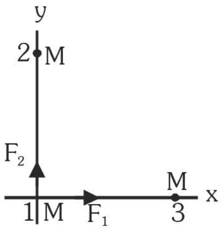
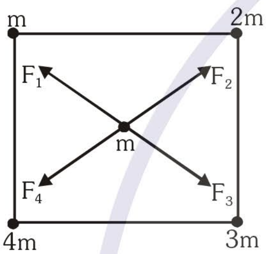
SOLUTIONS
GRAVITATION

1. $F_1 = F_2 = \dfrac{G(1)(1)}{(0.2)^2} = \dfrac{6.67\times10^{-11}}{0.04} = 1.67\times10^{-9}$



$\vec{F}_{net} = F_1(\hat{i}) + F_2(\hat{j}) = F(\hat{i}+\hat{j}) = 1.67\times10^{-9}(\hat{i}+\hat{j})$ N

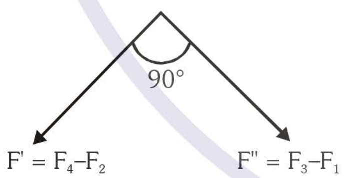
2.



$F_1 = \dfrac{Gmm}{(a/\sqrt{2})^2},\ F_2 = \dfrac{Gm2m}{(a/\sqrt{2})^2}$

$F_3 = \dfrac{Gm3m}{(a/\sqrt{2})^2},\ F_4 = \dfrac{Gm4m}{(a/\sqrt{2})^2}$

So resultant of forces will be :



$F' = \dfrac{Gm4m}{(a/\sqrt{2})^2} - \dfrac{Gm2m}{(a/\sqrt{2})^2},\ F'' = \dfrac{Gm3m}{(a/\sqrt{2})^2} - \dfrac{Gmm}{(a/\sqrt{2})^2}$

$F' = \dfrac{2Gm^2}{a^2/2},\qquad F'' = \dfrac{2Gm^2}{a^2/2}$

$F' = \dfrac{4Gm^2}{a^2},\qquad F'' = \dfrac{4Gm^2}{a^2}$

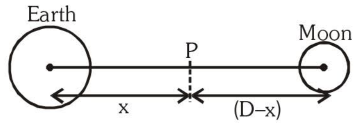
Now they are at 90°

So resultant force = $4\sqrt{2}\dfrac{Gm^2}{2}$

3. More work is done against the gravitational pull of the earth.

4. Action and reaction force are equal in magnitude.

5.



At point 'p' for gravitational field to be zero field due to earth = field due to moon

$\Rightarrow \dfrac{GM_e}{x^2} = \dfrac{GM_m}{(D-x)^2} \Rightarrow \dfrac{81M_m}{x^2} = \dfrac{M_m}{(D-x)^2}$

$\Rightarrow \dfrac{x}{D-x} = 9 \Rightarrow 9(D-x) = x \Rightarrow x = \dfrac{9D}{10}$

6. $g = \dfrac{GM_e}{R_e^2}$

$g_{mars} = \dfrac{G(0.1)M_e}{(0.5)^2R_e^2} = \dfrac{0.4GM_e}{R_e^2} = 0.4g$

7. $t = \sqrt{\dfrac{2h}{g}} = 1$ sec; $t' = \sqrt{\dfrac{2h}{g'}} = \sqrt{6}\sqrt{\dfrac{2h}{g}} = \sqrt{6}$ sec

8. $g_{height} = g/4 = \dfrac{g}{\left(1+\dfrac{h}{R_e}\right)^2}$

$\Rightarrow h = R_e = 6400$ km

9. Weight is due to gravitational pull of earth which varies with distance as $\dfrac{1}{r^2}$. Weight becomes zero when gravitational pull of both planets become equal & opposite After that its weight again increase to 240 N due to mars.

10. $g_{eff} = g\left(1-\dfrac{2h}{R}\right) = g\left(1-\dfrac{32\times2}{6400}\right) = g(1-0.01)$

$g_{eff} = 0.99g\ ms^{-2}$

11. Inside the earth $g \propto r$ and outside the earth

$g \propto \dfrac{1}{r^2}$

12. $g = \frac{GM_E}{R_E^2}$

where M_E and R_E is the mass and radius of the earth respectively.

