SOLUTIONS

PROPERTIES OF MATTER & FLUID MECHANICS

1. $\ell_{max} = \dfrac{\text{Breaking stress}}{\rho g} = \dfrac{15.8\times10^{8}}{7.9\times10^{3}\times10} = 20\ \text{km}$

2. Maximum force in the string is $F_{max} = mg + \dfrac{mv^2}{\ell}$

$\because\quad v=\sqrt{2g\ell}\,,\quad F_{max} = 3\ mg$

$\Delta\ell = \dfrac{F\ell}{YA} = \dfrac{3mg\ell}{YA} = \dfrac{3\times40\times5}{2\times10^{11}\times2\times10^{-6}} = 1.5\ \text{mm}$

3. Energy $U = \dfrac{1}{2Y}$ (stress)^2 volume

Stress $= \dfrac{F}{A} = \dfrac{40}{4\times10^{-4}} = 10^5\ \text{N/m}^2$,

Volume $= A\ell$

$U = \dfrac{1}{2\times10^{11}}\times(10^5)^2\times4\times10^{-4}\times0.2 = 4\ \mu J$

4. $\Delta\ell \propto \ell^2$

5. $\text{Slope} = \dfrac{7\text{mm}/1\text{m}}{7000\text{kN}/\text{m}^2} = \dfrac{1}{Y} \Rightarrow Y = 1\times10^9\ \text{N}/\text{m}^2$

6. Total weight at height $\dfrac{L}{4}$ from its lower end $= W_T$

W_T = weight suspended + weight of $\dfrac{L}{4}$ of wire

$W_T = W_1 + \dfrac{W}{4}$

$\text{Stress} = \dfrac{F}{\text{Area}} = \dfrac{W_T}{S}$

$\text{Stress} = \dfrac{\left(W_1+\dfrac{W}{4}\right)}{S}$

7. From graph

$\text{Slope} = \dfrac{F}{\Delta\ell}$

also $Y = \dfrac{FL}{A\Delta\ell}$

$\text{Slope} = \dfrac{F}{\Delta\ell} = Y\dfrac{A}{L}$

$\Rightarrow$ more slope more Y

So $\quad Y_A > Y_B$

8. Potential energy per unit volume is

$= \dfrac{1}{2}\times\dfrac{(\text{Stress})^2}{Y} \Rightarrow \dfrac{U_1}{U_2} = \dfrac{F_1^2}{A_1^2}\times\dfrac{A_2^2}{F_2^2} = \dfrac{r_2^4}{r_1^4} = 16:1$

9. $\dfrac{F}{A} = \dfrac{Y\Delta L}{L}$

$F = \dfrac{YA}{L}\cdot\Delta L$

$\boxed{\dfrac{F}{\Delta L}\propto A}$

more the slope, more will be area.

10. $Y = \dfrac{\frac{F}{A}}{\frac{\Delta\ell}{\ell}} \quad\Rightarrow\quad \Delta\ell = \dfrac{F\ell}{AY}$

But $V = A\ell$ so $A = \dfrac{V}{\ell}$ (V = volume)

Therefore $\Delta\ell = \dfrac{F\ell^2}{VY} \propto \ell^2$

11. $Y = \dfrac{F\ell}{A\Delta\ell} \quad \because V = A\ell \quad \text{so} \quad \ell = \dfrac{V}{A}$

So $F = \dfrac{YA\Delta\ell}{\ell} = \dfrac{YA^2\Delta\ell}{V} \propto A^2$

$\dfrac{F_1}{F_2} = \left(\dfrac{A_1}{A_2}\right)^2$

$\Rightarrow \dfrac{2F}{F_2} = \left(\dfrac{A}{2A}\right)^2 = \dfrac{1}{4}$

$\Rightarrow F_2 = 8F$

12. $\left(-\dfrac{\Delta V}{V}\right) = \dfrac{0.004}{100}$

$\Delta P = K\times\left(-\dfrac{\Delta V}{V}\right) = 2100\times10^6\times\dfrac{0.004}{100}$

