SOLUTIONS
WAVE MOTION & DOPPLER'S EFFECT

5. $y = 4\sin\left[\pi\left(\frac{t}{5} - \frac{x}{9}\right) + \frac{\pi}{6}\right]$ compare this equation by

$y = a \sin(\omega t - kx)$

$\omega = \frac{\pi}{5}$ $\quad k = \frac{\pi}{9}$

$a = 4$ $\quad \frac{T}{2} = 5$ $\quad \frac{\lambda}{2} = 9$

$T = 10$ s, $\lambda = 18$ cm

7. Wave is a disturbance, which carries energy and momentum without transport of mass of matter.

8. Velocity of wave = $\frac{600}{2}$ = 300 m/s

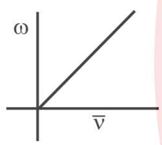
Frequency = 500 Hz, Wavelength $\lambda = \frac{3}{5}$ m

Number of wavelength = $\frac{600}{3/5}$ = 1000

9. $\omega = 2\pi n$

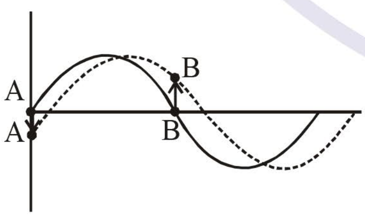
$\omega \propto n$ $\quad (2\pi = \text{const})$

$\omega \propto \frac{1}{\lambda}$ $\quad \bar{v} = \frac{1}{\lambda}$



$\Rightarrow \quad \omega \propto \bar{v}$

10.



from figure A is moving down & B is moving up and the phase at A is greater the phase at B.

11. $\frac{d}{v_P} - \frac{d}{v_S} = 4 \times 60$

$\Rightarrow \quad \frac{d}{4.5} - \frac{d}{8} = 240$

d = 2468.6 km $\approx$ 2500 k

12. $y = 3\sin\frac{\pi}{2}(50t - x)$

Particle velocity = $\frac{\partial y}{\partial t} = 3\left(\frac{\pi}{2} \times 50\right)\cos\frac{\pi}{2}(50t - x)$

Maximum particle velocity = 75π m/s

Wave velocity $v = \frac{\omega}{k} = \frac{50}{1} = 50$ m/s

Required ratio = $\frac{75\pi}{50} = \frac{3}{2}\pi$

OR

$\frac{(v_P)_{max.}}{V_{max.}} = \frac{A\omega}{\omega/k} = kA = \frac{\pi}{2} \times 3 = \frac{3\pi}{2}$

13. $k = \frac{2\pi}{\lambda} = \frac{2\pi}{4/5} = 7.85$

$v = \frac{\omega}{k} \Rightarrow \omega = vk = 128 \times 7.85 = 1005$

and moves in + x direction

14. $\frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

15. frequency does not depends upon medium, velocity of sound decreases when it goes from water to air. $\quad v = f\lambda \quad (\because \quad f = \text{const.})$

so λ also decreases

16. Velocity of sound

$v = \sqrt{\frac{\gamma P}{\rho}}$ $\quad$ as $\quad \rho_d > \rho_m \therefore \upsilon_m > \upsilon_d$

17. $v = \sqrt{\frac{T}{m}} \Rightarrow \frac{\omega}{K} = \sqrt{\frac{T}{m}} \Rightarrow \frac{\left(\frac{1}{0.04}\right)}{\left(\frac{1}{0.50}\right)} = \sqrt{\frac{T}{0.04}}$

$\Rightarrow \frac{0.50}{0.04} \times \frac{0.50}{0.04} = \frac{T}{0.04} \quad \therefore T = \frac{50}{4} \times 0.5 = 6.25 \text{ N}$

18. $v = \sqrt{\frac{T}{m}}, \quad T = 0.1 \times 10 = 1\text{N}, \quad m = \frac{0.1}{2.5}$

Velocity at upper point $v = \sqrt{1 \times 25}$

$v = 5$ m/s

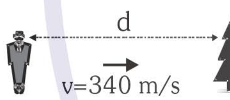
Now velocity at 0.5 m distance from lower point -

$v = \sqrt{\frac{T}{m}} \quad \Rightarrow \quad T = \frac{1}{2.5} \times 0.5 = \frac{1}{5}\text{N}, \; m = \frac{1}{25}$