$M_E = \frac{g}{G} R_E^2$

13. As $g' = \frac{g}{\left(1+\frac{h}{R}\right)^2}$

$\frac{g}{2} = \frac{g}{\left(1+\frac{h}{R}\right)^2} \Rightarrow \left(1+\frac{h}{R}\right)^2 = 2$

$\Rightarrow \frac{h}{R} = \sqrt{2}-1 \Rightarrow h = R\left(\sqrt{2}-1\right)$

14. $g' = g - R_e\omega^2\cos^2\lambda \Rightarrow g' < g$

except for $\lambda = 90°$ i.e. for poles.

15. There is no atmosphere on the moon, means no buoyant force in upward direction so it will fall down with acceleration g/6.

16. As $g = \frac{GM}{R^2}$

$\frac{\Delta g}{g}\% = \frac{-2\Delta R}{R}\%$

$\frac{\Delta g}{g}\% = -2(-1)\% = +2\%$

17. m decreased by 1% for h height

$g_h = g\left[1+\frac{h}{R}\right]^{-2}$

$\frac{\Delta g_h}{g} = \frac{-2h}{R}$

for depth.

$g_d = g\left[1-\frac{d}{R}\right]$

$\frac{\Delta g_d}{g} = \frac{-d}{R}$ (here d = h)

$\frac{\Delta g_h}{g} = \frac{2\Delta g_d}{g}$

$\frac{\Delta g_d}{g} = \frac{1}{2}\Delta g_h$

18. S_2 is correct because whatever be the g, the same force is acting on both the pans. Using a spring balance, the value of g is greater at the pole. Therefore mg at the pole is greater. S_4 is correct. S_2 and S_4 are correct.

19. $g = \frac{GM}{R^2} = \frac{GM_p}{\left(\frac{D_p}{2}\right)^2} = \frac{4GM_p}{D_p^2}$

20. $g = \left(\frac{GM_e}{R_e^3}\right)r$ for $0 < r \le R_e \Rightarrow \mathbf{g \propto r}$

$g = \frac{GM_e}{r^2}$ for $r \ge R_e \Rightarrow \mathbf{g \propto \frac{1}{r^2}}$

21. $m_s' = \frac{m_s}{6}$ & G' = 6 G

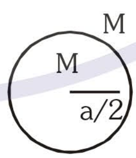
$g_E = \frac{G'M_E}{R^2} = \frac{6GM_E}{R^2} = 6g$

∴ Raindrops will fall faster, Walking on the ground would become more difficult, Time period of a simple pendulum on the earth would decrease.

22. $I_g = -\frac{GM}{R^2}$, $V = -\frac{GM}{R}$,

$V = I_g R = 6 \times 8 \times 10^6 = 4.8 \times 10^7$

23. $V = -\frac{GM}{(a/2)} - \frac{GM}{a} = -\frac{3GM}{a}$



24. Gravitational potential

$V = \frac{-GM}{r}$ for r > R

$V = \frac{-GM}{R}$ for $r \in [0, R]$

Potential remains constant inside the hollow sphere.

25. $V = \frac{-GM}{R+h} = -5.4 \times 10^7$ ..... (1)

and $g = \frac{GM}{(R+h)^2} = 6$ ..... (2)

dividing (1) and (2)

$\Rightarrow R+h = \frac{5.4\times10^7}{6}$

$\Rightarrow$ R + h = 9000 km so h = 2600 km

26. $U = -\frac{GM_e m}{R} - \frac{GM_m m}{r} = -\frac{G(81M_m)m}{R} - \frac{GM_m m}{r}$

$= -GmM_m\left(\frac{81}{R}+\frac{1}{r}\right)$

27. To escape from the earth total energy of the body should be zero KE+ PE = 0

$\frac{1}{2}mv^2 - \frac{GMm}{5R_e} = 0 \Rightarrow KE_{min} = \frac{mgR_e}{5}$

28. $V_{escape} = \sqrt{2gR}$

$\frac{V_1}{V_2} = \sqrt{\frac{g_1R_1}{g_2R_2}} = \sqrt{pq}$

29. Escape velocity = $\sqrt{\frac{2GM}{R}}$ = c = speed of light

$R = \frac{2GM}{c^2} = \frac{2\times6.6\times10^{-11}\times5.98\times10^{24}}{(3\times10^8)^2} = 10^{-2}$ m

