SOLUTIONS
COLLISION & CENTER OF MASS

1. $m_1 = M$ (Solid sphere)

$r_{x_1} = 0 \qquad r_{y_1} = a$

Hollow sphere $\quad m_2 = M$

$r_{x_2} = 0 \qquad r_{y_2} = 0$

Disc. $\quad m_3 = M$

$r_{x_3} = a \qquad r_{y_3} = 0$

$$r_{x_{cm}} = \frac{m_1 r_{x_1} + m_2 r_{x_2} + m_3 r_{x_3}}{m_1 + m_2 + m_3} = \frac{0 + 0 + aM}{3M} = \frac{a}{3}$$

$$r_{y_{cm}} = \frac{m_1 r_{y_1} + m_2 r_{y_2} + m_3 r_{y_3}}{m_1 + m_2 + m_3} = \frac{Ma + 0 + 0}{3M} = \frac{a}{3}$$

So co-ordinate of mass of system will be

$$\left(\frac{a}{3}, \frac{a}{3}\right)$$

2. $$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3}{m_1 + m_2 + m_3}$$

$\vec{r}_{cm} = 0$

$\Rightarrow \quad m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 = 0$

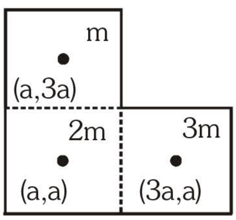
$$\vec{r}\,'_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + m_4\vec{r}_4}{m_1 + m_2 + m_3 + m_4}$$

$$(\hat{i} + 2\hat{j} + 3\hat{k}) = \frac{0 + 4\alpha(\hat{i} + 2\hat{j} + 3\hat{k})}{1 + 2 + 3 + 4} \Rightarrow \alpha = \frac{5}{2}$$

3. $$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2} = \frac{5(5\hat{i} + 4\hat{j}) + 10(4\hat{i} + 3\hat{j})}{5 + 10}$$

$$= \frac{65\hat{i} + 50\hat{j}}{15} = \frac{13\hat{i} + 10\hat{j}}{3}$$

4.



$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3}{m_1 + m_2 + m_3}$$

$$= \frac{m(a\hat{i} + 3a\hat{j}) + 2m(a\hat{i} + a\hat{j}) + 3m(3a\hat{i} + a\hat{j})}{m + 2m + 3m}$$

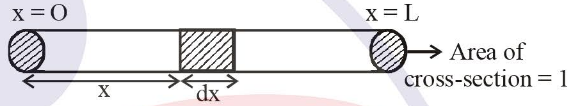
$$= \frac{12a\hat{i} + 8a\hat{j}}{6} = 2a\hat{i} + \frac{4a}{3}\hat{j}$$

5. $$h_{cm} = \frac{H}{4} = \frac{20}{4} = 5 \text{ cm}$$

6. $$x_{cm} = \frac{\int x\, dm}{\int dm} = \frac{\int_0^L x(kx^2 dx)}{\int_0^L kx^2 dx}$$

$$= \frac{\int_0^L x^3 dx}{\int_0^L x^2 dx} = \frac{L^4}{4} \times \frac{3}{L^3} = \frac{3L}{4}$$

7.



$$x_{cm} = \frac{\int_0^L x\,dm}{\int_0^L dm} = \frac{\int_0^L x(\rho_0 x^2/L^2)A\,dx}{\int_0^L (\rho_0 x^2/L^2)A\,dx} = \frac{3L^4}{4L^3} = \frac{3L}{4}$$

8. Centre of mass of solid hemisphere from its base

is $= \dfrac{3R}{8} = \dfrac{3 \times 24}{8} = 9$ cm

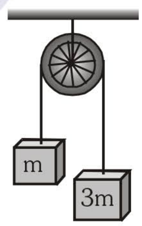
9. If external force is zero there will be no shift in COM.

