SOLUTIONS

OSCILLATIONS (SHM)

1. Force constant

$K = m\omega^2$

$K = m(2\pi n)^2$

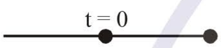
$K = m4\pi^2 n^2$

2. $\frac{d^2x}{dt^2} = -\left(\frac{k}{m}\right)x$

3. $a + 16\pi^2 x = 0$

$\Rightarrow \quad \omega^2 = 16\pi^2 \Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{4\pi} = 0.5\,s$

4. S.H.M. is periodic function of unique frequency and finite amplitude.

5. 

$t = \frac{T}{4}\left(\frac{\pi}{2}\text{Angle}\right)$

$\frac{\pi}{6}$ phase will develop when particle move from mean position towards amplitude position. It mean particle move in + × direction

$x = a \sin \omega t$

$x = a \sin \theta$

$x = a\sin\frac{\pi}{6} = a\sin 30° = \frac{a}{2}$

6. Time required between mean to half of amplitude $\left(\frac{a}{2}\right)$ position in t_1 is $= \frac{T}{12}$

Time required between $\left(\frac{a}{2}\right)$ to a position is $t_2 = \frac{T}{6}$

$\frac{t_1}{t_2} = \frac{\frac{T}{12}}{\frac{T}{6}} = \frac{1}{2} = 1:2$

7. $x = A\sin\omega t$

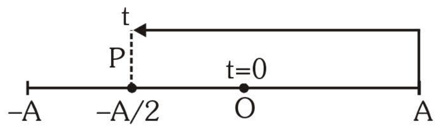
If t = 1 second then $x = A\sin\frac{2\pi}{T}\times 1$

or $x_I = A\sin\frac{2\pi}{8} = \frac{A}{\sqrt{2}}$

but in two second it will reach to extreme position, so distance travelled in next second

$x_{II} = A - \frac{A}{\sqrt{2}} = \frac{(\sqrt{2}-1)A}{\sqrt{2}}$

ratio of distance $\frac{A/\sqrt{2}}{(\sqrt{2}-1)A/\sqrt{2}} = \frac{1}{\sqrt{2}-1}$

8. 

Total length in one cycle = 4A

So, $\frac{5}{8}\cdot(4A) = \frac{5A}{2}$

$\Rightarrow$ Particle travels from 0 to P $\Rightarrow \Delta\phi = 210°$

$t = \frac{T}{360°} \times 210° = \frac{7T}{12}$

9. For first particle $x = A/2 \Rightarrow \theta_1 = 30°$

For second particle

$x = -A/2 \Rightarrow \theta_2 = 2\pi - 30°$

$\Delta\phi = \theta_2 - \theta_1 = 300°$ or $60°$

(as phase difference of ϕ or $2\pi - \phi$ is same)

10. $f = \frac{1}{\text{Time period}} \Rightarrow f = \frac{1}{0.04} = 25$ Hz

11. $v_{max} = \omega A = 0.4$

$A_{max} = A = 10$ cm

So, $\omega = 4$ rad/sec

and $T = \frac{2\pi}{\omega} = \frac{\pi}{2}$ second

12. $V_{max} = A\omega$

13. In one complete oscillation the displacement is zero, so average velocity is also zero.

14. $x = A\sin(\omega t + \phi)$

$\because$ At t = 1 second $x = 0$

$0 = A\sin(\omega\times 1 + \phi)$ So $\phi = -\omega$

Now $V = A\omega\cos(\omega t + \phi)$

At t = 2 second $V = \frac{1}{4}$ m/s

So $\frac{1}{4} = A \times \omega \cos(\omega \times 2 - \omega)$

$\frac{1}{4} = A\omega\cos\omega$

by putting values

$\frac{1}{4} = A \times \frac{2\pi}{6}\cos\frac{2\pi}{6}$ So $A = \frac{3}{2\pi}$ m

15. $y_1 = 0.1\sin(100\pi t + \pi/3)$

$\Rightarrow V_1 = 0.1 \times 100\pi \cos\left(100\pi t + \frac{\pi}{3}\right)$

and $y_2 = 0.1\cos 100\pi t$

$\Rightarrow V_2 = -0.1 \times 100\,\pi \sin 100\pi t$

or $V_2 = 10\,\pi \sin(100\pi t + \pi)$

Now $\phi_{12} = \phi_1 - \phi_2 = \left[100\pi t + \frac{\pi}{3} + \frac{\pi}{2}\right] - [100\pi t + \pi]$

$= -\frac{\pi}{6}$ rad.

16. Amplitude = same

$\Delta\phi$ = const $\Rightarrow$ T = same

So, $v_{max} = \omega A$ = same

17. $|\vec{a}| = \omega^2 |\vec{x}| \Rightarrow 20 = \omega^2(5)$

So, ω = 2 rad/s.

