$ is equal to :- **[JEE(Main)-2017]**

(1) $\frac{4}{3}$ (2) $\frac{1}{3}$ (3) $-\frac{2}{3}$ (4) $-\frac{1}{3}$

15. Let $y = y(x)$ be the solution of the differential equation $\sin x \frac{dy}{dx} + y \cos x = 4x$, $x \in (0, \pi)$. If $y\left(\frac{\pi}{2}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to : **[JEE(Main)-2018]**

(1) $\frac{-8}{9\sqrt{3}}\pi^2$ (2) $-\frac{8}{9}\pi^2$

(3) $-\frac{4}{9}\pi^2$ (4) $\frac{4}{9\sqrt{3}}\pi^2$

16. The differential equation $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ determines a family of circles with- **[IIT-2007]**

(1) variable radii and a fixed centre at (0, 1)

(2) variable radii and a fixed centre at (0, –1)

(3) fixed radius 1 and variable centres along the x-axis

(4) fixed radius 1 and variable centres along the y-axis

17. Let f be a real-valued differentiable function on R (the set of all real numbers) such that f(1) = 1. If the y-intercept of the tangent at any point P(x,y) on the curve y = f(x) is equal to the cube of the abscissa of P, then the value of f(–3) is equal to :- **[IIT-2010]**

(1) –3 (2) 3

(3) 9 (4) –9

18. Let $f : [1,\infty) \to [2,\infty)$ be a differentiable function such that f(1) = 2. If $6\int_1^x f(t)dt = 3x\,f(x) - x^3$ for all $x \geq 1$, then the value of $f(2)$ is **[IIT 2011]**

***19.** If y(x) satisfies the differential equation $y' - y\tan x = 2x \sec x$ and $y(0) = 0$, then **[IIT-2012]**

(1) $y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{8\sqrt{2}}$

(2) $y'\left(\frac{\pi}{4}\right) = \frac{\pi^2}{18}$

(3) $y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{9}$

(4) $y'\left(\frac{\pi}{3}\right) = \frac{4\pi}{3} + \frac{2\pi^2}{3\sqrt{3}}$

20. A curve passes through the point $\left(1, \frac{\pi}{6}\right)$. Let the slope of the curve at each point (x, y) be $\frac{y}{x} + \sec\left(\frac{y}{x}\right), x > 0$. Then the equation of the curve is **[JEE(Adv.)-2013]**

(1) $\sin\left(\frac{y}{x}\right) = \log x + \frac{1}{2}$

(2) $\operatorname{cosec}\left(\frac{y}{x}\right) = \log x + 2$

(3) $\sec\left(\frac{2y}{x}\right) = \log x + 2$

(4) $\cos\left(\frac{2y}{x}\right) = \log x + \frac{1}{2}$

21. Let $f : \left[\frac{1}{2}, 1\right] \to R$ (the set of all real numbers) be a positive, non-constant and differentiable function such that $f'(x) < 2f(x)$ and $f\left(\frac{1}{2}\right) = 1$. Then the value of $\int_{1/2}^{1} f(x)dx$ lies in the interval **[JEE(Advanced) 2013]**

(1) (2e – 1, 2e) (2) (e – 1, 2e – 1)

(3) $\left(\frac{e-1}{2}, e-1\right)$ (4) $\left(0, \frac{e-1}{2}\right)$

22. The function $y = f(x)$ is the solution of the differential equation

$$\frac{dy}{dx}+\frac{xy}{x^2-1}=\frac{x^4+2x}{\sqrt{1-x^2}}$$ in $(-1,1)$ satisfying $f(0) = 0$.

Then $\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x)dx$ is **[JEE(Advanced)-2014]**

(1) $\frac{\pi}{3}-\frac{\sqrt{3}}{2}$ (2) $\frac{\pi}{3}-\frac{\sqrt{3}}{4}$

(3) $\frac{\pi}{6}-\frac{\sqrt{3}}{4}$ (4) $\frac{\pi}{6}-\frac{\sqrt{3}}{2}$

***23.** If $f : \mathbb{R} \to \mathbb{R}$ is a differentiable function such that $f'(x) > 2f(x)$ for all $x \in \mathbb{R}$, and $f(0) = 1$, then **[JEE(Advanced)-2017]**

(1) $f(x) > e^{2x}$ in $(0,\infty)$

(2) $f(x)$ is decreasing in $(0,\infty)$

(3) $f(x)$ is increasing in $(0,\infty)$

(4) $f'(x) < e^{2x}$ in $(0,\infty)$

***24.** Let $f : (0, \pi) \to \mathbb{R}$ be a twice differentiable function such that

$$\lim_{t\to x}\frac{f(x)\sin t - f(t)\sin x}{t-x} = \sin^2 x$$ for all $x \in (0, \pi)$.

If $f\left(\frac{\pi}{6}\right) = -\frac{\pi}{12}$, then which of the following statement(s) is (are) TRUE ? **[JEE(Advanced)-2018]**

(1) $f\left(\frac{\pi}{4}\right)=\frac{\pi}{4\sqrt{2}}$

(2) $f(x)<\frac{x^4}{6}-x^2$ for all $x \in (0, \pi)$

(3) There exists $\alpha \in (0, \pi)$ such that $f'(\alpha) = 0$

(4) $f''\left(\frac{\pi}{2}\right)+f\left(\frac{\pi}{2}\right)=0$

25. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function with $f(0) = 0$. If $y = f(x)$ satisfies the differential equation $\frac{dy}{dx}=(2+5y)(5y-2)$, then the value of $\lim_{x\to-\infty} f(x)$ is _____ . **[JEE(Advanced)-2018]**

* **Marked Questions are multiple answer**

PREVIOUS YEARS QUESTIONS — **ANSWER KEY** — Exercise-II

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	3	4	4	3	2	1	4	4	2	3
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	1	1	1	2	2	3	3	Bonus	1,4	1
Que.	21	22	23	24	25					
Ans.	4	2	1,3	2,3,4	0.4					