SOLUTIONS
ROTATIONAL MOTION

1. $\frac{ML^2}{12}+\frac{MR^2}{4}=\frac{MR^2}{2} \Rightarrow \frac{L^2}{12}=\frac{R^2}{4} \Rightarrow L=\sqrt{3}\ R$

2. $I = I_{disc} + I_{Masses} = \frac{MR^2}{2} + 4mR^2$

3. As disc is lying in the x-z plane, so applying perpendicular axis theorem. $I_x + I_z = I_y$

$80 + I_z = 100 \quad \Rightarrow I_z = 100 - 80 = 20\ kg\ m^2$

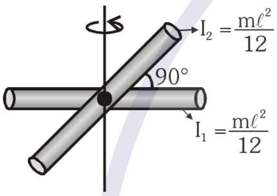
4. as $\quad I = MR^2 \quad$ or $\quad I \propto R^2$

so graph between I and R will be a parabola.

5. $I = m(a)^2 + m(a)^2 + m(0)^2 + m(2a)^2 \Rightarrow I = 6ma^2$

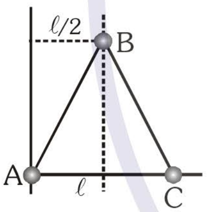
6. $I = MK^2 \Rightarrow I = 10(0.2)^2 = 0.4\ kg\text{-}m^2$

7.



So $I = I_1 + I_2 = \frac{m\ell^2}{12}+\frac{m\ell^2}{12}=\frac{m\ell^2}{6}$

8.



$I = I_A + I_B + I_C \quad \Rightarrow \quad I = 0 + m\left(\frac{\ell}{2}\right)^2 + m\ell^2 = \frac{5}{4}m\ell^2$

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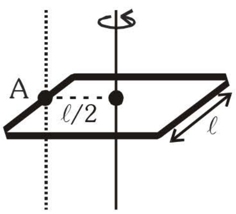


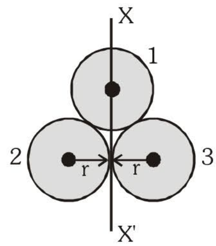
Moment of inertia of rad A about given axis, I_A

so, $\quad I_A = I_C + Md^2$

$$I_A = \frac{M\ell^2}{12} + M\left(\frac{\ell}{2}\right)^2 = \frac{M\ell^2}{12}+\frac{M\ell^2}{4}$$

$\Rightarrow \quad I_A = \frac{M\ell^2}{3} \quad$ so, $\quad I = 4 \times I_A = \frac{4}{3}M\ell^2$

10.



$I_{xx'} = I_1 + I_2 + I_3$

$= \frac{2}{3}mr^2 + \left(\frac{2}{3}mr^2 + mr^2\right) + \left(\frac{2}{3}mr^2 + mr^2\right)$

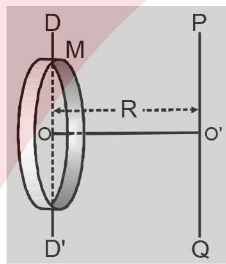
$\Rightarrow I_{xx'} = 2mr^2 + 2mr^2 = 4mr^2$

11. given $I_{solid\ sphere} = I_{hollow\ sphere}$

$\Rightarrow \quad \frac{2}{5}Mr_1^2 = \frac{2}{3}Mr_2^2 \Rightarrow \frac{r_1^2}{r_2^2} = \frac{5}{3}$

$\Rightarrow \quad \frac{r_1}{r_2} = \sqrt{5} : \sqrt{3}$

12.

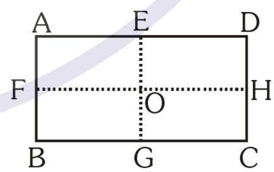


Applying parallel axis theorem

$I_{PQ} = I_C + Md^2$

$\Rightarrow \quad I_{PQ} = \frac{MR^2}{2} + M(R)^2 = \frac{3}{2}MR^2$

13. The moment of inertia is minimum about FH because mass distribution is at minimum distance from FH.

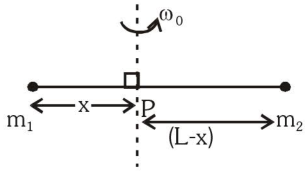


14. as, $\quad K = \frac{1}{2}I\omega^2 = \frac{1}{2}(I \times \omega) \times \omega$

$\Rightarrow \quad K = \frac{1}{2}L\omega \quad$ or $\quad L = \frac{2K}{\omega}$

so, $\quad \frac{L_2}{L_1} = \frac{K_2}{K_1} \times \frac{\omega_1}{\omega_2} = \frac{1}{2} \times \frac{1}{2}$

$\Rightarrow \quad \frac{L_2}{L_1} = \frac{1}{4} \quad$ or $\quad L_2 = \frac{L}{4}$

15. 

