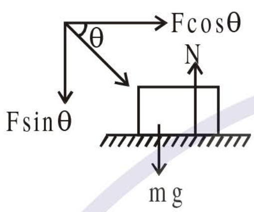
SOLUTIONS
WORK, ENERGY & POWER

1. $\vec{d} = \vec{r}_2 - \vec{r}_1$

$\Rightarrow \vec{d} = \hat{i} + \hat{j} - \hat{k}$ & $\vec{F} = (\hat{i} + 2\hat{j} + 3\hat{k})$

So $W = \vec{F}.\vec{d} = 0$

2. $N = Mg + F\sin\theta$



$\vec{F}_{Net} = \mu_k(mg + F\sin\theta)$

Work $= -\mu_k(mg + F\sin\theta)\, s$

3. Work done = Area under F – d curve

4. $dW = kx^2 dx \cos 60°$

$\therefore \quad WD = \frac{k}{2}\int_{x_1}^{x_2} x^2 dx = \frac{k}{6}\left(x_2^3 - x_1^3\right)$

5. $x = 3t - 4t^2 + t^3$

$v = \frac{dx}{dt} = 3 - 8t + 3t^2$

$WD = \frac{1}{2}mv_4^2 - \frac{1}{2}mv_0^2 = \Delta kE$

$WD = \frac{1}{2}\left(\frac{30}{1000}\right)\left[(19)^2 - (3)^2\right] = 5.285\ J$

6. $WD = \int_0^5 (3x^2 + 2x - 7)dx$

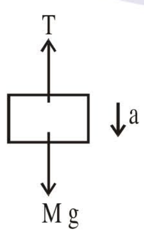
$= 125 + 25 - 35 = 115\ J$

7. Work will be zero as angle between force & displacement is 90°

8. For the block

$Mg - T = M(g/4)$

$\Rightarrow T = \frac{3}{4}Mg$



So, Work $= \frac{3}{4}Mg(h)\cos 180° = -\frac{3}{4}Mgh$

9. $s = \frac{t^2}{4} \qquad v = \frac{t}{2}$

$t = 0, u = 0$

$t = 2, v = 1$

$\therefore \quad WD = \Delta KE = \frac{1}{2} \times 6 \times 1 = 3J$

10. Work will be zero as force is perpendicular to displacement.

11. WD is independent of path for conservative forces.

12. $\vec{s} = \vec{r}_f - \vec{r}_i = 4\hat{i} - \hat{j} + 3\hat{k}$

$W = \vec{F}.\vec{s} = \left(2\hat{i} + 3\hat{j}\right).\left[4i - \hat{j} + 3\hat{k}\right] = 8 - 3 = 5J$

13. As we are pulling the bucket with constant velocity and leakage is at constant rate. We can take average mass

$W = m_{avg}gh = \left(\frac{15+9}{2}\right) \times 10 \times 15 = 1800J$

14. Since force is constant so work done is path independent. Hence $W_1 = W_2$

15. For conservative force in a closed loop W = 0

$WD_{PQ} + WD_{QR} + WD_{RP} = 0$

$8 + 2 + WD_{RP} = 0$

$\therefore \quad WD_{PR} = 10\ J$

16. $K \propto p^2 \Rightarrow \frac{\Delta K}{K} = \frac{2\Delta p}{p}$ So $\frac{\Delta p}{p} = \frac{1}{2}\frac{\Delta K}{K} = 2\%$

17. $E_k = \frac{p^2}{2m} \Rightarrow \sqrt{E_K} \times \frac{1}{p} = \text{constant}$

$\therefore$ graph is rectangular hyperbola

18. $K \propto p^2$

So if p become 'n' times & K becomes n^2 times

19. $p \propto \sqrt{K}$

$\Rightarrow \frac{p_1}{p_2} = \sqrt{\frac{K_1}{K_2}} \Rightarrow p_2 = \sqrt{2}p_1$ as $K_2 = 2K_1$

