

CONTINUITY, DIFFERENTIABILITY & DIFFERENTIATIONS - PYQ

- 1.** If $f(x) = \begin{cases} x & x \in \mathbb{Q} \\ -x & x \notin \mathbb{Q} \end{cases}$, then f is continuous at- [AIEEE 2002]
- (1) Only at zero (2) only at 0, 1
 (3) all real numbers (4) all rational numbers
- 2.** If $y = (x + \sqrt{1+x^2})^n$ then $(1+x^2)y_2 + xy_1 =$ [AIEEE-2002]
- (1) ny^2 (2) n^2y
 (3) n^2y^2 (4) None of these
- 3.** If for all values of x & y ; $f(x+y) = f(x)f(y)$ and $f(5) = 2$, $f(0) = 3$, then $f'(5)$ is- [AIEEE-2002]
- (1) 3 (2) 4 (3) 5 (4) 6
- 4.** If $f(x) = \begin{cases} xe^{-\left(\frac{1}{|x|} + \frac{1}{x}\right)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ then $f(x)$ is- [AIEEE 2003]
- (1) discontinuous everywhere
 (2) continuous as well as differentiable for all x
 (3) continuous for all x but not differentiable at $x=0$
 (4) neither differentiable nor continuous at $x=0$
- 5.** Let $f(x) = \frac{1-\tan x}{4x-\pi}$, $x \neq \frac{\pi}{4}$, $x \in \left[0, \frac{\pi}{2}\right]$, If $f(x)$ is continuous in $\left[0, \frac{\pi}{2}\right]$, then $f\left(\frac{\pi}{4}\right)$ is- [AIEEE 2004]
- (1) 1 (2) $1/2$
 (3) $-1/2$ (4) -1
- 6.** The set of points where $f(x) = \frac{x}{1+|x|}$ is differentiable [AIEEE-2006]
- (1) $(-\infty, -1) \cup (-1, \infty)$ (2) $(-\infty, \infty)$
 (3) $(0, \infty)$ (4) $(-\infty, 0) \cup (0, \infty)$
- 7.** If $x^m \cdot y^n = (x+y)^{m+n}$, then $\frac{dy}{dx}$ is [AIEEE-2006]
- (1) $\frac{x+y}{xy}$ (2) xy (3) $\frac{x}{y}$ (4) $\frac{y}{x}$
- 8.** The function $f : \mathbb{R}/\{0\} \rightarrow \mathbb{R}$ given by $f(x) = \frac{1}{x} - \frac{2}{e^{2x}-1}$ can be made continuous at $x=0$ by defining $f(0)$ as- [AIEEE 2007]
- (1) 2 (2) -1 (3) 0 (4) 1

- 9.** Let y be an implicit function of x defined by $x^{2x} - 2x^x \cot y - 1 = 0$. Then $y'(1)$ equals [AIEEE-2009]
- (1) $\log 2$ (2) $-\log 2$
 (3) -1 (4) 1
- 10.** Let $f : (-1, 1) \rightarrow \mathbb{R}$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$. Let $g(x) = [f(2f(x) + 2)]^2$. Then $g'(0)$ [AIEEE-2010]
- (1) 4 (2) -4
 (3) 0 (4) -2
- 11.** If function $f(x)$ is differentiable at $x = a$ then [AIEEE-2011]
- $$\lim_{x \rightarrow a} \frac{x^2 f(a) - a^2 f(x)}{x - a}$$
- (1) $2a f(a) + a^2 f'(a)$ (2) $-a^2 f'(a)$
 (3) $af(a) - a^2 f'(a)$ (4) $2af(a) - a^2 f'(a)$
- 12.** $\frac{d^2x}{dy^2}$ equal to [AIEEE-2011]
- (1) $\left(\frac{d^2y}{dx^2}\right)^{-1}$ (2) $\left(\frac{d^2y}{dx^2}\right)^{-1} \left(\frac{dy}{dx}\right)^{-3}$
 (3) $\left(\frac{d^2y}{dx^2}\right) \left(\frac{dy}{dx}\right)^{-2}$ (4) $-\left(\frac{d^2y}{dx^2}\right) \left(\frac{dy}{dx}\right)^{-3}$
- 13.** The values of p and q for which the function
- $$f(x) = \begin{cases} \frac{\sin(p+1)x + \sin x}{x}, & x < 0 \\ q, & x = 0 \\ \frac{\sqrt{x+x^2} - \sqrt{x}}{x^{\frac{3}{2}}}, & x > 0 \end{cases}$$
- is continuous for all x in \mathbb{R} , are :- [AIEEE 2011]
- (1) $p = -\frac{3}{2}$, $q = \frac{1}{2}$ (2) $p = \frac{1}{2}$, $q = \frac{3}{2}$
 (3) $p = \frac{1}{2}$, $q = -\frac{3}{2}$ (4) $p = \frac{5}{2}$, $q = \frac{1}{2}$
- 14.** The function $f(x) = [x] \cos\left(\frac{(2x-1)\pi}{2}\right)$, $[]$ denotes the greatest integer function, is discontinuous at- [AIEEE-2012]
- (1) all x
 (2) all integer points
 (3) no x
 (4) x which is not an integer

15. If $f(x) = a |\sin x| + b e^{|x|} + c |x|^3$, where $a, b, c \in \mathbb{R}$, is differentiable at $x = 0$, then

[AIEEE-2012 (Online)]

- (1) $c = 0, a = 0, b$ is any real number
- (2) $a = 0, b$ and c are any real numbers
- (3) $b = 0, c = 0, a$ is any real number
- (4) $a = 0, b = 0, c$ is any real number

16. If $x + |y| = 2y$ then y as a function of x , at $x = 0$ is

[AIEEE-2012 (Online)]

