SOLUTIONS
CIRCULAR MOTION

1. $(\theta) = 2t^3 + 0.5$

$\omega = \frac{d\theta}{dt} = 6t^2$

at $t = 2$ s

$\omega = 6 \times 4 = 24$ rad/s

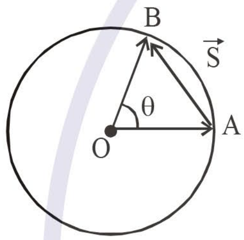
2. $\omega = \frac{\Delta\theta}{\Delta t} \Rightarrow \omega = \frac{2\pi}{60} = \frac{\pi}{30}$ rad/s

3. $\vec{v} = \vec{\omega} \times \vec{r}$

$$\vec{v} = \begin{vmatrix} i & j & k \\ 3 & -4 & 1 \\ 5 & -6 & 6 \end{vmatrix}$$

$\vec{v} = -18\hat{i} - 13\hat{j} + 2\hat{k}$ m/s

4.



$\theta = \omega t$

$\vec{S} = \overrightarrow{OB} - \overrightarrow{OA}$

$S = \sqrt{R^2 + R^2 + 2R^2 \cos(\pi - \omega t)}$

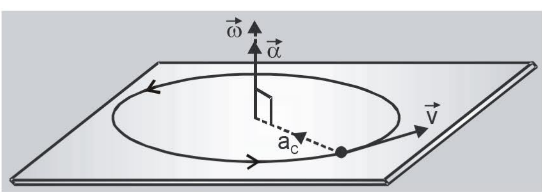
$S = 2R \sin\left(\frac{\omega t}{2}\right)$

5. $a_C = \frac{v^2}{r} = \frac{4}{r^2} \Rightarrow v^2 = \frac{4}{r}$

$\Rightarrow v = \frac{2}{\sqrt{r}}$

Momentum $p = mv = \frac{2m}{\sqrt{r}}$

6.



7. $(F_C)_{heavier} = (F_C)_{lighter}$

$\Rightarrow \frac{2mV^2}{r} = \frac{m(nV)^2}{2r} \Rightarrow n^2 = 4 \Rightarrow n = 2$

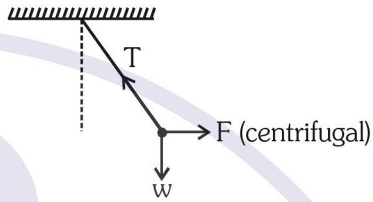
8. Speed is constant

$a_t = 0$

$a_{cp} = \frac{v^2}{r} = \text{constant}$

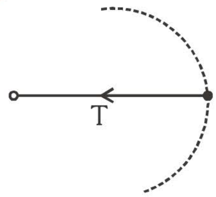
$|a_{net}| = \text{constant}$

9.



10. $a_{cp} = \frac{v^2}{R} \Rightarrow a_t = \frac{dv}{dt} = a \Rightarrow a_{net} = \sqrt{a_t^2 + a_{cp}^2}$

11. f_{net} = centripetal force



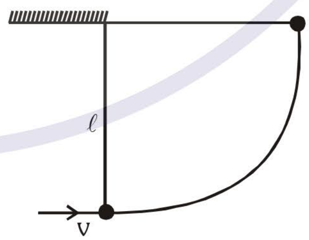
$T = \frac{mv^2}{r}$

12. $v = 72$ km/h $\Rightarrow \frac{v^2}{rg} = \tan\theta$

$v = \frac{72 \times 5}{18} = 20$ m/s $\Rightarrow \tan\theta = \frac{400}{20 \times 10}$

$\tan\theta = 2 \Rightarrow \theta = \tan^{-1}2$

13.



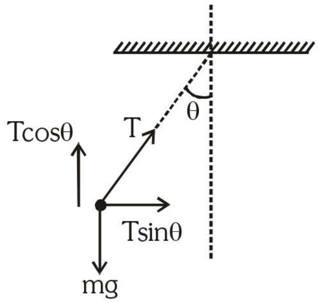
by COME

$\frac{1}{2}mv^2 = mg\ell + O \Rightarrow v = \sqrt{2g\ell}$

14. Using centre of Mass

$F_{cp} = m\omega^2 r = \frac{m\omega^2\ell}{2}$

15.



$T\sin\theta = mv^2/r$

$T\cos\theta = mg$

$$\tan\theta = \frac{v^2}{rg}$$

$$\tan\theta = \frac{100}{10\times10} = 1$$

$\theta = 45°$

16. by COME

$$v = \sqrt{2gl\sin\theta}$$

$$a_T = \sqrt{a_{cp}^2 + a_{t^2}}$$

$$a_{cp} = \frac{v^2}{r} = \frac{2g\ell\sin\theta}{\ell}$$

$= 2g\sin\theta$

and $a_t = g\cos\theta$

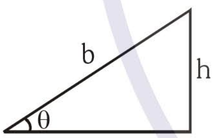
$$a_{net} = \sqrt{(g\cos\theta)^2 + (2g\sin\theta)^2}$$

$$= g\sqrt{1+3\sin^2\theta}$$

17. to complete circle

$v = \sqrt{5gr}$ by COME $h \geq \frac{5}{2}r$

18.