The position of point P on rod through which the axis should pass so that the work required to set the rod rotating with minimum angular velocity ω_0 is their centre of mass

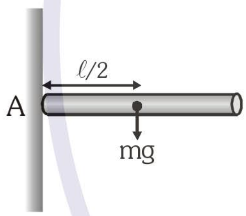
so $m_1x = m_2(L-x) \Rightarrow x = \dfrac{m_2L}{m_1+m_2}$

17. Torque $\vec{\tau} = \vec{r} \times \vec{F}$

$$\therefore \vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0.5 & 1 \\ 2 & 0 & -3 \end{vmatrix}$$

$$\vec{\tau} = \hat{i}[-1.5-0] - \hat{j}[6-2] + \hat{k}[0-1]$$

$$\vec{\tau} = \left(-1.5\hat{i} - 4\hat{j} - \hat{k}\right)\text{N}-\text{m}$$

18. 

Torque, $\tau_A = mg \times \dfrac{\ell}{2}$ and $I_A = \dfrac{m\ell^2}{3}$

as, $\tau_A = I_A \times \alpha$

or $\alpha = \dfrac{\tau_A}{I_A} = \dfrac{\frac{mg\ell}{2}}{\frac{m\ell^2}{3}} \Rightarrow \alpha = \dfrac{3g}{2\ell}$

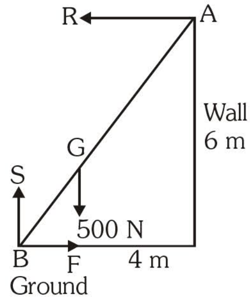
19. Here α = 2 revolutions/s^2 = 4π rad/s^2

$I = \dfrac{1}{2}MR^2 = \dfrac{1}{2}(50)(0.5)^2 = \dfrac{25}{4}$ Kg-m^2

As $\tau = I\alpha$ so $TR = I\alpha$

$\Rightarrow T = \dfrac{I\alpha}{R} = \dfrac{\left(\frac{25}{4}\right)(4\pi)}{(0.5)}N = 50\pi$ N = 157 N

20. The ladder touches the wall at A and the normal reaction is denoted by R. The whole system is in equilibrium. Take moments of forces about B.



$R \times 6 = 500 \times \dfrac{4}{3}$

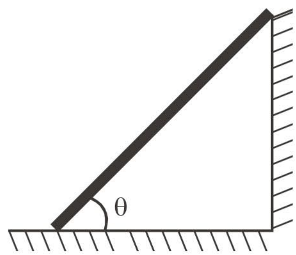
or $R = \dfrac{500 \times 4}{6 \times 3} = \dfrac{1000}{9}$ or R = 111 N.

21. Angular acceleration will be more if M.I will be less. $(\vec{\tau} = I\vec{\alpha})$

22. Torque = $\dfrac{\text{Change in angular momentum}}{\text{time}}$

$\tau = \dfrac{L_2 - L_1}{t} = \dfrac{4A_0 - A_0}{4}$

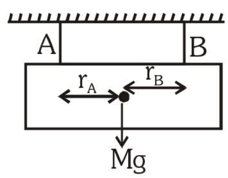
$\Rightarrow \tau = \dfrac{3A_0}{4}$

23. 

Increasing angle causes equilibrium.

24. (i) Given, the centre of mass is closer is closer to B than A

$\therefore r_A > r_B$



Torque $\vec{\tau} = \vec{r} \times \vec{F}$

For rotational equilibrium

$\tau_A = \tau_B$

$T_Ar_A = T_Br_B$

(ii) $T_Ar_A = T_Br_B \Rightarrow \dfrac{T_A}{T_B} = \dfrac{r_B}{r_A}$

$\therefore r_A > r_B \Rightarrow T_B > T_A$

25. For same tension in cable, more counter torque will be provided in (A) $\Rightarrow$ Pattern A is more sturdy

26. $f = 60 \text{ rpm} = \frac{60}{60} \text{rps} = 1 \text{ rps};$ $\quad I = 2 \text{ kg-m}^2$