30. when particle falls from infinity

$PE_i + KE_i = PE_f + KE_f$

$0 + 0 = \frac{-GMm}{R} + \frac{1}{2}mv_1^2$

$v_1 = \sqrt{\frac{2GM}{R}}$

when particle falls from a height

$PE_1 + KE_i = PE_f + kE_f$

$-\frac{GMm}{11R} + 0 = -\frac{GMm}{R} + \frac{1}{2}mv_2^2$

$\sqrt{\frac{10}{11}\times\frac{2GM}{R}} = v_2$

$\frac{v_1}{v_2} = \sqrt{\frac{11}{10}}$

31. Escape velocity $v = \sqrt{\frac{2GM_e}{R}}$ is independent of angle at projection

32. As $v_{esc} \propto \sqrt{\frac{M}{R}}$

Hence $\frac{v_2}{v_1} = \sqrt{\frac{M_2}{M_1}}\sqrt{\frac{R_1}{R_2}}$

$\frac{v_2}{v_1} = \sqrt{100}\sqrt{\frac{1}{4}} = 10\times\frac{1}{2} = 5$

$v_2 = 11.2 \times 5 = 56$ km/sec

33. $-\frac{GM_e m}{R} + \frac{1}{2}mK^2V_e^2 = -\frac{GM_e m}{r}$

$\frac{1}{2}K^2V_e^2 = GM_e\left(\frac{1}{R}-\frac{1}{r}\right)$

but $V_e = \sqrt{\frac{2GM_e}{R}}$

$\frac{1}{2}K^2\frac{2GM_e}{R} = GM_e\left(\frac{1}{R}-\frac{1}{r}\right)$

$\frac{K^2}{R} = \frac{1}{R} - \frac{1}{r} \Rightarrow \frac{1}{r} = \frac{1}{R} - \frac{k^2}{R} = \frac{1-k^2}{R} \Rightarrow r = \frac{R}{1-K^2}$

34. Escape Velocity = $\sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G}{R}\cdot\left(\frac{4}{3}\pi R^3\rho\right)} \propto R\sqrt{\rho}$

$\therefore$ Ratio, $\frac{v_e}{v_p} = 1 : 3\sqrt{3}$

35. $F \propto \frac{1}{r^m}$; $F = \frac{C}{r^m}$

This force will provide the required centripetal force Therefore

$m\omega^2 r = \frac{C}{r^m}$; $\omega^2 = \frac{C}{mr^{m+1}}$

$T = \frac{2\pi}{\omega} \Rightarrow T \propto r^{(m+1)/2}$

36. $\because$ total mechanical energy $E = -\frac{GMm}{2r}$

$\therefore \frac{E_1}{E_2} = \frac{m_1}{m_2} \times \frac{r_2}{r_1} = \frac{3}{1} \times \frac{4r}{r} = \frac{12}{1}$

37. $W = E_2 - E_1 = \frac{GMm}{2}\left(\frac{1}{2R} - \frac{1}{3R}\right) = \frac{GMm}{12R}$

38. Because its period is equal to the period of rotation of the earth about its axis so it will remain in the same place with respect to the earth.

39. $TE_1 = -\frac{GMm}{2(R+R)} = -\frac{GMm}{4R}$

$TE_2 = -\frac{GMm}{2(R+7R)} = -\frac{GMm}{16R}$

$\frac{TE_1}{TE_2} = \frac{4}{1}$

$\frac{KE_1}{KE_2} = \frac{-\frac{GMm}{4R}}{-\frac{GMm}{16R}} = \frac{4}{1}$

40. $K.E. = \frac{GMm}{2r}$ ⇒ Kinetic energies are unequal

$T = \frac{2\pi r^{3/2}}{\sqrt{GM}}$ ⇒ Time period are equal

$P.E. = -\frac{GMm}{r}$ ⇒ Potential energies are unequal

$v = \sqrt{\frac{GM}{r}}$ ⇒ Orbital speeds are equal

41. Speed of satellite $V = \sqrt{\frac{GM}{r}}$

$\Rightarrow \frac{V_B}{V_A} = \sqrt{\frac{r_A}{r_B}} = \sqrt{\frac{4R}{R}} = 2 \Rightarrow V_B = (3V)(2) = 6V$