- (1) Neither continuous nor differentiable
- (2) Continuous as well as differentiable
- (3) Differentiable but not continuous
- (4) Continuous but not differentiable

17. If $f(x) = \sin(\log x)$ and $y = f\left(\frac{2x+3}{3-2x}\right)$, then $\frac{dy}{dx}$ equals

[AIEEE-2012 (Online)]

(1) $\frac{12}{(3-2x)^2} \cos\left[\log\left(\frac{2x+3}{3-2x}\right)\right]$

(2) $\sin\left[\log\left(\frac{2x+3}{3-2x}\right)\right]$

(3) $\frac{12}{(3-2x)^2} \sin\left[\log\left(\frac{2x+3}{3-2x}\right)\right]$

(4) $\frac{12}{(3-2x)^2}$

18. If $y = \sec(\tan^{-1}x)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to

[JEE (Main)-2013]

(1) $\frac{1}{\sqrt{2}}$

(2) $\frac{1}{2}$

(3) 1

(4) $\sqrt{2}$

19. If the function.

$$g(x) = \begin{cases} k\sqrt{x+1}, & 0 \leq x \leq 3 \\ mx+2, & 3 < x \leq 5 \end{cases}$$

is differentiable, then value of $k + m$ is

[JEE (Main)-2015]

(1) $\frac{10}{3}$

(2) 4

(3) 2

(4) $\frac{16}{r}$

20. For $x \in \mathbb{R}$, $f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then :

[JEE (Main)-2016]

- (1) g is differentiable at $x = 0$ and $g'(0) = -\sin(\log 2)$
- (2) g is not differentiable at $x = 0$
- (3) $g'(0) = \cos(\log 2)$
- (4) $g'(0) = -\cos(\log 2)$

21. If for $x \in \left(0, \frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$

is $\sqrt{x} \cdot g(x)$, then $g(x)$ equals :-

[JEE (Main)-2017]

(1) $\frac{3}{1+9x^3}$ (2) $\frac{9}{1+9x^3}$ (3) $\frac{3x\sqrt{x}}{1-9x^3}$ (4) $\frac{3x}{1-9x^3}$

22. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ is a function which is defined by $f(x) = \max\{x, x^3\}$ set of points on which $f(x)$ is not differentiable is-

[IIT-2001]

- (1) $\{-1, 1\}$ (2) $\{-1, 0\}$
- (3) $\{0, 1\}$ (4) $\{-1, 0, 1\}$

23. Which of the following function is differentiable at $x = 0$?

[IIT-2001]

- (1) $\cos(|x|) + |x|$ (2) $\cos(|x|) - |x|$
- (3) $\sin(|x|) + |x|$ (4) $\sin(|x|) - |x|$

24. Let y be a function of x , such that $\log(x+y) - 2xy = 0$, then $y'(0)$ is-

[IIT-2004]

- (1) 0 (2) 1
- (3) $1/2$ (4) $3/2$

25. If $x \cos y + y \cos x = \pi$, then $y''(0) =$

[IIT-2005]

- (1) π (2) $-\pi$
- (3) 0 (4) 1

26. If $f'(x) = -f(x)$ and $g(x) = f(x)$

and $F(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)^2$

and given that $F(5) = 5$, then $F(10)$ is-

[IIT-2006]

- (1) 15 (2) 0
- (3) 5 (4) 10

*27. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(x+y) = f(x) + f(y), \forall x, y \in \mathbb{R}$.

If $f(x)$ is differentiable at $x = 0$, then [IIT-2011]

- (1) $f(x)$ is differentiable only in a finite interval containing zero
- (2) $f(x)$ is continuous $\forall x \in \mathbb{R}$
- (3) $f'(x)$ is constant $\forall x \in \mathbb{R}$
- (4) $f(x)$ is differentiable except at finitely many points

*28. If $f(x) = \begin{cases} -x - \frac{\pi}{2}, & x \leq -\frac{\pi}{2} \\ -\cos x, & -\frac{\pi}{2} < x \leq 0 \\ x - 1, & 0 < x \leq 1 \\ \ln x, & x > 1 \end{cases}$ then

[IIT-2011]

- (1) $f(x)$ is continuous at $x = -\frac{\pi}{2}$
- (2) $f(x)$ is not differentiable at $x = 0$
- (3) $f(x)$ is differentiable at $x = 1$
- (4) $f(x)$ is differentiable at $x = -\frac{3}{2}$

29. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ be respectively given by $f(x) = |x| + 1$ and $g(x) = x^2 + 1$. Define $h : \mathbb{R} \rightarrow \mathbb{R}$ by

$$h(x) = \begin{cases} \max\{f(x), g(x)\} & \text{if } x \leq 0, \\ \min\{f(x), g(x)\} & \text{if } x > 0. \end{cases}$$

The number of points at which $h(x)$ is not differentiable is

[JEE (Advanced)-2014]

- (1) 1
- (2) 2
- (3) 3
- (4) 0

30. Let $y(x) = \cos(3 \cos^{-1} x)$, $x \in [-1, 1]$, $x \neq \pm \frac{\sqrt{3}}{2}$.

Then $\frac{1}{y(x)} \left\{ (x^2 - 1) \frac{d^2 y(x)}{dx^2} + x \frac{dy(x)}{dx} \right\}$ equals

[JEE (Advanced)-2014]

- (1) 1
- (2) 2
- (3) 8
- (4) 9

* Marked Questions are multiple answer

PREVIOUS YEARS QUESTIONS			ANSWER KEY				Exercise-II			
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	1	2	4	3	3	2	4	4	3	2
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	4	4	1	3	4	4	3	1	3	3
Que.	21	22	23	24	25	26	27	28	29	30
Ans.	2	4	4	2	1	3	2,3	1,2,3,4	3	4