$v = \sqrt{\frac{1}{5} \times \frac{25}{1}} = \sqrt{5} = 2.24$ m/s

19. Let distance between cliff and mountain be d

$1 = \frac{d}{340} + \frac{d}{340} \quad \Rightarrow \quad d = 170\text{m}$



distance $= \frac{340}{2} = 170$m

20. $I = \frac{1}{2}\rho v a^2 \omega^2 \Rightarrow I \propto n^2 a^2$

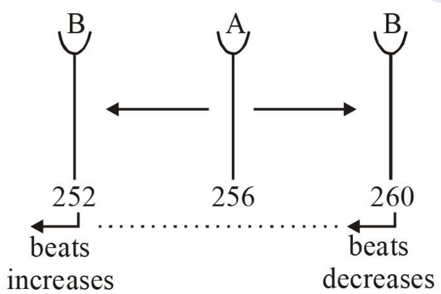
21. Average frequency $= \frac{499 + 501}{2} = 500$Hz

Beat frequency = 501 – 499 = 2Hz

22. n, n + 8 .... n + 120

n + 120 = 2n , n = 120

23.



So answer is 252, and 260 is also when more wax is applied on B.

24.

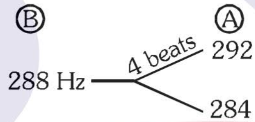
A(256 Hz)

4beats → (B) 252 → On waxing → 250 Hz → By removing same amount → 252 Hz

4beats → (B) 260 → On waxing → 250 Hz → By removing litle amount → 252 Hz

$\therefore$ frequency of B = 260 Hz

25.



After loading wax, frequency of A decrease. So beats frequency will also decrease.

Ans. would be 292

26. Number of beats $= \frac{508\pi}{2\pi} - \frac{500\pi}{2\pi}$

$= 254 - 200 = 4$

and $\frac{I_{max}}{I_{min}} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right) = \left(\frac{4+5}{4-5}\right)^2 = \frac{81}{1}$

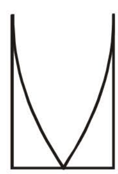
27. Number of beats $\Delta n = n_1 - n_2$

$2 = \frac{v}{\lambda_1} - \frac{v}{\lambda_2} \quad \Rightarrow \quad 2 = \frac{v}{2} - \frac{v}{2.02}$

$2 = \frac{v}{202} \Rightarrow v = 404$

28. For closed pipe

$n = \frac{v}{4\ell} = 512$

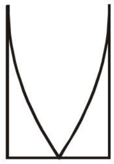


For open pipe

$n = \frac{v}{2\ell} = 1024$

n = 1024 Hz.

29. $n = \dfrac{v}{4\ell}$



$n = 264$ Hz

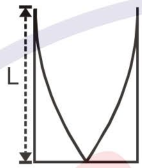
$v = 330$ m/s

Then

$\ell = 31.25$ m

30. $n = \dfrac{v}{4\ell}$

By increasing temperature, velocity of sound & length both increases, so



$n = \dfrac{V + v}{4(L + \ell)}$

31. $\dfrac{v}{4\ell}$ = fundamental frequency = 1500 Hz

Normal person can listen frequency 20,000 Hz.

1500x = 20,000 , x = 13 (No. of Harmonic)

Then overtones and frequency in case of closed organ pipe, ratio of frequency

1,	3,	5,	7,	9,	11,	13
	I	II	III	IV	V	VI

Then 6 overtones.

32. $n_1 = \dfrac{v}{4\ell_1}$, $n_2 = \dfrac{v}{4\ell_2}$

$\ell_1 = 0.75$ m $\ell_2 = 0.770$ m

$n_1 - n_2 = 3 \Rightarrow \dfrac{v}{4}\left(\dfrac{1}{\ell_1} - \dfrac{1}{\ell_2}\right) = 3 \Rightarrow v = 346.5$ m/s

33. $\dfrac{v}{4\ell} = 260$, $\ell = 31.7$ cm

34. For open organ pipe

$n_1 = \dfrac{v}{2\ell}$

for COP $n_2 = \dfrac{v}{4(\ell/2)} = \dfrac{2v}{4\ell} \Rightarrow n_2 = \dfrac{v}{2\ell}$, $n_1 = n_2$

35. 425 : 595 : 765

5 : 7 : 9

ratio of frequency is odd

pipe → closed end.

425 is frequency of 2^{nd} overtone

$\dfrac{5v}{4\ell} = 425 \Rightarrow \dfrac{5 \times 340}{4\ell} = 425$, $\ell = 1$m

36. $\lambda = \dfrac{v}{n} = \dfrac{330}{330} = 1\text{m} = 100$ cm

for first resonance length of air column

$\ell = \dfrac{\lambda}{4} = 25$ cm

second resonance $\ell_2 = \dfrac{3\lambda}{4} = 75$ cm

third resonance $\ell_3 = \dfrac{5\lambda}{4} = 125$ cm which is not possible

minimum length of water column

$= 120 - 75 = 45$ cm

37. For second overtone (3^{rd} harmonic) in open organ pipe,

$\dfrac{3\lambda}{2} = \ell_0 \Rightarrow \lambda = \dfrac{2\ell_0}{3}$

for first overtone (3^{rd} harmonic) in closed organ pipe,

$\dfrac{3\lambda}{4} = \ell_C \Rightarrow \lambda = \dfrac{4\ell_C}{3} = \dfrac{4L}{3}$