$\Rightarrow p_2 = 1.41\ p_1$

So momentum will increase by 41.4%

20. Work = Change in kinetic energy

$$= E_f - E_i = \frac{1}{2}m(v_f^2 - v_i^2)$$

$$W = \frac{1}{2}(2)(0^2 - 20^2) \Rightarrow W = -400\text{ J}$$

21. Till x = 2m, area under the curve F – d is zero so W.D. is zero therefore KE remains same at x = 2m, v = 4 m/s

Force = –4 N, mass of body = 2 kg

$$\therefore \text{acceleration (a)} = \frac{-4N}{2kg} = -2\ m/s^2.$$

This reduces velocity.

Now $v^2 = u^2 + 2as$.

$\therefore v^2 = (4)^2 + 2 \times (-2) = (16 - 4) = 12$

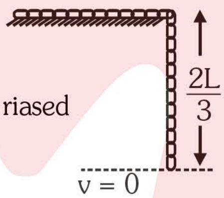
at x = 3 m and onwards.

$$\therefore \text{Kinetic energy} = \frac{1}{2} \times 2 \times 12 = 12\text{ J}$$

22. Work done = mgh

m = mass of hanging part

$$= \frac{M}{L} \cdot \left(\frac{2L}{3}\right) = \frac{2M}{3}$$



the height by which the COM riased

$$= \left(\frac{2L}{3}\right) \cdot \frac{1}{2} = \frac{L}{3}$$

$$WD = mgh = \frac{2M}{3} g \cdot \frac{L}{3} = \frac{2MgL}{9}$$

23. Given : $U = \dfrac{20xy}{z}$

For a conservative field

$\vec{F} = -\vec{\nabla}U$

Where, $\vec{\nabla} = \hat{i}\dfrac{\partial}{\partial x} + \hat{j}\dfrac{\partial}{\partial y} + \hat{k}\dfrac{\partial}{\partial k}$

$$\therefore \vec{F} = -\left[\hat{i}\frac{\partial U}{\partial x} + \hat{j}\frac{\partial U}{\partial y} + \hat{k}\frac{\partial U}{\partial z}\right]$$

$$= -\left[\hat{i}\frac{\partial}{\partial x}\left(\frac{20xy}{z}\right) + \hat{j}\frac{\partial}{\partial y}\left(\frac{20xy}{z}\right) + \hat{k}\frac{\partial}{\partial z}\left(\frac{20xy}{z}\right)\right]$$

$$= -\left[\hat{i}\left(\frac{20y}{z}\right) + \left(\frac{20x}{z}\right)\hat{j} + \left(-\frac{20xy}{z^2}\right)\hat{k}\right]$$

$$= -\left(\frac{20y}{z}\right)\hat{i} - \left(\frac{20x}{z}\right)\hat{j} + \frac{20xy}{z^2}\hat{k}$$

24. $U = \dfrac{A}{r^{12}} - \dfrac{B}{r^6}$

$\dfrac{dU}{dr} = 0$ at Equlibrium

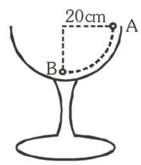
$$\therefore \frac{-12A}{r^{13}} - \frac{(-6)B}{r^7} = 0\ ;\ \frac{6}{r^7}\left[\frac{-2A}{r^6} + B\right] = 0$$

$$r = \left(\frac{2A}{B}\right)^{1/6}$$

$\therefore$ In Eq U is given by

$$U = \frac{A}{\left(\frac{2A}{B}\right)^2} - \frac{B}{\frac{2A}{B}} = \frac{B^2}{4A} - \frac{B^2}{2A} = -\frac{B^2}{4A}$$

25.