$= 84\ \text{kPa}$

13. Bulk modulus $K = \dfrac{\text{Volume Stress}}{\text{Volume Strain}}$

$K = \dfrac{\rho gh}{\Delta V/V} = \dfrac{10^3\times9.8\times400}{0.01/100} = 39.2\times10^9\ \text{N}/\text{m}^2$

14. $B = \dfrac{\Delta P}{-\dfrac{\Delta V}{V}}$, $\dfrac{\Delta V}{V} = \dfrac{3\Delta R}{R}$

$B = \dfrac{\Delta P}{\dfrac{-3\Delta R}{R}} \Rightarrow -\dfrac{\Delta R}{R} = \dfrac{P}{3B}$ $(\Delta P = P)$

15. $Y = 2\eta\,(1 + \sigma)$
$2.4\eta = 2\eta\,(1 + \sigma)$
$\sigma = 0.2$

16. $\dfrac{F_A}{F_B} = \dfrac{P_A A_A}{P_B A_B} = \dfrac{\rho_A g h_A A_A}{\rho_B g h_B A_B} = 1$

17. Hydraulic lift, hydraulic jack, hydraulic press works on Pascal's law.

18. The pressure at any level is the same in both arms of the U-tube. That is the pressure at the interface sides are the same.

i.e. $P_1 = P_2$

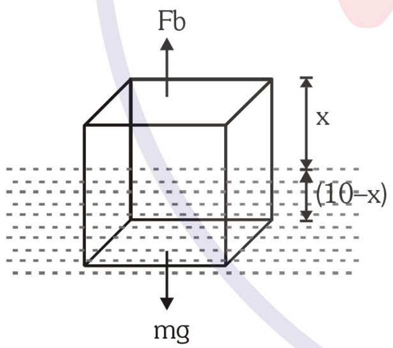
or $\rho_{oil} g(d+\ell) = \rho_{water} g\ell$

$\rho_{oil} \times 9.8 \times (0.0125 + 0.135) = 1000 \times 9.8 \times 0.135$

$\therefore\ \rho_{oil} = \dfrac{1000 \times 9.8 \times 0.135}{9.8 \times (0.0125 + 0.135)}$

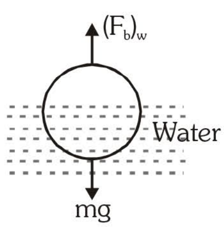
$= \dfrac{135}{0.1475} = 915\ \text{kg/m}^3$

19.

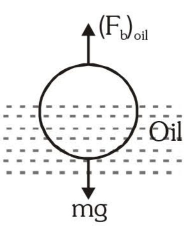


$F_b = mg$
$\rho_L V_d g = \rho_B V_B g$
$(1)\,(10)\,(10)\,(10 - x) = 800$
$x = 2$ cm
Outside volume $= 2 \times 10 \times 10 = 200\ \text{cm}^3$

20.



$V_d = V/3$



$V_d = V/4$

$(F_b)_w = (F_b)_{oil}$
$\rho_w V_d g = \rho_{oil} V_d\ g$

$(1)\,\dfrac{V}{3} g = \rho_{oil} \dfrac{V}{4} g \Rightarrow \rho_{oil} = \dfrac{4}{3}\ \text{gm/cc}$

21. Due to upthrust on the block, its apparent wt. will reduce & hence the reading of spring balance A will be less then 2 kg. But since the liquid in the beaker exerts buoyant force on the block, the block will also exert reaction force on the liquid in the downward direction and the reading of balance B will increase due to this additional force.

22. Liquid with low density float over high density.

23. When coin is removed the downward force an block decrease, which will reduce the volume submerged also the removing volume will filled by water so $h\downarrow$.