42. For the satellite revolving around earth

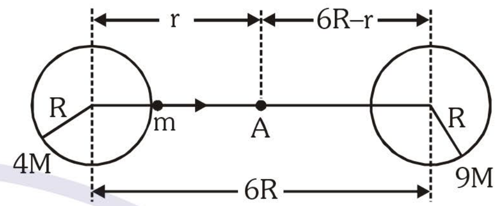
$$v_0 = \sqrt{\frac{GM_e}{(R_e + h)}} = \sqrt{\frac{GM_e}{R_e\left(1 + \frac{h}{R_e}\right)}} = \sqrt{\frac{gR_e}{1 + \frac{h}{R_e}}}$$

substituting the values

$v_0 = \sqrt{60 \times 10^6}$ m/s

$v_0 = 7.76 \times 10^3$ m/s = 7.76 km/s

43. The gravitational force on projectile of mass m at a distance r from a planet of mass 4M is zero



$\therefore \frac{G(4M)m}{r^2} = \frac{G(9M)m}{(6R-r)^2}$

or $4(6R-r)^2 = 9r^2$ or $2(6R - r) = 3r$ or $r = 2.4$ R.

44. First law of motion i.e. law of inertia.

45. As $T^2 \propto r^3$

$\frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2} = \left(\frac{9R}{R}\right)^{3/2} = 27$

$T_2 = 27 \times 2\pi\sqrt{\frac{R}{g}}$

46. For system to be bounded
Net energy i.e. (KE + PE) must be negative

47. $T = 2\pi\sqrt{\frac{r^3}{GM}} \Rightarrow T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$

$\Rightarrow h = \left[\frac{GMT^2}{4\pi^2}\right]^{1/3} - R$

48. $T^2 = \frac{4\pi^2}{GM}r^3$

$\left(\frac{T_1}{T_2}\right)^{2/3} = \frac{r_1}{r_2}$ $[\therefore\ T_1 = T_2]$

$\Rightarrow \frac{r_1}{r_2} = 1$

49. Orbital velocity of satellite, $v = \sqrt{\frac{GM}{R}} = v_0$

$\therefore v' = \sqrt{\frac{GM}{R + \frac{R}{2}}} = \sqrt{\frac{2GM}{3R}} = \sqrt{\frac{2}{3}}v_0$.

50. Total energy $= -\dfrac{GM_e m}{2(R+h)}$

$\because \quad g_0 = \dfrac{GM_e}{R^2} \quad \Rightarrow \quad M_e = \dfrac{g_0 R^2}{G}$

$\therefore$ Energy $= -\dfrac{m g_0 R^2}{2(R+h)}$

51. $T^2 = \dfrac{4\pi^2}{GM} a^3$

$T^2 = \dfrac{4\pi^2}{GM}\left(\dfrac{r_p + r_A}{2}\right)^3 = \dfrac{\pi^2}{2GM}(r_p + r_A)^3$

mvr = constant $\Rightarrow$ vr = constant

$\therefore \quad v_A r_A = v_p r_p$

$\therefore \quad r_A > r_p \quad \therefore \quad v_A < v_p$

52. According to Kepler's law

$\Rightarrow \dfrac{A_{SCD}}{t_1} = \dfrac{A_{SAB}}{t_2} \Rightarrow \dfrac{2A_{SAB}}{t_1} = \dfrac{A_{SAB}}{t_2} \Rightarrow t_1 = 2t_2$

53. From conservation of angular momentum

$mv_1r_1 = mv_2r_2 \Rightarrow \dfrac{v_1}{v_2} = \dfrac{r_2}{r_1}$

54. For satellite S moving elliptical orbit around the earth net force will be towards centre of the earth. (like centripetal force in circular motion)

55. Apply angular momentum conservation law

$mv_1r_1 = mv_2r_2$

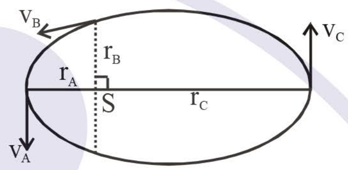
$\Rightarrow \quad 60 \times 1.6 \times 10^{12} = v_2 \times 8 \times 10^{12}$

$v_2 = 12$ m/sec

56. Since the angular momentum of the satellite about the earth is conserved.

$\therefore \vec{L}_a = \vec{L}_P$

58.



As $\perp$ = mvr = constant

and $r_C > r_B > r_A$

so $v_A > v_B > v_C$