10. $$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}$$

$$a_1 = \left(\frac{15 - 5}{15 + 5}\right)g = \frac{g}{2}, \qquad \vec{a}_1 = -\frac{g}{2}\hat{j},\ \vec{a}_2 = \frac{g}{2}\hat{j}$$

$$a_{cm} = \frac{\frac{1}{2}(-15 + 5)g}{20} = \frac{-10}{4} = \frac{-5}{2} m/s^2$$

11. $$a = \frac{(3m - m)}{3m + m} g = \frac{g}{2}$$



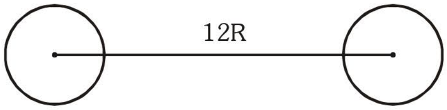
$$\vec{a}_{cm} = \frac{3m\vec{a}_1 + m\vec{a}_2}{3m + m}$$

Both mass have same magnitude of accleration but in opposite direction $\vec{a}_1 = -\vec{a}_2 = a$ Let

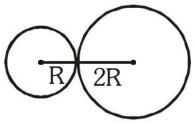
$$a_{cm} = \left(\frac{3m - m}{4m}\right) \times \frac{g}{2} = \frac{g}{4}$$

12. Centre of mass is towards heavier mass and bottom piece has more mass in comparison to the handle piece.

13.



Initial distance between their centers = 12 R



At time of collision the distance between their centers = 3R

So total distance travelled by both=12R–3R = 9R

Since the bodies move under mutual forces, center of mass will remain stationary so

$m_1x_1 = m_2x_2$

$mx = 5m(9R - x)$

$x = 45R - 5x$

$6x = 45R$

$x = \frac{45R}{6}$

$\boxed{x = 7.5R}$

14. $x_{CM} = \frac{m_1x_1 - m_2x_2}{m_1 - m_2}$

$m_1 = \frac{4}{3}\pi R^3\rho,\ m_2 = \frac{4}{3}\pi a^3\rho$

$x_1 = 0, \quad x_2 = b$

putting value $x_{CM} = \frac{-ba^3}{R^3 - a^3}$

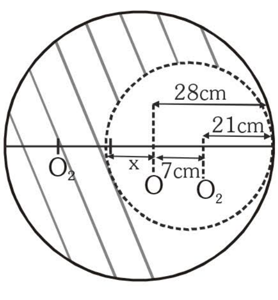
15. $Y_{CM} = \frac{4m \times 0 - m\ a/2}{4m - m} = -\frac{ma}{2(3m)} = -\frac{a}{6}$

16. $x_{CM} = \frac{m_1x_1 - m_2x_2}{m_1 - m_2}$, $m_1 = \pi R^2\rho$, $m_2 = \frac{\pi R^2}{4}\rho$

also $x_1 = 0,\ x_2 = -\frac{R}{2}$

putting value $x_{CM} = \frac{R}{6}$

17.



Let mass of circular plate be = M

Mass per unit area $= \frac{M}{\pi(28)^2}$

Mass of part removed $= \frac{M}{\pi(28)^2} \times \pi(21)^2 = \frac{9M}{16}$

Mass of remaining part $= \frac{7M}{16}$

Now $\frac{7M}{16} \times x = \frac{9M}{16} \times 7$

x = 9 cm

The centre of mass of the remaining portion from the centre of plate is 9 cm.

18. By COLM

$m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 = 0$

$m_1 = \frac{1}{4}\text{kg},\ m_2 = \frac{1}{4}\text{kg},\ m_3 = \frac{1}{2}\text{kg}$

also $v_1 = 30\hat{i}$ m/s, $v_2 = 30\hat{j}$ m/s

$\vec{v}_3 = -\left(\frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_3}\right)$ $v_3 = 15\sqrt{2}$ m/s

19. By COLM

$Mu = mu + Mv \Rightarrow v = \frac{(M-m)u}{M}$

20. $M_1 \times 8v = M_2v$

$M_2 = 8M_1$ $\left(\because M = \frac{4}{3}\pi r^3\rho, M \propto r^3\right)$

$\therefore \quad r_2 = 2r_1 \quad \Rightarrow \quad \frac{r_1}{r_2} = \frac{1}{2}$

21. Let plank shifted by x then

$\Delta x_{CM} = 0$, as there is no external force on the system.