Work done by gravitational force

= force ×component of displacement along force

$$= mg(R) = \left(\frac{2}{1000}\right) \times (9.8) \times \left(\frac{20}{100}\right)$$

$$= 392 \times 10^{-5}\text{ J} = 3.92\text{ mJ}$$

26. $U = \dfrac{1}{2}mv^2 \quad \therefore \quad m = \dfrac{2U}{v^2}$

27. Energy dissipated = kinetic energy – potential enrgy

$$\Rightarrow E = \frac{1}{2}mv^2 - mgh$$

$$\Rightarrow E = \frac{1}{2} \times 0.5 \times (14)^2 - (0.5)\ (9.8)\ (8.0)$$

$$\Rightarrow E = (49 - 39.2)\text{ J} \Rightarrow E = 9.8\text{ J.}$$

28. $\dfrac{KE_1}{KE_2} = \dfrac{m_1gh}{m_2gh} = \dfrac{4}{8} = \dfrac{1}{2}$

29. Force $= \frac{-A}{R^2}$

$\therefore$ Potential energy $= -\int_{\infty}^{R} F dR = \frac{-A}{R}$

K.E. $= \frac{1}{2}\frac{A}{R}$ by $F_{centripetal} = \frac{mv^2}{R}$

T.E. $= \frac{-A}{2R}$

30. Work done by the gravity $(W_g) = mgh$

$= 10^{-3} \times 10 \times 10^3 = 10$ J

By work–energy theorem $= W_g + W_{res} = \Delta KE$

$$10 + W_{res} = \frac{1}{2} \times 10^{-3} \times (50)^2$$

$W_{res} = -8.75$ J

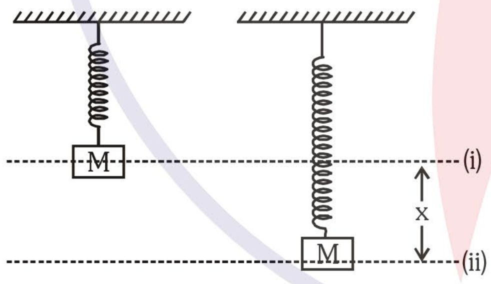
31. $\frac{1}{2} \times 0.5 \times (1.5)^2 = \frac{1}{2} \times 50 \times x^2$

$\frac{0.5 \times (1.5)^2}{50} = x^2$ $\quad x = 0.15$m

32. WD is +ve by gravity and –ve by spring on body.

$\therefore$ $\quad WD = mg(h + d) - \frac{1}{2}kd^2$

33.



By conservation of energy

TE at (i) = Mgx

TE at (ii) $= \frac{1}{2}kx^2$

$Mgx = \frac{1}{2}kx^2 = \boxed{x = \frac{2Mg}{k}}$

34. T = kx for spring

$\text{Energy} = \frac{1}{2}kx^2 = \frac{1}{2}k\frac{T^2}{k^2} = \frac{T^2}{2k}$

35. $P_1 = \frac{WD_1}{T_1}$

$P_2 = \frac{WD_2}{T_2}$ $\quad \therefore \frac{P_1}{P_2} = \frac{2}{3} \times \frac{7}{10} = \frac{7}{15}$

36. For the block moving in upward direction

$T - 10g = 10a \Rightarrow T = 10(g + g/2) = 150$ N

$s = \frac{1}{2}\left(\frac{g}{2}\right)(4)^2 = 40$ m

$P = \frac{W}{t} = \frac{T.s}{t} = \frac{150 \times 40}{4} = 1500W$

37. $P = mav \Rightarrow P = m\left(v\frac{dv}{dx}\right).v$

$\Rightarrow mv^2dv = Pdx \quad \Rightarrow \frac{mv^3}{3} = px \Rightarrow v \propto x^{1/3}$

38. Amount of water flowing per unit time $\frac{dm}{dt} = Av\rho$

v = velocity of flow, A is area of cross–section,
ρ = density of liquid
To get n times water in the same time,

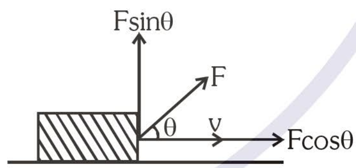
$\left(\frac{dm}{dt}\right)' = n\frac{dm}{dt} \Rightarrow A\,v'\rho = nAv\rho \quad \Rightarrow v' = nv$

$F = \frac{vdm}{dt} \Rightarrow F' = v'\frac{dm'}{dt} = n^2 v\frac{dm}{dt} = n^2F$

To gets n times water, force must be increased n^2 times.