$w = w_0 + at \quad \Rightarrow \quad 0 = 2p + a(60)$

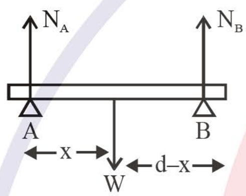
$\alpha = -\frac{2\pi}{60} = -\frac{\pi}{30} \quad \therefore$ (Retardation)

$\tau = I\alpha \quad \Rightarrow \quad \tau = 2 \times \frac{\pi}{30} = \frac{\pi}{15}$

27. By torque balancing about B

$N_A(d) = W(d-x)$

$\Rightarrow N_A = \frac{W(d-x)}{d}$



28. Particle at periphery will have both radial and tangential acceleration

$a_t = R\alpha = 0.5 \times 2 = 1 \text{ m/s}^2$

$\omega = \omega_0 + \alpha t = 0 + 2 \times 2 = 4 \text{ rad/sec}$

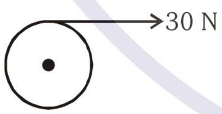
$a_c = \omega^2 R = (4)^2 \times 0.5 = 16 \times 0.5 = 8 \text{ m/s}^2$

$a_{total} = \sqrt{a_p^2 + a_c^2} = \sqrt{1^2 + 8^2} \approx 8 \text{m/s}^2$

***In this question we have assumed the point to be located at periphery of the disc.**

29. $\tau = I\alpha$

$RF = mR^2\alpha$



$\alpha = \frac{F}{mR} = \frac{30}{3 \times \frac{50}{100}} = 20 \text{ rad/s}^2$

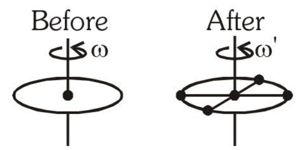
30. $\omega_1 = \frac{2\pi}{2} = \pi \text{ rad/s.}$

By conservation of Angular momentum

$I_1\omega_1 = I_2\omega_2$

$\Rightarrow \quad 100\,\pi = [100 + 50(2)^2]\omega \Rightarrow \omega = \frac{\pi}{3}$

31.



Applying conservation of angular momentum :-

$I_1\omega_1 = I_2\omega'$

$Mr^2 \times \omega = (Mr^2 + 4mr^2)\omega'$

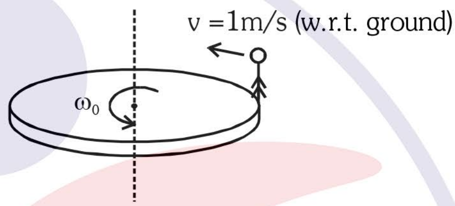
so, $\quad \omega' = \frac{Mr^2\omega}{r^2(M+4m)} = \frac{M\omega}{M+4m}$

32. As the angular momentum is conserved

$\therefore I_1\omega_1 = I_2\omega_2$ or $mr^2 3\omega = m(r/2)^2\omega_2$

$\omega_2 = 12\omega.$

33.



MI of platform = 200 kg-m^2,

MI of man = mR^2 = 200 kg-m^2

For system (platform + man) by using COAM

$I\omega_0 = mvR \Rightarrow \omega_0 = \frac{50 \times 1 \times 2}{200} = \frac{1}{2} \text{ rad/s}$

Angular velocity of man w.r.t. platform

$= \frac{v}{R} + \omega_0 = \frac{1}{2} + \frac{1}{2} = 1 \text{ rad/s}$

Time taken by the man to complete one revolution

$= \frac{2\pi \text{ rad}}{1 \text{ rad/s}} = 2\pi \text{ s}$

34. $|\vec{L}| = mvr_\perp = m(v\cos 45°)\left[\frac{v^2 \sin^2 45°}{2g}\right]$

$= \frac{mv^3}{4\sqrt{2}g}$

35. $J = I\omega$ where 'I' is constant. So graph will be straight line.

36. Centripetal force, $F_C = \frac{mv^2}{r} \quad$ ...(i)

and angular momentum, $L = mvr$

so, $\quad v = \frac{L}{mr}$

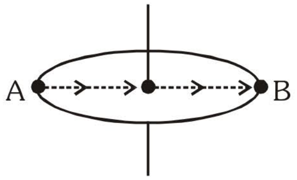
puting value of v in (i),

$F_C = \frac{m}{r} \times \left(\frac{L}{mr}\right)^2$

$\Rightarrow \quad F_C = \frac{m}{r} \times \frac{L^2}{m^2r^2} = \frac{L^2}{mr^3}$

37. On loosing atmosphere, moment of inertia of earth decreases so, ω increases and time period T decreases.

$$\left\{\because T = \frac{2\pi}{\omega}\right\}$$

38. 