So, $\dfrac{2\ell_0}{3} = \dfrac{4L}{3} \Rightarrow \ell_0 = 2L$

38. Difference between any two consecutive frequencies of COP $= \dfrac{2v}{4\ell} = 500 - 460 = 40$ Hz

$\Rightarrow \dfrac{v}{4\ell} = 20\text{Hz}$

So fundamental frequency = 20 Hz

39. $V = 2f(\ell_2 - \ell_1)$

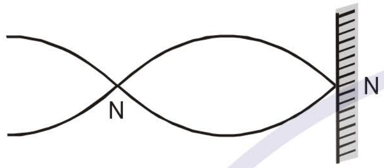
$= 2(320) \times (0.93 - 0.40)$

$= (640) \times (0.53) = 339.2$ m/s

40. First minimum resonating length for closed organ pipe $= \dfrac{\lambda}{4} = 40\,\text{cm}$

$\therefore$ Next larger length of air column $= \dfrac{3\lambda}{4} = 120\,\text{cm}$

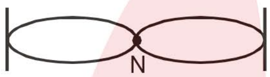
41. $n = 100$ Hz., Given $\dfrac{\lambda}{2} = 10$ cm



$v = n\lambda = 100 \times 0.2$ $\quad$ $v = 20$ m/ s

42. Plucking distance

$x = \dfrac{\ell}{2p} = \dfrac{\ell}{4}$



touching distance $= \ell/2$

43. The frequency of the piano string may be 508 or 516 Hz.

As frequency $\propto \sqrt{\text{Tension}}$ so answer will be 508 Hz.

44. $n \propto \sqrt{T} \Rightarrow \dfrac{\Delta n}{n} = \dfrac{1}{2}\dfrac{\Delta T}{T} \Rightarrow \dfrac{\Delta T}{T} = 2\left(\dfrac{\Delta n}{n}\right)$

$$= 2\left(\frac{6}{600}\right) = 0.02$$

45. Let unknown frequancy be n

Case-I $\quad$ n can be 250 – 4, 250 + 4

246 Hz or $\boxed{254}$Hz

Case-II $\quad$ 2n can be 513 – 5, 513 + 5

$\therefore$ n can be $\dfrac{508}{2}, \dfrac{518}{2}$

i.e. $\boxed{254}$, 259

254 Hz satisfy both condition.

46. Total length of string $\ell = \ell_1 + \ell_2 + \ell_3$

But frequency $\propto \dfrac{1}{\text{length}}$

so $\dfrac{1}{n} = \dfrac{1}{n_1} + \dfrac{1}{n_2} + \dfrac{1}{n_3}$

47. All harmonic frequencies are obtained for string fixed at both ends.

$\therefore$ $r_0 = 420$ Hz – 315 Hz = 105 Hz

48. $n = \dfrac{1}{2L}\sqrt{\dfrac{T}{m}}$

$n \propto \sqrt{T}$

$\dfrac{n_1}{n_2} = \sqrt{\dfrac{T}{1.02T}}$

$\dfrac{n}{n+15} = \sqrt{\dfrac{T}{1.02T}} = \sqrt{\dfrac{1}{1.02}}$

$n = 1500$ Hz

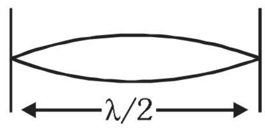
and $v \propto \sqrt{T} \Rightarrow \dfrac{\Delta v}{v} = \dfrac{1}{2}\dfrac{\Delta T}{T} \Rightarrow \dfrac{\Delta v}{v} \times 100 = 1\%$

so (2) is incorrect

49. $y = \cos 2\pi t \sin 2\pi x$, $\quad k = 2\pi$

$\lambda = 1$m

So, minimum length of string $= \lambda/2 = 0.5$ m



50. $y = a \cos(\omega t - kx)$

$x = 0$ is a node

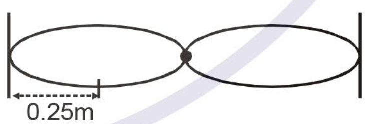
So rigid end reflection

$y = a \cos(\omega t + kx + \pi)$

$y = -a \cos(\omega t + kx)$

$[\cos(180 + \theta) = -\cos\theta]$

51.