$(M + m)\ x_{plank} - (m)\ (L - x_{plank}) = 0$

$x_{plank} = \frac{mL}{M+m}$

22. Let plank moved by x in right side.

but $\Delta x_{cm} = 0$

$150(x) + 50(10 + x) = 100\ (10 - x)$

$3x + (10 + x) = 20 - 2x$

$6x = 10 \Rightarrow x = \frac{5}{3}$m

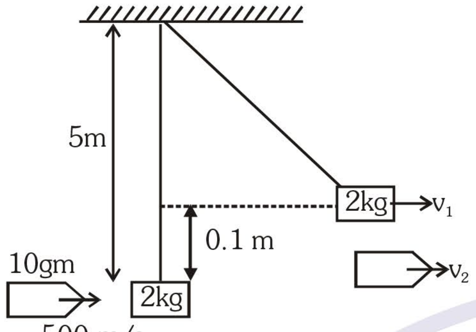
23. $mv = MV\ ;\ \frac{1}{2}MV^2 = \frac{1}{2}kd^2$

$\therefore V = \sqrt{\frac{kd^2}{M}}\ \therefore v = \frac{M}{m}\sqrt{\frac{kd^2}{M}} = \frac{d}{m}\sqrt{kM}$.

24. $mv = (M + m)v_f$

$$v_f = \left(\frac{m}{m+M}\right)v$$

25.



Applying momentum conservation

$$\frac{10}{1000}\times 500 + 0 = 2\times v_1 + \frac{10}{1000}\times v_2$$

$\Rightarrow$ $5 = 2v_1 + 0.01v_2$ ......(1)

Applying work energy theorem for block

$W = \Delta KE$

$$\Rightarrow 2\times 10\times 0.1 = \frac{1}{2}\times 2\times {v_1}^2$$

$$\Rightarrow v_1 = \sqrt{2} = 1.4 \text{ m/s}$$

Putting the value of v_1 in equation (1)

$5 = 2\times 1.4 + 0.01\, v_2 \Rightarrow v_2 = 220$ m/s

26. $$V = \frac{m_1v_1 - m_2v_2}{m_1+m_2} = \frac{10\times 10 - 20\times 15}{10+20}$$

$$= \frac{100-300}{30} = -\frac{200}{30} = -\frac{20}{3} = -6.6 \text{ m/s}$$

27. Velocities get inter changed.

$u_1 = 6$ m/s ; $u_2 = 0$

$\therefore$ $v_1 = 0$; $v_2 = 6$ m/s

28. Since both bodies are identical and collision is elastic. Therefore velocities will be interchanged after collision.

$v_A = -3$ m/s and $v_B = 5$ m/s

29. $e^2 = \dfrac{16}{25}$ $\Rightarrow$ $e = 4/5$

30. if 60% is lost

Total E = mgh

Remaining E = $\dfrac{40}{100}$mgh

So $mgH = \dfrac{40}{100}mgh$ $\Rightarrow$ H = 4 m

31. $u_1 = 6$ m/s, $u_2 = -6$ m/s, $e = 0.5$

$$v_1 = \frac{m_1 - em_2}{m_1+m_2}u_1 + \frac{(1+e)m_2}{m_1+m_2}u_2$$

$$= \left(\frac{1-0.5}{2m}\right)6m + \frac{(1+0.5)m}{2m}(-6)$$

$$= \frac{0.5\times 6}{2} + \frac{1.5}{2}(-6) = 1.5 - 4.5 = -3 \text{ m/s}$$

32. Momontum is conserved in all collision.

33. By COLM

$mu + 0 = mv_1 + mv_2$ ..... (1)

$e = \dfrac{v_2 - v_1}{u - 0}$ ..... (2)

$\Rightarrow v_2 - v_1 = eu$ ..... (3)

from (1)

$u = v_1 + v_2$ ..... (4)

putting (4) in (1)

$v_2 - v_1 = e(v_1 + v_2)$

$\Rightarrow v_2 - ev_2 = v_1 + ev_1$

$$\frac{v_1}{v_2} = \frac{1-e}{1+e}$$

34. Let a ball fall from a height (h) and let it touch the ground with a velocity v taking time (t) to reach the ground.

Let v_1, v_2, v_3....... be the velocities immediately after first, second, third......collisions with the ground.

