DEFINITE INTEGRATION-PYQ

1. The value of the integral, $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}\,dx$ is- **[AIEEE-2006]**

(1) $\frac{3}{2}$ (2) 2

(3) 1 (4) $\frac{1}{2}$

2. $\int_{-3\pi/2}^{-\pi/2} [(x+\pi)^3 + \cos^2(x+3\pi)]\,dx$ is equal to- **[AIEEE-2006]**

(1) $(\pi^4/32) + (\pi/2)$ (2) $\pi/2$

(3) $(\pi/4) - 1$ (4) $\pi^4/32$

3. $\int_0^{\pi} x f(\sin x)\,dx$ is equal to- **[AIEEE-2006]**

(1) $\pi\int_0^{\pi} f(\sin x)\,dx$ (2) $\frac{\pi}{2}\int_0^{\pi/2} f(\sin x)dx$

(3) $\pi\int_0^{\pi/2} f(\cos x)\,dx$ (4) $\pi\int_0^{\pi} f(\cos x)dx$

4. The value of $\int_1^a [x]f'(x)\,dx$, $a > 1$, where [x] denotes the greatest integer not exceeding x is- **[AIEEE-2006]**

(1) [a] f(a) – {f(1) + f(2) + … + f([a])}

(2) [a] f([a]) – {f(1) + f(2) + … + f(a)}

(3) a f([a]) – {f(1) + f(2) + … + f(a)}

(4) a f(a) – {f(1) + f(2) + … + f([a])}

5. Let $F(x) = f(x) + f\left(\frac{1}{x}\right)$, where $f(x) = \int_1^x \frac{\log t}{1+t}\,dt$. Then F(e) equals- **[AIEEE-2007]**

(1) $\frac{1}{2}$ (2) 0

(3) 1 (4) 2

6. The solution for x of the equation $\int_{\sqrt{2}}^{x} \frac{dt}{t\sqrt{t^2-1}} = \frac{\pi}{12}$ is- **[AIEEE-2007]**

(1) 2 (2) π

(3) $\sqrt{3}/2$ (4) $2\sqrt{2}$

7. Let $I = \int_0^1 \frac{\sin x}{\sqrt{x}}\,dx$ and $J = \int_0^1 \frac{\cos x}{\sqrt{x}}\,dx$. Then which one of the following is true ? **[AIEEE-2008]**

(1) $I > \frac{2}{3}$ and $J > 2$ (2) $I < \frac{2}{3}$ and $J < 2$

(3) $I < \frac{2}{3}$ and $J > 2$ (4) $I > \frac{2}{3}$ and $J < 2$

8. $\int_0^{\pi} [\cot x]\,dx$, where [.] denotes the greatest integer function, is equal to - **[AIEEE-2009]**

(1) –1 (2) $-\frac{\pi}{2}$ (3) $\frac{\pi}{2}$ (4) 1

9. Let p(x) be a function defined on R such that $p'(x) = p'(1 - x)$, for all $x \in [0, 1]$, $p(0) = 1$ and $p(1) = 41$. Then $\int_0^1 p(x)\,dx$ equals :- **[AIEEE-2010]**

(1) $\sqrt{41}$ (2) 21

(3) 41 (4) 42

10. The value of $\int_0^1 \frac{8\log(1+x)}{1+x^2}\,dx$ is :- **[AIEEE-2011]**

(1) $\frac{\pi}{2}\log 2$ (2) log 2

(3) π log 2 (4) $\frac{\pi}{8}\log 2$

11. Let [.] denote the greatest integer function then the value of $\int_0^{1.5} x[x^2]\,dx$ is :- **[AIEEE-2011]**

(1) $\frac{5}{4}$ (2) 0 (3) $\frac{3}{2}$ (4) $\frac{3}{4}$

***12.** If $g(x) = \int_0^x \cos 4t\,dt$, then $g(x + \pi)$ equals : **[AIEEE-2012]**

(1) g(x) . g(π) (2) $\frac{g(x)}{g(\pi)}$

(3) g(x) + g(π) (4) g(x) – g(π)

13. **Statement-I :** The value of the integral $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$ is equal to $\frac{\pi}{6}$. **[JEE (Main)-2013]**

Statement-II : $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$.

(1) Statement-I is true, Statement-II is true; statement-II is a **correct** explanation for Statement-I.
(2) Statement-I is true, Statement-II is true; statement-II is **not** a correct explanation for Statement-I.
(3) Statement-I is true, Statement-II is false.
(4) Statement-I is false, Statement-II is true.

14. The integral $\int_0^{\pi} \sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}}dx$ equals : **[JEE(Main)-2014]**

(1) p – 4 (2) $\frac{2\pi}{3}-4-4\sqrt{3}$

(3) $4\sqrt{3}-4$ (4) $4\sqrt{3}-4-\frac{\pi}{3}$

15. The integral **[JEE (Main) 2015]**

$$\int_2^4 \frac{\log x^2}{\log x^2+\log(36-12x+x^2)}\,dx$$

is equal to :
(1) 1 (2) 6 (3) 2 (4) 4

16. $\lim_{n\to\infty}\left(\frac{(n+1)(n+2)....3n}{n^{2n}}\right)^{1/n}$ is equal to :- **[JEE (Main) 2016]**

(1) 3 log3 – 2 (2) $\frac{18}{e^4}$

(3) $\frac{27}{e^2}$ (4) $\frac{9}{e^2}$

17. The integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$ is equal to :- **[JEE (Main) 2017]**

(1) –1 (2) –2 (3) 2 (4) 4

18. The value of $a\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x}\,dx$ is : **[JEE (Main) 2018]**

(1) $\frac{\pi}{2}$ (2) 4π (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{8}$