Initially as ant move towards axis I decreases so ω increases.

After crossing axis I increases and ω decreases.

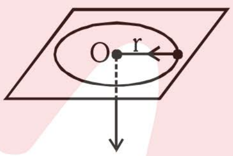
so, first ω increases then ω decreases.

39. Applying conservation of angular momentum :-

$I_1\omega = (I_1 + I_2)\omega'$ so, $\omega' = \frac{I_1\omega}{(I_1 + I_2)}$

40. As the string is pulled downwards, tension in the string, always pass through point O.

So, torque of tension about O will be zero and angular momentum of object about O will remain constant.



also, $K = \frac{L^2}{2I}$

$\Rightarrow K \propto \frac{1}{I} \propto \frac{1}{mr^2}$ $\{\therefore$ L is constant$\}$

so, if r becomes 2r then K becomes $\frac{K}{4}$

41. $\tau_{ext} = 0 \Rightarrow \frac{dL}{dt} = 0 \Rightarrow L = \text{constant}$

42. Rotational kinetic energy of a body,

$K_{rot} = \frac{1}{2}I\omega^2$

or, $I\omega^2 = 2K_{rot}$ or $\omega = \sqrt{\frac{2K_{rot}}{I}}$ ....(i)

Angular momentum of a body $L = I\omega$

or, $L = I\sqrt{\frac{2K_{rot}}{I}} = \sqrt{2IK_{rot}}$ [From (i)]

43. Frequency of rotation = n Hz

so, $\omega = 2\pi n$

and kinetic energy, $K = \frac{1}{2}I\omega^2$

so, $K = \frac{1}{2} \times \frac{mL^2}{3} \times (4\pi^2 \times n^2)$

$\Rightarrow K = \frac{2}{3}mL^2\pi^2n^2$

44. $K_A = K_B \Rightarrow \frac{L_A^2}{2I_A} = \frac{L_B^2}{2I_B}$

As $I_B > I_A$ So, $L_A^2 < L_B^2 \Rightarrow L_A < L_B$

45. $E_{sphere} = \frac{1}{2}I_s\omega^2 = \frac{1}{2} \times \frac{2}{5}MR^2 \times \omega^2$

$E_{disc} = \frac{1}{2}I(2\omega)^2 = \frac{1}{2} \times \frac{MR^2}{2} \times 4\omega^2$

$\frac{E_{sphere}}{E_{disc}} = \frac{1}{5}$

46. COAM : $I\omega_1 + I\omega_2 = 2I\omega \Rightarrow \omega = \frac{\omega_1 + \omega_2}{2}$

$(K.E.)_i = \frac{1}{2}I\omega_1^2 + \frac{1}{2}I\omega_2^2$

$(K.E.)_f = \frac{1}{2} \times 2I\omega^2 = I\left(\frac{\omega_1 + \omega_2}{2}\right)^2$

Loss in K.E. = $(K.E.)_i - (K.E)_f = \frac{I}{4}(\omega_1 - \omega_2)^2$

47. W = loss in KE = $\frac{1}{2}I\omega^2 \propto I$

$I_A = \frac{2}{5}MR^2 = 0.4MR^2$

$I_B = \frac{1}{2}MR^2 = 0.5MR^2$

$I_C = MR^2$

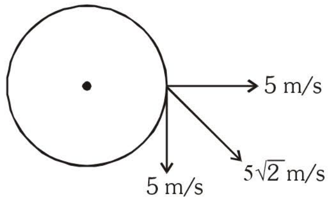
$\therefore W_C > W_B > W_A$

48. $\frac{K_{Rotation}}{K_{Total}} = \frac{\frac{K^2}{R^2}}{1 + \frac{K^2}{R^2}}$ For disc, $\frac{K^2}{R^2} = \frac{1}{2}$,

so, fraction $= \frac{\frac{1}{2}}{1 + \frac{1}{2}} = 1:3$

49. Different parts of the rolloing wheel move with the same linear and angular speed. The magnitude of the linear velocity of the points at the extermities of the horizontal diameter of the wheel is equal to