$n = \dfrac{p}{2\ell}\sqrt{\dfrac{T}{m}} = \dfrac{2}{2\ell}\sqrt{\dfrac{20 \times 10^3}{0.5}} = 200$ Hz

52. $\dfrac{n_1}{n_2} = \dfrac{\ell_2}{\ell_1}\sqrt{\dfrac{T_1}{T_2} \times \dfrac{r_2^2}{r_1^2} \times \dfrac{d_2}{d_1}}$

$\dfrac{n_1}{n_2} = \dfrac{35}{36} \Rightarrow n_1 = \dfrac{35 \times n_2}{36} = \dfrac{35 \times 360}{36} = 350$

beats frequency $= n_2 - n_1 = 10$ Hz

53. $v = \frac{5}{4\ell}\sqrt{\frac{9}{m}} = \frac{3}{4\ell}\times\sqrt{\frac{M}{m}}$

$\sqrt{M} = 5$

Hence $\boxed{\sqrt{M} = 5}$

54. When sounded with 440 Hz,

Frequency of guitar = 440 + 5, 440 – 5

When sounded with 436 Hz,

Frequency of guitar = 436 + 9, 436 – 9

so frequency of guitar = 445 Hz

55. $v = \frac{\omega}{K} = \frac{240\pi}{\left[\frac{4\pi}{5\times10^{-2}}\right]} = 300\times10^{-2} = 3$

$v = \sqrt{\frac{T}{m}}$ $\quad\because\ m = \frac{4\times10^{-3}}{10^{-2}} = 4\times10^{-1}$

$3 = \sqrt{\frac{T}{0.4}} \Rightarrow 9\times0.4 = T$

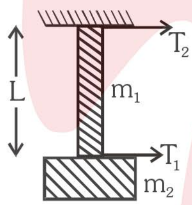
T = 3.6 N

56. $T_1 = m_2 g$

$T_2 = (m_1 + m_2)g$

Velocity $\propto \sqrt{T}$

$\Rightarrow \lambda \propto \sqrt{T}$



$\Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{\sqrt{T_1}}{\sqrt{T_2}} \Rightarrow \frac{\lambda_2}{\lambda_1} = \sqrt{\frac{m_1+m_2}{m_2}} = \sqrt{\frac{3}{2}}$

57. $1950 = n \times \left(\frac{340\text{-}15}{340+15}\right)$

$n = \frac{1950\times355}{325} = 2130$ Hz

58. $\lambda = \frac{v}{\nu} = \frac{340}{170} = m$

$\lambda = 2m$

$\therefore$ Distance between 2 portions of minimum intensity

$= \frac{\lambda}{2} = 1$ m

59. ω = 20 rad/s ,v = ωr = 20 × 0.50= 10 m/s

Minimum frequency = $385 \times \left(\frac{340}{340+10}\right)$

$= 385 \times \frac{340}{350} = 374$ Hz

60.

There is no relative motion between source and observer so no doppler effect.

61. n = 165 Hz, $v_s = v_o$ = 5 m/s, v = 335 m/s

$n' = n\left(\frac{v+v_o}{v-v_s}\right) \Rightarrow n' = 165 \times \left(\frac{340}{330}\right)$ Hz

n' = 170 Hz then number of beats

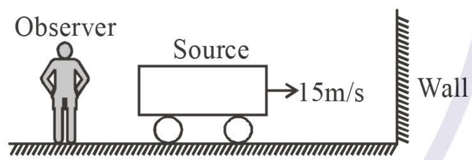
(170 – 165) = 5

OR

$\Delta n = \frac{2v_0}{v-v_0}n$

62. $b = \Delta n = \frac{2nv}{V_{sound}}$

63.



frequency at wall is n′

$n' = \frac{v}{v-v_s}n_0$

$n' = \frac{330}{330-30}(900) = \frac{330\times900}{300} = 990$ Hz

Since the observer and wall are stationary so frequency of echo heard by observer will also be 838 Hz.

64. $\Delta\lambda = \frac{v}{c}\lambda \quad \Rightarrow \frac{\Delta\lambda}{\lambda} = \frac{0.4}{100}$

$\therefore \frac{0.4}{100} = \frac{v}{3\times10^8} \quad \therefore \quad v = 1.2\times10^6$ m/s

65. As speed of light is independent of relative motioin b/w source and observer.

67. $n' = n\left(\frac{v + v_0}{v}\right) = f\left(\frac{v + \frac{v}{5}}{v}\right) \Rightarrow n' = 1.2\ f$

Only observer move. There will be no change in the λ.

68. $\Delta\lambda = \lambda_0 \frac{v}{c} \Rightarrow 1\text{nm} = 600\text{nm}\frac{v}{c}$

$\Rightarrow v = 5 \times 10^5\ \text{m/s}$