19. **[IIT-2007]**

Column-I	Column-II
(A) $\int_{-1}^{1}\frac{dx}{1+x^2}$	(P) $\frac{1}{2}\log\left(\frac{2}{3}\right)$
(B) $\int_0^1\frac{dx}{\sqrt{1-x^2}}$	(Q) $2\log\left(\frac{2}{3}\right)$
(C) $\int_2^3\frac{dx}{1-x^2}$	(R) $\frac{\pi}{3}$
(D) $\int_1^2\frac{dx}{x\sqrt{x^2-1}}$	(S) $\frac{\pi}{2}$

***20.** If $I_n = \int_{-\pi}^{\pi}\frac{\sin nx}{(1+\pi^x)\sin x}dx$, n = 0, 1, 2, ..., then **[IIT-2009]**

(1) $I_n = I_{n+2}$ (2) $\sum_{m=1}^{10} I_{2m+1} = 10\pi$

(3) $\sum_{m=1}^{10} I_{2m} = 0$ (4) $I_n = I_{n+1}$

21. Let f: R $\to$ R be a continuous function which satisfies $f(x) = \int_0^x f(t)dt$ .Then the value of f(ln 5) is: **[IIT-2009]**

(1) 0 (2) 1
(3) 2 (4) 5

22. The value of $\lim_{x\to0}\frac{1}{x^3}\int_0^x \frac{t\ell n(1+t)}{t^4+4}dt$ is **[IIT-2010]**

(1) 0 (2) $\frac{1}{12}$ (3) $\frac{1}{24}$ (4) $\frac{1}{64}$

23. Let f be the function on $[-\pi,\pi]$ given by f(0)= 9 and $f(x) = \sin\left(\frac{9x}{2}\right)\Big/\sin\left(\frac{x}{2}\right)$ for $x \neq 0$. The value of $\frac{2}{\pi}\int_{-\pi}^{\pi} f(x)dx$ is :- **[IIT-2010]**

(1) –4 (2) 0 (3) 4 (4) 5

24. The value of $\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \frac{x\sin x^2}{\sin x^2 + \sin(\ln 6 - x^2)}dx$ is

[IIT-2011]

(1) $\frac{1}{4}\ln\frac{3}{2}$ (2) $\frac{1}{2}\ln\frac{3}{2}$ (3) $\ln\frac{3}{2}$ (4) $\frac{1}{6}\ln\frac{3}{2}$

25. The value of the integral

$\int_{-\pi/2}^{\pi/2}\left(x^2 + \ln\frac{\pi+x}{\pi-x}\right)\cos x\,dx$ is **[IIT-2012]**

(1) 0 (2) $\frac{\pi^2}{2}-4$ (3) $\frac{\pi^2}{2}+4$ (4) $\frac{\pi^2}{2}$

26. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2\cos x}{1+e^x}dx$ is equal to

[JEE(Advanced)-2016]

(1) $\frac{\pi^2}{4}-2$ (2) $\frac{\pi^2}{4}+2$

(3) $\pi^2 - e^{\frac{\pi}{2}}$ (4) $\pi^2 + e^{\frac{\pi}{2}}$

27. Let f : R → R be a differentiable function such that f(0) = 0, $f\left(\frac{\pi}{2}\right) = 3$ and f'(0) = 1. If

$g(x) = \int_x^{\frac{\pi}{2}}\left[f'(t)\,\text{cosec}\,t - \cot t\,\text{cosec}\,t\,f(t)\right]dt$

for $x \in \left(0, \frac{\pi}{2}\right]$, then $\lim_{x\to 0} g(x) =$

[JEE(Advanced)-2017]

***28.** If $I = \sum_{k=1}^{98}\int_k^{k+1}\frac{k+1}{x(x+1)}dx$, then

[JEE(Advanced)-2017]

(1) $I < \frac{49}{50}$ (2) $I < \log_e 99$

(3) $I > \frac{49}{50}$ (4) $I > \log_e 99$

29. If $g(x) = \int_{\sin x}^{\sin(2x)} \sin^{-1}(t)\,dt$, then

[JEE(Advanced)-2017]

(1) $g'\left(\frac{\pi}{2}\right) = -2\pi$ (2) $g'\left(-\frac{\pi}{2}\right) = 2\pi$

(3) $g'\left(\frac{\pi}{2}\right) = 2\pi$ (4) $g'\left(-\frac{\pi}{2}\right) = -2\pi$

30. For each positive integer n, let

$y_n = \frac{1}{n}(n+1)(n+2)...(n+n)^{1/n}$

For $x \in \mathbb{R}$, let [x] be the greatest integer less than or equal to x. If $\lim_{n\to\infty} y_n = L$, then the value of [L] is _____ **[JEE(Advanced)-2018]**

31. The value of the integral

$\int_0^{\frac{1}{2}} \frac{1+\sqrt{3}}{\left((x+1)^2(1-x)^6\right)^{\frac{1}{4}}}dx$

is _____ .

[JEE(Advanced)-2018]

* **Marked Questions are multiple answer**

PREVIOUS YEARS QUESTIONS	ANSWER KEY							Exercise-II	
Que.	1	2	3	4	5	6	7	8	9
Ans.	**1**	**2**	**3**	**1**	**1**	**1**	**2**	**2**	**2**
Que.	10	11	12	13	14	15	16	17	18
Ans.	**3**	**4**	**3,4**	**4**	**4**	**1**	**3**	**3**	**3**
Que.	19					20	21	22	23
Ans.	(A) → (S); (B) → (S); (C) → (P); (D)→ (R)					**1,2,3**	**1**	**2**	**3**
Que.	24	25	26	27	28	29	30	31	
Ans.	**1**	**2**	**1**	**2**	**2,3**	**Bonus**	**1**	**2**	