24. $W_{APP} = W - F_B$ $(F_B = \text{force of buoyant})$

$W_{APP} = V\rho g - V\rho_1 g$

$= V\rho g\left[1 - \dfrac{\rho_1}{\rho}\right]$

$W_{APP} = W\left[1 - \dfrac{\rho_1}{\rho}\right]$

25. For wooden block $\rho_{Block} < \rho_{Water}$

So $F_B > Mg$, hence it will rise up with an constant acceleration.

26. In floating condition

$W = Th$

Let V_1 volume immersed in lower liquid then

$V\rho g = (V - V_1)\rho_1 g + V_1\rho_2 g$

$V(\rho - \rho_1) = V_1(\rho_2 - \rho_1)$

$\dfrac{V_1}{V} = \dfrac{\rho - \rho_1}{\rho_2 - \rho_1} = \dfrac{\rho_1 - \rho}{\rho_1 - \rho_2}$

27. $P_1 = P_2$

$\dfrac{F_1}{A_1} = \dfrac{F_2}{A_2} \Rightarrow \dfrac{10}{\pi r_1^2} = \dfrac{F_2}{\pi r_2^2}$

$F_2 = 90\ N$

28. Writing equation of continuity for the tube & the holes

$\Rightarrow\ A_{tube} v_{tube} = nA_{hole} v_{hole}$

$\pi R^2 \times v = n\pi r^2 v' \quad \Rightarrow \quad v' = \dfrac{vR^2}{nr^2}$

29. 

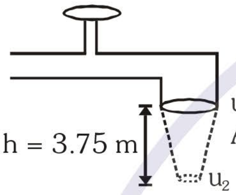
Av = constant

$\pi R^2 V = n\pi r^2 v_1 \Rightarrow v_1 = \frac{VR^2}{nr^2}$

30. Rate of flow = Av = same everywhere in pipe

$\Rightarrow \quad v = \frac{3800 \times 10^{-3} m^3}{60s} \times \frac{1}{\pi(6.4 \times 10^{-2})^2}$

$\Rightarrow \quad v = 5$ m/s

31. 

$u_2^2 = u_1^2 + 2gh$

$u_2^2 = 25 + 2 \times 10 \times 3.75$

$u_2 = 10$ m/s

$A_1 u_1 = A_2 u_2 \Rightarrow A_2 = 5 \times 10^{-5}$ m^2

32. $F = (\Delta P)A = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)A$

$F = \frac{1}{2} \times 1.2\left((40)^2 - 0\right) \times 250$

$= 2.4 \times 10^5$ N upwards

33. AV = constant

If A ↓ then speed ↑ and pressure ↓

34. $F = \Delta PA = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)A$

$F = \frac{1}{2}\rho\left[\left(\sqrt{5}v\right)^2 - v^2\right]A = 2\rho v^2 A$

35. For maximum range of y = h

36. Volume flowing per second = Av

$= A\sqrt{2gh} = 10^{-4}\sqrt{2 \times 10 \times 5} = 10^{-3}$ m^3/s

37. Scent sprayer is based on Bernoulli's theorem.

38. $A_1 v_1 = A_2 v_2$

$L^2 \times \sqrt{2gy} = \pi R^2 \times \sqrt{2g4y}$

$R = \frac{L}{\sqrt{2\pi}}$

39. $\eta = \frac{\text{Shear stress}}{\text{Strain Rate}} = \frac{F/A}{V/\ell} = \frac{mg\ell}{AV}$

ℓ is thickness of film

$\eta = \frac{0.01 \times 9.8 \times 0.3 \times 10^{-3}}{0.1 \times 0.085} = 3.45 \times 10^{-3}$ Pa − s

40. $F_v = \eta A \frac{v}{\ell} \quad \Rightarrow \quad \ell = \frac{\eta Av}{F_v}$

$\ell = \frac{0.01 \times 10^{-1} \times 1 \times 4}{2} = 2$ mm

41. $v_T = \frac{2}{9}\frac{r^2 \rho g}{\eta}$ (ρ of air neglect)

