APPLICATION OF DERIVATIVE - PYQ

1. A spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of ice is 5 cm, then the rate of which the thickness of ice decreases, is- **[AIEEE - 2005]**

(1) $\frac{1}{36\pi}$ cm/min (2) $\frac{1}{18\pi}$ cm/min

(3) $\frac{1}{54\pi}$ cm/min (4) $\frac{5}{6\pi}$ cm/min

2. Let f be differentiable for all x. If f(1) = –2 and f'(x) ≥ 2 for x ∈ [1, 6] then- **[AIEEE-2005]**

(1) f(6) ≥ 8 (2) f(6) < 8

(3) f(6) < 5 (4) f(6) = 5

3. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minimum at- **[AIEEE-2006]**

(1) x = –2 (2) x = 0

(3) x = 1 (4) x = 2

4. A triangular park is enclosed on two sides by a fence and on the third side by a straight river bank. The two sides having fence are of same length x. The maximum area enclosed by the park is- **[AIEEE-2006]**

(1) $\sqrt{\frac{x^3}{8}}$ (2) $\frac{1}{2}x^2$ (3) πx^2 (4) $\frac{3}{2}x^2$

5. If p and q are positive real numbers such that $p^2 + q^2 = 1$, then the maximum value of (p + q) is- **[AIEEE-2007]**

(1) 2 (2) $\frac{1}{2}$ (3) $\frac{1}{\sqrt{2}}$ (4) $\sqrt{2}$

6. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in- **[AIEEE-2007]**

(1) (π/4, π/2) (2) (–π/2, π/4)

(3) (0, π/2) (4) (–π/2, π/2)

(4) Decreasing in (–1, ∞) and increasing in (–∞, –1)

7. A value of C for which the conclusion of Mean values theorem holds for the function $f(x) = \log_e x$ on the interval [1, 3] is- **[AIEEE-2007]**

(1) $2\log_3 e$ (2) $\frac{1}{2}\log_e 3$

(3) $\log_3 e$ (4) $\log_e 3$

8. Suppose the cubic $x^3 - px + q$ has three real roots where p > 0 and q > 0. Then which of the following holds ? **[AIEEE-2008]**

(1) The cubic has minima at $\sqrt{\frac{p}{3}}$ and maxima at $-\sqrt{\frac{p}{3}}$

(2) The cubic has minima at $-\sqrt{\frac{p}{3}}$ and maxima at $\sqrt{\frac{p}{3}}$

(3) The cubic has minima at both $-\sqrt{\frac{p}{3}}$ & $\sqrt{\frac{p}{3}}$

(4) The cubic has maxima at both $\sqrt{\frac{p}{3}}$ & $-\sqrt{\frac{p}{3}}$

9. The shortest distance between line y – x = 1 and curve $x = y^2$ is :- **[AIEEE-2009]**

(1) $\frac{8}{3\sqrt{2}}$ (2) $\frac{4}{\sqrt{3}}$ (3) $\frac{\sqrt{3}}{4}$ (4) $\frac{3\sqrt{2}}{8}$

10. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$, that is parallel to the x-axis, is :- **[AIEEE-2010]**

(1) y = 0 (2) y = 1

(3) y = 2 (4) y = 3

11. Let f : R → R be defined by

$$f(x) = \begin{cases} k - 2x, & \text{if} \quad x \le -1 \\ 2x + 3, & \text{if} \quad x > -1 \end{cases}$$

If f has a local minimum at x = – 1, then a possible value of k is : **[AIEEE-2010]**

(1) 1 (2) 0

(3) $-\frac{1}{2}$ (4) –1

12. Let $f(x) = x e^{x(1-x)}$, then f(x) is- **[AIEEE-2012 (Online)]**

(1) Increasing on [–1/2, 1]

(2) Decreasing on R

(3) Increasing on R

(4) Decreasing on [–1/2, 1]

13. A spherical balloon is filled with 4500π cubic meters of helium gas. If a leak in the balloon causes the gas to escape at the rate of 72π cubic meters per minute, then the rate (in meters per minute) at which the radius of the balloon decreases 49 minutes after the leakage began is **[AIEEE - 2012]**

(1) 9/2 (2) 9/7

(3) 7/9 (4) 2/9

14. If metallic circular plate of radius 50 cm is heated so that its radius increases at the rate of 1 cm per hour, then the rate at which the area of the plate increases (in cm^2/hr) is : **[AIEEE - 2012 (Online)]**

(1) 5π (2) 10π

(3) 100π (4) 50π

15. The intercepts on x-axis made by tangents to the curve, $y = \int_0^x |t|\,dt$, $x \in R$, which are parallel to the line $y = 2x$, are equal to **[JEE(MAIN)-2013]**

(1) ± 1 (2) ± 2

(3) ± 3 (4) ± 4

16. The real number k for which the equation $2x^3 + 3x + k = 0$ has two distinct real roots in [0, 1] **[JEE(MAIN) 2013]**

(1) lies between 1 and 2.

(2) lies between 2 and 3.