18. x = 1 cm a = 1 cm/s²

V = 1 cm/s

$a = \omega^2 x \Rightarrow \omega^2 = 1 \Rightarrow T = 2\pi$

19. $a = -\omega^2 x$,

So graph is straight line with negative slope

20. $x = 5.0\cos(2\pi t + \pi) = -5.0\cos 2\pi t$

Velocity = $\frac{dx}{dt} = 10\pi \sin 2\pi t$

Acceleration = $\frac{d^2x}{dt^2} = 20\pi^2 \cos 2\pi t$

$\therefore$ At t = 1.5 sec,

$x = -5.0\cos 3\pi = -5.0(-1) = 5$

Velocity = $\frac{dx}{dt} = 10\pi(\text{zero}) = 0$

Acceleration = $\frac{d^2x}{dt^2} = 20\pi^2(-1) = -20\pi^2$

21. $a_{max} = -\omega^2 A$

So $\frac{a_{max}}{a'_{max}} = \frac{\omega^2}{(\omega')^2} = \frac{(500)^2}{(5000)^2} = \frac{1}{100}$

22. $\alpha = \omega^2 A$

$\beta = \omega A \quad \therefore \frac{\alpha}{\beta} = \omega \Rightarrow \frac{\alpha}{\beta} = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi\beta}{\alpha}$

23. Amplitude A = 3 cm

When particle is at x = 2 cm,

its |velocity| = |acceleration|

i.e., $\omega\sqrt{A^2 - x^2} = \omega^2 x \Rightarrow \omega = \frac{\sqrt{A^2 - x^2}}{x}$

$T = \frac{2\pi}{\omega} = 2\pi\left(\frac{3}{4}\right) = \frac{3\pi}{2}$

24. PE = $\frac{1}{4}$(TE)

$\frac{1}{2}kx^2 = \frac{1}{4}\left(\frac{1}{2}kA^2\right) \Rightarrow \boxed{x = \pm\frac{A}{2}}$

25. $U_{max} = \frac{1}{2}kx^2$

$1 = \frac{1}{2}k(0.2)^2 \Rightarrow k = 50$

and $T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow T = 2\pi\sqrt{\frac{4}{50}} = \frac{2\pi\sqrt{2}}{5}$ sec

26. $F = -\frac{dU}{dx} = \frac{-d}{dx}(8x^2) \Rightarrow -16x$

(F) = −16(−2) = 32 Dyne.

27. $K_{max} = \frac{1}{2}KA^2$

$(KE)_P = \frac{1}{2}K_{max} = \frac{1}{2}K(A^2 - x^2)$

$\Rightarrow x = \frac{A}{\sqrt{2}}$ or $\theta = 45°$

So, time = $\frac{T}{360°} \times 45° = T/8$

28. T.E. = mgh + $(K.E.)_{max}$

9 = 5 + $(K.E.)_{max}$

$(K.E.)_{max}$ = 4 J This is KE at mean position

29. $KE = \frac{1}{2}KA^2\cos^2\omega t$

$\because PE_{max} = TE = \frac{1}{2}KA^2 = K_0$

30. $\langle E \rangle = \langle V \rangle = \frac{1}{4}KA^2$

31. Energy at rest position is 3J this is potential energy due to height, mgh = 3J.

Mean K.E. = 4 J

So Maximum K.E. = 2(Mean K.E.) = 8J

T.E. = mgh + $(K.E.)_{max}$ = 3 + 8 = 11 J

32. $$T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{R}+\frac{1}{R}\right)}} \Rightarrow T = 2\pi\sqrt{\frac{R}{2g}} = \frac{84.6}{\sqrt{2}}\text{min}$$

or T = 59.8 min

33. $$T' = 2\pi\sqrt{\frac{\ell}{g_{eff}}}$$

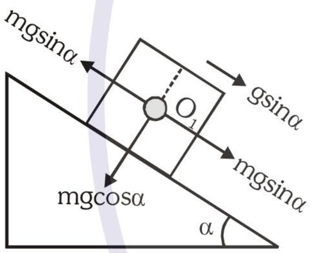
$$T' = 2\pi\sqrt{\frac{\ell}{g+g/3}} = \frac{\sqrt{3}}{2}T$$

34. $A = a \Rightarrow x = a/2 \Rightarrow \theta = 30°$

$-a$ O $a/2$ a

So, $v = v_{max}\cos 30° = \dfrac{\pi\sqrt{3}a}{T}$

35.