$\eta = \frac{2}{9}\frac{r^2 \rho g}{v_T} = \frac{2}{9} \times \frac{(10^{-1})^2 \times 1.47 \times 980}{2.1}$

= 1.52 poise

43. Final velocity is terminal velocity which is independent from 'h'

$v_T = \frac{2r^2}{9}\frac{(\rho - \sigma)}{\eta}g$

44. $v_T = \frac{2r^2}{9}\frac{(\rho - \sigma)}{\eta}g$

$v_T = 0.2$ m/s for gold

ρ = 19.5 kg/m^3 (gold)

σ = 1.5 kg /m^3

From above data we can find value of $\frac{r^2}{n}$

putting in 2nd case for silver

$\rho_2 = 10.5$ kg/m^3

$\boxed{v_T = 0.1 m/s}$

45. $v_T = \frac{2}{9}\frac{r^2(\rho - \sigma)}{\eta}$

$\frac{v_1}{v_2} = \frac{(\rho - \sigma_1)\eta_2}{(\rho - \sigma_2)\eta_1}$

$v_2 = \frac{V_1[\rho - \sigma_{gly}]\eta_{water}}{[\rho - \sigma_{water}]\eta_{gly}}$

$v_2 = 3.98 \times 10^{-4}$ cm/s

46. By using $A_1 v_1 = A_2 v_2$

we have 80(3) = 40(v)

$\Rightarrow$ v = 6 m/s

47. $2T\ell = W$

$$T = \frac{W}{2\ell} = \frac{1.5\times10^{-2}}{2\times0.6} = 0.0125\ N/m$$

49. Elastic membrane is formed on the surface of water due to surface tension. This help's spider & insects to move and run on the surface of water.

50. Detergent decreases the oil-water surface tension and helps in removing dirty greasy stains.

51. W.D. $= 2 \times 4\pi T\left(r_2^2 - r_1^2\right)$

If initial volume = V

then final volume = 8V then radius will become 2R

So $\quad W.D. = 2\times4\pi T\left((2R)^2 - R^2\right) = 24\pi R^2 T$

52. $$\frac{SE_i}{SE_f} = \frac{(n)^{1/3}}{1} = \frac{(2)^{1/3}}{1}$$

53. W.D. $= 2T\Delta A$

$= 2 \times 3 \times 10^{-2} \times (30 - 25) \times 10^{-4}$

$= 30 \times 10^{-6}$ J

54. $\Delta E = 4\pi R^2 T\left(n^{1/3} - 1\right)$

$$= 4\pi\frac{D^2}{4}T\,[(27)^{1/3} - 1] = 2\pi D^2 T$$

55. $W = T\Delta A$

$W = TA$

as $\Delta A = A$

56. $(P_{ex})_1 = 2(P_{ex})_2$

$$\frac{4T}{r_1} = 2\left(\frac{4T}{r_2}\right) \Rightarrow \frac{r_1}{r_2} = \frac{1}{2}$$

$$\frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

57.

58. For soap film we have two free surface

$W = T \times 2\Delta A$

$= 3\times10^{-2}\times 2[4\times5 - 4\times4]\times10^{-4}$

$= 24\times10^{-6}$ J

59. $W = T(2\Delta A) \qquad \{\Delta A = (20 - 14)\ cm^2\}$

$$\Rightarrow\ T = \frac{W}{2\Delta A} = \frac{1.5\times10^{-4}}{2\times6\times10^{-4}} = 0.125\ Nm^{-1}$$

60. $P_{excess} = \dfrac{4T}{r}$

$$\rho gh = \frac{4T}{r} \Rightarrow 10^3\times10\times4\times10^{-3} = \frac{4T}{16\times10^{-3}}$$