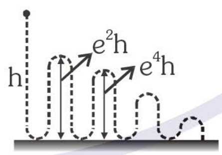
Height Attained by the Ball After the 'n'th Rebound

$v_1 = ev \Rightarrow \sqrt{2gh_1} = e\sqrt{2gh} \Rightarrow h_1 = e^2h,$

$v_2 = e^2v \Rightarrow \sqrt{2gh_2} = e^2\sqrt{2gh}\,,\ h_2 = e^4h.$

Similarly $\boxed{h_n = e^{2n}h}$

35.



Total distance $= h + 2e^2h + 2e^4h + ...$

$= h + 2e^2h(1 + e^2 + e^4 + ...)$

$$= h + 2e^2h\times\frac{1}{1-e^2}$$

$$= h\left[1 + \frac{2e^2}{1-e^2}\right] = \left(\frac{1+e^2}{1-e^2}\right)h$$

36. According to the law of conservation of momentum

$$m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2 \quad ......(i)$$

Here $\vec{u}_1, \vec{u}_2$ = Initial velocities of m_1 and m_2

$\vec{v}_1, \vec{v}_2$ = Final velocities of m_1 and m_2

Change in velocity of m_1 = $\vec{v}_1 - \vec{u}_1$

Change in velocity of m_2 = $\vec{v}_2 - \vec{u}_2$

$$m_1\vec{v}_1 - m_1\vec{u}_1 = -m_2\vec{v}_2 + m_2\vec{u}_2$$

or $m_1(\vec{v}_1 - \vec{u}_1) = m_2(-\vec{v}_2 + \vec{u}_2)$

or $\dfrac{\vec{v}_1 - \vec{u}_1}{-\vec{v}_2 + \vec{u}_2} = \dfrac{m_2}{m_1}$

$\therefore \dfrac{\text{changes in velocity of } m_1}{\text{changes in velocity of } m_2} = \dfrac{m_2}{m_1}$

37. It is not possible that after collision one ball moves along the original line of motion while the other ball moves along some angle (α) with original line. The momentum perpendicular to original line of motion cannot be conserved in this situation.

Initial momentum along perpendicular direction = zero

Final momentum along perpendicualr direction = $m_2v_2 \sin\alpha$. Hence momentum is not conserved. Hence the situation is physically impossible.

38. $e = \sqrt{\dfrac{h_f}{h_i}} = \sqrt{\dfrac{1.8}{5}} = 0.6 = \dfrac{3}{5}$

loss in velocity = $1 - \dfrac{3}{5} = \dfrac{2}{5}$

39. Max. compression happen when both have same speed

$\therefore \quad 2 \times 10 + 5 \times 3 = 7 \times v$

$\therefore \quad v = \dfrac{35}{7} = 5 \text{ m/s}$

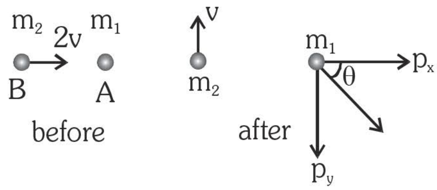
Energy conservation

$$\frac{1}{2}\times 2 \times 10^2 + \frac{1}{2}\times 5 \times 3^2 = \frac{1}{2}\times 7 \times 5^2 + \frac{1}{2}1120x^2$$

$\Rightarrow \quad x^2 = 0.0625$

$\therefore \quad x = 0.25$ m

40. **Case-I**



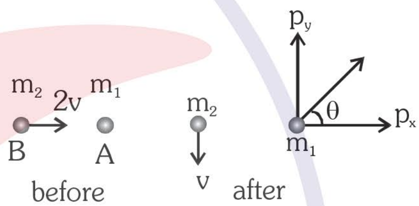
$p_x = m_2 2v$

$p_y = m_2 v$

$\therefore \quad \tan\theta = \dfrac{p_y}{p_x} = -\dfrac{1}{2}$

Case-II

similarly

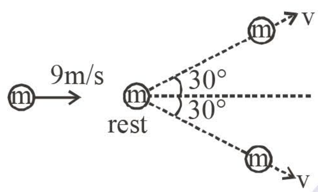


$p_x = m_2 v$

$p_y = -m_2 v$

$\therefore \quad \tan\theta = \dfrac{p_y}{p_x} = \dfrac{1}{2}$

41. By COLM



$m \times 9 = 2[mv \cos 30°]$

$v = 3\sqrt{3}\, m/s$