When pendulum is observed from vehicle frame

$$g_{eff} = g\cos\alpha \Rightarrow T = 2\pi\sqrt{\frac{L}{g\cos\alpha}}$$

36. $$T \propto \sqrt{\ell} \Rightarrow \frac{T'}{T} = \sqrt{\frac{1.44}{1}}$$

$\Rightarrow$ T' = 1.2T $\Rightarrow$ T' = 72 sec

37. $$T_1 = 2\pi\sqrt{\frac{1.21}{g}} \Rightarrow T_2 = 2\pi\sqrt{\frac{1}{g}}$$

$\Rightarrow T_1 = 1.1\,T_2$ or $10T_1 = 11\,T_2$

This means 10 oscillation of larger pendulum will take same time as 11 oscillation of samller pendulum and after that they will be in plane.

OR

They will be in same phase when long pendulum completes N and short pendulum completes (N +1) oscillation.

So $N\sqrt{\ell_{long}} = (N+1)\sqrt{\ell_{short}}$

$\Rightarrow N\sqrt{1.21} = \sqrt{(N+1)}\sqrt{1}$

$\Rightarrow$ 1.1 N = N + 1 $\Rightarrow$ N = 10

38. $|\vec{a}| = \omega^2 x$

$\Rightarrow 16 = \omega^2\, 4 \Rightarrow \omega^2 = 4$

$\Rightarrow \omega = 2 \Rightarrow \dfrac{2\pi}{T} = 2 \Rightarrow T = \pi$ s

39. $$f_1 = \frac{1}{2\pi}\sqrt{\frac{K/2}{m}} \quad \{K_{eq} = K/2\}$$

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{2K/3}{m}} \quad \left\{K_{eq} = \frac{2K\cdot K}{2K+K} = \frac{2K}{3}\right\}$$

$$\frac{f_1}{f_2} = \frac{\sqrt{3}}{2}$$

40. $$V_{max_1} = \sqrt{\frac{k_1}{m}}\cdot A_1 \Rightarrow V_{max_2} = \sqrt{\frac{k_2}{m}}A_2$$

Since $V_{max_1} = V_{max_2} \Rightarrow \dfrac{A_1}{A_2} = \sqrt{\dfrac{k_2}{k_1}}$

41. $$t_1 = 2\pi\sqrt{\frac{m}{k_1}}$$

$$t_2 = 2\pi\sqrt{\frac{m}{k_2}}$$

$$t = 2\pi\sqrt{\frac{m}{k_1+k_2}} \quad \{k_{eq} = k_1 + k_2\}$$

Solving $t = \dfrac{t_1 t_2}{\sqrt{t_1^2 + t_2^2}}$

42. In series combination

$$\frac{1}{K_S} = \frac{1}{K_1} + \frac{1}{K_2} \Rightarrow K_S = \frac{K_1K_2}{K_1+K_2}$$

43. Both springs are connected in parallel

so K_{eq} = 2k + k = 3k

frequency $n = \dfrac{1}{2\pi}\sqrt{\dfrac{3k}{m}}$

44. Effective value of g is zero in artificial satellite.

45. $T = 2\pi\sqrt{\frac{m}{k}}$

$3 = 2\pi\sqrt{\frac{m}{k}}$ ...(1)

$5 = 2\pi\sqrt{\frac{m+2}{k}}$ ...(2)

$\frac{(1)^2}{(2)^2} \Rightarrow \frac{9}{25} = \frac{m}{m+2} \Rightarrow m = \frac{18}{16} = \frac{9}{8}$

46. In 20 seconds the amplitude becomes 1/3

So in 40 seconds the amplitude $= \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$ of initial.

47. Amplitude of vibrations remain constant in case of free, maintained & forced vibrations.

48. $T_{st} = 2\pi\sqrt{\frac{R_e}{g}} = 84.6$ min

$T_{ma} = 2\pi\sqrt{\frac{R_e}{g}} = 84.6$ min

$T_{SP} = 2\pi\sqrt{\frac{R_e}{g_{eff}}} = 2\pi\sqrt{\frac{R_e}{g}} = 84.6$ min

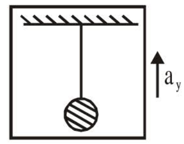
$T_{is} = 2\pi\sqrt{\frac{R_e}{g}} = 84.6$ min

49. $a_{max} = \omega^2 A = g$ {when N = 0}

$\Rightarrow A = \frac{g}{\omega^2} = \frac{10}{\pi^2} = 1\text{m}$

and $V_{max} = \omega A = 3.14$ m/s

50. $\uparrow y = Kt^2$



$a_y = \frac{d^2y}{dt^2} = 2K = 2 \text{ m/s}^2$

So, $g_{eff} = g + a_y = 12 \text{ m/s}^2$

$T_1 = 2\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{\frac{\ell}{10}}$

$T_2 = 2\pi\sqrt{\frac{\ell}{g_{eff}}} = 2\pi\sqrt{\frac{\ell}{12}}$ So, $\frac{T_1^2}{T_2^2} = \frac{12}{10} = \frac{6}{5}$