T = 160 dyne/cm

61. $P_1 - P_2 = \dfrac{4T}{r}$

$$\Rightarrow \frac{4T}{r_1} - \frac{4T}{r_2} = \frac{4T}{r}$$

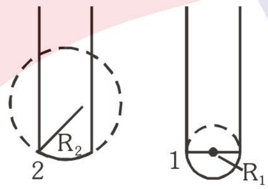
$$\Rightarrow r = \frac{r_1 r_2}{r_2 - r_1}$$

$$r = \frac{r_1 r_2}{r_2 - r_1} = \frac{4\times5}{5-4}\text{cm.} = 20\text{ cm.}$$

62. Excess pressense inside a bubble is

$$P_{Ex} = \frac{4T}{r}$$

$$P_{Ex} \propto \frac{1}{r}$$



$(R_2 > R_1)$

$r_1 < r_2$

So, $(P_1 > P_2)$

63. For hemispherical

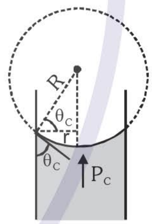
$R = r$

i.e. $\cos\theta = 1$

$\theta = 0°$

as $R\cos\theta = r$

in capillary tube



$$R = \frac{r}{\cos\theta}$$

64. $h = \dfrac{2T\cos\theta}{\rho g r}$

As r, h, T are same, $\dfrac{\cos\theta}{\rho}$ = constant

$$\Rightarrow \frac{\cos\theta_1}{\rho_1} = \frac{\cos\theta_2}{\rho_2} = \frac{\cos\theta_3}{\rho_3}$$

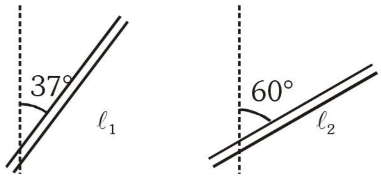
As $\rho_1 > \rho_2 > \rho_3$

$\Rightarrow \cos\theta_1 > \cos\theta_2 > \cos\theta_3 \Rightarrow \theta_1 < \theta_2 < \theta_3$

As water rises so θ must be acute

So, $0 \le \theta_1 < \theta_2 < \theta_3 < \pi/2$

65.



$h = \ell_1 \cos 37°$ $\quad h = \ell_2 \cos 60°$

Height will remain same

$\ell_1 \cos 37° = \ell_2 \cos 60°$

$$\ell_1 \frac{4}{5} = \ell_2 \frac{1}{2}$$

$$\frac{\ell_1}{\ell_2} = \frac{5}{8}$$

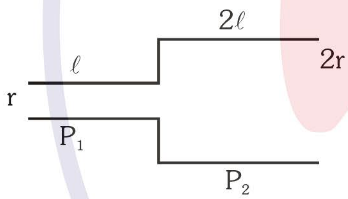
66. $h = \dfrac{2T\cos\theta_c}{\rho r g}$ $\quad \because \theta_C = 0°$

$$r = \frac{2T}{\rho h g} = \frac{2 \times 75 \times 10^{-3}}{10^3 \times 0.03 \times 10} = 0.5\,\text{mm}$$

67. $m \propto r$

$$\frac{m_1}{m_2} = \frac{r_1}{r_2} \Rightarrow m_2 = 2M$$

68. Rate flow $Q = \dfrac{\pi P r^4}{8\eta\ell}$



In series combination rate flow is same

$Q_1 = Q_2$

$$\frac{\pi P_1 r^4}{8\eta\ell} = \frac{\pi P_2 (2r)^4}{8\eta(2\ell)}$$

$$\frac{P_1}{P_2} = \frac{8}{1}$$

69. $\because g_{eff} = 0$

Then height = 50 cm

70. When water rise upto a height h then mass of liquid rise $m = (\pi r^2 h)\rho$

$\because$ total mass be located at centre of mass.

then potential energy $U = mg\dfrac{h}{2} = (\pi r^2 H\rho)g\dfrac{h}{2}$

by Zurin law $r \times h$ = constant

$\therefore$ U = constant

71. $mg = (2\pi r)T\cos\theta$

$6.28 \times 10^{-4} = 2\pi(r) \times (5 \times 10^{-2}) \times 1$

$r = 2 \times 10^{-13}$ m