(3) lies between –1 and 0

(4) does not exist

17. If f and g are differentiable functions in [0, 1] satisfying $f(0) = 2 = g(1)$, $g(0) = 0$ and $f(1) = 6$, then for some $c \in]0, 1[$: **[JEE(Main)–2014]**

(1) $2f'(c) = g'(c)$ (2) $2f'(c) = 3g'(c)$

(3) $f'(c) = g'(c)$ (4) $f'(c) = 2g'(c)$

18. If $x = -1$ and $x = 2$ are extreme points of $f(x) = \alpha \log |x| + \beta x^2 + x$ then : **[JEE(Main)–2014]**

(1) $\alpha = -6$, $\beta = \frac{1}{2}$

(2) $\alpha = -6$, $\beta = -\frac{1}{2}$

(3) $\alpha = 2$, $\beta = -\frac{1}{2}$

(4) $\alpha = 2$, $\beta = \frac{1}{2}$

19. The normal to the curve, $x^2 + 2xy - 3y^2 = 0$, at (1, 1) : **[JEE(MAIN)-2015]**

(1) meets the curve again in the third quadrant

(2) meets the curve again in the fourth quadrant

(3) does not meet the curve again

(4) meets the curve again in the second quadrant

20. Let $f(x)$ be a polynomial of degree four having extreme values at $x = 1$ and $x = 2$.

If $\lim_{x\to 0}\left[1+\frac{f(x)}{x^2}\right] = 3$, then $f(2)$ is equal to : **[JEE(Main)–2015]**

(1) 0 (2) 4

(3) –8 (4) –4

21. Consider $f(x) = \tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right)$, $x \in \left(0, \frac{\pi}{2}\right)$.

A normal to $y = f(x)$ at $x = \frac{\pi}{6}$ also passes through the point : **[JEE(MAIN)-2016]**

(1) $\left(\frac{\pi}{4}, 0\right)$ (2) (0, 0)

(3) $\left(0, \frac{2\pi}{3}\right)$ (4) $\left(\frac{\pi}{6}, 0\right)$

22. The point (s) on the curve $y^3 + 3x^2 = 12y$ where the tangent is vertical, is (are) **[IIT-2002]**

(1) $\left(\pm\frac{4}{\sqrt{3}}, -2\right)$ (2) $\left(\pm\sqrt{\frac{11}{3}}, 1\right)$

(3) (0, 0) (4) $\left(\pm\frac{4}{\sqrt{3}}, 2\right)$

23. According to mean value theorem in the interval $x \in [0, 1]$ which of the following does not follow - **[IIT-2003]**

(1) $f(x) = \begin{cases} \frac{1}{2} - x, & x < \frac{1}{2} \\ \left(\frac{1}{2} - x\right)^2, & x \geq \frac{1}{2} \end{cases}$

(2) $f(x) = \begin{cases} \frac{\sin x}{x}; & x \neq 0 \\ 1 \;; & x = 0 \end{cases}$

(3) $f(x) = x|x|$

(4) $f(x) = |x|$

24. Let f, g and h be real-valued functions defined on the interval [0, 1] by $f(x) = e^{x^2} + e^{-x^2}$, $g(x) = xe^{x^2} + e^{-x^2}$ and $h(x) = x^2e^{x^2} + e^{-x^2}$. If a, b and c denote, respectively, the absolute maximum of f, g and h on [0, 1], then **[IIT-2010]**

(1) $a = b$ and $c \neq b$

(2) $a = c$ and $a \neq b$

(3) $a \neq b$ and $c \neq b$

(4) $a = b = c$

25. Let f be a function defined on R (the set of all real numbers) such that

$f'(x) = 2010(x-2009)(x-2010)^2(x-2011)^3(x-2012)^4$, for all $x \in R$. If g is a function defined on R with values in the interval $(0,\infty)$ such that $f(x) = \ell n(g(x))$, for all $x \in R$, then the number of points in R at which g has a local maximum is - **[IIT-2010]**

(1) 1 (2) 2

(3) 3 (4) None

26. If $f(x) = x^{3/2}(3x - 10)$, $x \geq 0$, then $f(x)$ is increasing in :- **[IIT 2011]**

(1) $(-\infty,0)\cup(0,\infty)$ (2) $[2,\infty)$

(3) $(-\infty,-1]\cup[1,\infty)$ (4) $(-\infty,0]\cup[2,\infty)$

27. Let p(x) be a real polynomial of least degree which has a local maximum at x = 1 and a local minimum at x = 3. If p(1) = 6 and p(3) = 2, then p'(0) is **[IIT-2012]**

(1) 7 (2) 8

(3) 9 (4) 10

***28.** If $f(x) = \int_0^x e^{t^2}(t-2)(t-3)\,dt$ for all $x \in (0,\infty)$, then-

(1) f has a local maximum at x = 2 **[IIT-2012]**

(2) f is decreasing on (2,3)

(3) there exists some $c \in (0,\infty)$ such that $f''(c) = 0$

(4) f has a local minimum at x = 3

29. The number of points in $(-\infty, \infty)$, for which $x^2 - x\sin x - \cos x = 0$, is **[JEE-Advanced 2013]**

(1) 6 (2) 4

(3) 2 (4) 0

30. The slope of the tangent to the curve $(y - x^5)^2 = x(1 + x^2)^2$ at the point (1,3) is **[JEE(Advanced)-2014]**

(1) 7 (2) 8

(3) 9 (4) 10

* **Marked Question is multiple answer**

PREVIOUS YEARS QUESTIONS — **ANSWER KEY** — Exercise-II

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	**2**	**1**	**4**	**2**	**4**	**2**	**1**	**1**	**4**	**4**
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	**4**	**1**	**4**	**2**	**1**	**4**	**4**	**3**	**2**	**1**
Que.	21	22	23	24	25	26	27	28	29	30
Ans.	**3**	**4**	**1**	**4**	**1**	**2**	**3**	**1,2,3,4**	**3**	**2**