$$\tan\theta \simeq \sin\theta = \frac{h}{b}$$

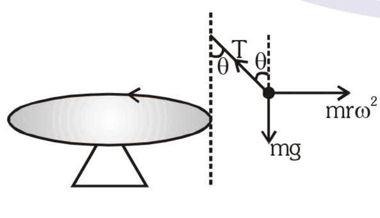
$$\therefore \quad \frac{h}{b} = \frac{v^2}{Rg}$$

$$\therefore \quad h = \frac{v^2 b}{Rg}$$

19. $T\cos\theta = mg$

$T\sin\theta = mr\omega^2$

$$\therefore \tan\theta = \frac{r\omega^2}{g}$$



20. For looping the loop minimum velocity at top point

$v = \sqrt{gL}$

time taken by particle

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times2L}{g}} = 2\sqrt{\frac{L}{g}}$$

$$\therefore \text{ horizontal range } x = vt = \sqrt{gL}\times2\sqrt{\frac{L}{g}} = 2L$$

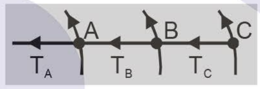
21. $\tan\theta = \frac{v^2}{rg}$

by ↑ speed by 10% speed becomes 1.1v

$$\therefore \quad \frac{v^2}{rg} = \frac{(1.1v)^2}{r'g}$$

$\therefore \quad r' = (1.1)^2 r = 24.2$ m

22.



angular velocity ω is same for all.

$T_C = m\omega^2 (3\ell)$

$T_B = T_C + m\omega^2 (2\ell) = m\omega^2(5\ell)$

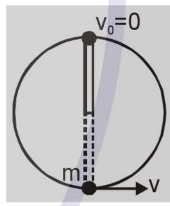
$T_A = T_B + m\omega^2 (\ell) = m\omega^2 (6\ell)$

$\therefore \quad T_C : T_B : T_A \quad :: 3 : 5 : 6$

23. At top most point speed of the body may be zero, because rod will support the body their

$$\therefore \quad \frac{1}{2}mv^2 = 0 + mg(2\ell)$$

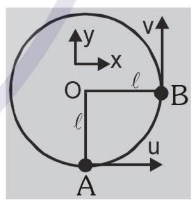
$$v = \sqrt{4g\ell}$$



24. $\vec{v} = v\hat{j}$ and $\vec{u} = u\hat{i}$

By COME between A and B

$$\frac{1}{2}mv^2 + mg\ell = \frac{1}{2}mu^2$$



$$\therefore \quad v = \sqrt{u^2 - 2g\ell}$$

$$\Delta\vec{v} = \vec{v} - \vec{u} = \sqrt{u^2 - 2g\ell}\,\hat{j} - u\hat{i}$$

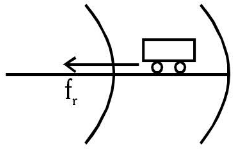
$$|\Delta\vec{v}| = \sqrt{(\sqrt{u^2-2g\ell})^2 + u^2} = \sqrt{2(u^2 - g\ell)}$$

25. In Balancing condition

$F_{centritugal} \leq$ Friction force

$mr\omega^2 \leq \mu mg$

$$r \leq \frac{\mu g}{\omega^2}$$

26. 

$f_r = F_{CP} = \dfrac{Mv^2}{R}$

But $f_r \le \mu Mg$

So $v^2 \le \mu_s Rg$

$v_{max} = \sqrt{\mu_s Rg}$

27. By using work-energy theorem, $W = \Delta KE$

$$\Rightarrow (ma_t)(4\pi R) = \frac{1}{2}mv^2 \Rightarrow a_t = \frac{\left(\frac{1}{2}mv^2\right)}{4\pi mR}$$

$$\Rightarrow a_t = \frac{8\times10^{-4}}{4\times3.14\times10\times10^{-3}\times6.4\times10^{-2}} = 0.1\ m/s^2$$

OR

$$\frac{1}{2}mv^2 = KE \Rightarrow \frac{1}{2}\left(\frac{10}{1000}\right)v^2 = 8\times10^{-4}$$

$$\Rightarrow v^2 = 16\times10^{-2} \Rightarrow v = 4\times10^{-1} = 0.4\ m/s$$

Now,

$v^2 = u^2 + 2a_t s \qquad (s = 4\pi R)$

$$\Rightarrow \frac{16}{100} = 0^2 + 2a_t\left(4\times\frac{22}{7}\times\frac{6.4}{100}\right)$$

$$\Rightarrow a_t = \frac{16}{100}\times\frac{7\times100}{8\times22\times6.4} = 0.1\ m/s^2$$

28. $\dfrac{v^2}{rg} = \tan(\phi-\theta)$

$$= \frac{\tan\phi - \tan\theta}{1-\tan\phi\tan\theta} = (\mu_s = \tan\theta)$$

$$\Rightarrow \quad v = \sqrt{rg\frac{\mu_s - \tan\theta}{1-\mu_s\tan\theta}} \qquad (\mu_s = \tan\phi)$$

OR

Check by dimensions.

29. Centripetal acceleration $= \dfrac{v^2}{R} = a\cos30°$

$$\Rightarrow \quad v = \sqrt{aR\cos30°} = \sqrt{7.5\times5\times\frac{\sqrt{3}}{2}} = 5.7 m/s$$