$5\sqrt{2}\ ms^{-1}$



50. As we know, $t = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}\left(1+\frac{K^2}{R^2}\right)}$

For sphere, $\frac{K^2}{R^2} = \frac{2}{5}$ & For disc, $\frac{K^2}{R^2} = \frac{1}{2}$

so, $\frac{t_{sphere}}{t_{disc}} = \frac{\frac{1}{\sin\theta}\sqrt{\frac{2h}{g}\left(1+\frac{2}{5}\right)}}{\frac{1}{\sin\theta}\sqrt{\frac{2h}{g}\left(1+\frac{1}{2}\right)}} = \sqrt{\frac{7/5}{3/2}} = \sqrt{14} : \sqrt{15}$

51. $v \propto \sqrt{a}$

$\frac{v}{v_R} = \sqrt{\frac{a}{a_R}}$ & $a_R = \frac{g\sin\theta}{1+\frac{I}{mR^2}} = \frac{g\sin\theta}{2}$

$a = g\sin\theta \Rightarrow v_R = \frac{v}{\sqrt{2}}$

52. Translational KE $= \frac{1}{2}mv^2 = E$

Total KE $= \frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right)$

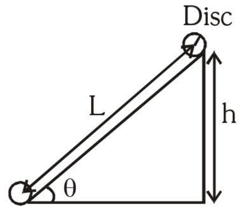
$= \frac{1}{2}mv^2(1+1) = 2E$

53. Given, $\frac{K_{Rotation}}{K_{Total}} \times 100 = 50\%$

$\Rightarrow \frac{\frac{K^2}{R^2}}{1+\frac{K^2}{R^2}} \times 100 = 50 \Rightarrow \frac{\frac{K^2}{R^2}}{1+\frac{K^2}{R^2}} = \frac{1}{2}$

so, $\frac{K^2}{R^2} = 1$, so it is a ring.

54.



Applying conservation of energy,

$$mgh = \frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right)$$

also, $h = L\sin\theta$ and $\frac{K^2}{R^2} = \frac{1}{2}$

so, $mgL\sin\theta = \frac{1}{2}mv^2\left(1+\frac{1}{2}\right)$

on solving, $v = \sqrt{\frac{4gL\sin\theta}{3}}$

55. t_1 = time for sliding $= \sqrt{\frac{2L}{a_s}}$

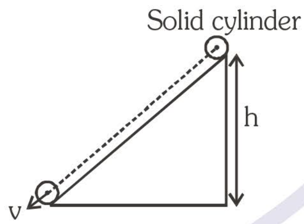
t_2 = time for rolling $= \sqrt{\frac{2L}{a_R}}$

$\Rightarrow t_1 : t_2 = \sqrt{a_R} : \sqrt{a_s}$

$a_R = \frac{mg\sin\theta}{m+\frac{I}{R^2}} = \frac{g\sin\theta}{2} = \frac{1}{2}a_s$

$\Rightarrow \quad t_1 : t_2 = 1 : \sqrt{2}$

56.



Applying conservation of energy :-

$mgh = K_{Total}$

so $mgh = \frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right)$

$mgh = \frac{1}{2}mv^2\left(1+\frac{1}{2}\right)$ on solving, $v = \sqrt{\frac{4}{3}gh}$

57. For rolling motion on an inclined plain acceleration is given by $a_R = \frac{g\sin\theta}{\left(1+\frac{K^2}{R^2}\right)}$

58. Applying conservation of energy.

$$\frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right)=mgh$$

$$\Rightarrow \frac{1}{2}mv^2\left(1+\frac{1}{2}\right)=mgh \quad \left\{\because \text{for disc } \frac{K^2}{R^2}=\frac{1}{2}\right\}$$

$$\Rightarrow h=\frac{3v^2}{4g}$$

59. At maximum compression of spring, total kinetic energy of rolling cylinder will be converted into potential energy of the spring.

so, $$\frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right)=\frac{1}{2}kx^2$$

$$\Rightarrow \frac{1}{2}12\times(4)^2\left[1+\frac{1}{2}\right]=\frac{1}{2}\times 200\times x^2$$

on solving, we get $x = 1.2$ m

60. acceleration = $\dfrac{g\sin\theta}{1+\dfrac{K^2}{R^2}}$

for disc ; $\dfrac{K^2}{R^2}=\dfrac{1}{2}=0.5$

for sphere ; $\dfrac{K^2}{R^2}=\dfrac{2}{5}=0.4$

$\Rightarrow a_{(sphere)} > a_{(disc)}$ $\quad \therefore$ sphere reaches first