39. Power = 100 × 10 × 100 = 100 kW

40.



Power $= \vec{F}.\vec{v} = Fv\cos\theta$

41. Here $m = \frac{dM}{dl}$

So, Rate of KE $= \frac{d(KE)}{dt}$

$\frac{d(KE)}{dt} = \frac{1}{2}\frac{dM}{dt}v^2$

Also,

$$\frac{dM}{dt} = \frac{dM}{dt}.\frac{dl}{dl} = \frac{dM}{dl}.\frac{dl}{dt} = mv$$

$$\frac{d(KE)}{dt} = \frac{1}{2}(mv)(v^2) = \frac{1}{2}mv^3$$

42. $P = Fv = mav \Rightarrow k = mv\dfrac{dv}{dt}$

By integrating the equation

$$\Rightarrow \int v\,dv = \int \frac{k}{m}dt$$

$$\Rightarrow \frac{v^2}{2} = \frac{k}{m}t \Rightarrow v = \sqrt{\frac{2k}{m}t}$$

$$a = \frac{dv}{dt} = \sqrt{\frac{2k}{m}}\left(\frac{1}{2}t^{-1/2}\right)$$

$$F = ma = m\left(\frac{1}{2}\right)\sqrt{\frac{2k}{mt}} \Rightarrow F = \sqrt{\frac{mk}{2t}}$$

43. $P = F.v = ma.v$

$$a = \frac{v_1}{t_1} \ \& \ v = 0 + \frac{v_1}{t_1}t$$

$$\text{So } P = m\left(\frac{v_1}{t_1}\right)\left(\frac{v_1}{t_1}t\right) \Rightarrow P = \frac{mv_1^2}{t_1^2}t$$

44. Force against which work done is

$$F = mg\sin\theta = 4 \times 9.8 \times \frac{1}{40} = 0.98 \text{ N}$$

speed $v = 40$ m/s

for 50% efficiency required power = 2 (F·v)

45. Mass of water $= 2238 \times 10^{-3} \times 10^{3}$
$= 2238$ kg

$\therefore$ Energy $= 2238 \times 10 \times 10 = mgh$

$$\therefore \quad \frac{2238 \times 30 \times 10}{T} = 1 \times 750 \text{ (T is time)}$$

$$\therefore \quad T = \frac{2238 \times 10 \times 30}{750} \text{ second} = 15 \text{ min.}$$

46. $a = \dfrac{v}{t_1}$ & $F = ma = \dfrac{mv}{t_1}$

$$s = \frac{1}{2}at^2 \Rightarrow s = \frac{1}{2}\frac{v}{t_1}t^2$$

$$W = F.s = \frac{1}{2}m\frac{v^2}{t_1^2}t^2$$

47. $F.v = P_0$

$$m\frac{dv}{dt} \times v = P_0$$

$$\frac{v^2}{2} = \frac{P_0 t}{m} \Rightarrow v \propto \sqrt{\frac{t}{m}} \propto t^{1/2}$$

$$a = \frac{dv}{dt} \propto t^{-1/2}$$

48. Pressure = 75 mm Hg

$$\text{Pumping rate } = \frac{dV}{dt} = \frac{10 \times 10^{-3}}{60} \ m^3/s$$

$$\text{Power of heart } = P.\frac{dV}{dt} = \rho gh \times \frac{dV}{dt}$$

$$= (13.6 \times 10^3 \text{ kg/m}^3)(10) \times (0.075) \times \frac{10 \times 10^{-3}}{60}$$

$$= \frac{13.6 \times 10 \times 0.075}{6} = 1.70 \text{ watt}$$

49. $\vec{F} = 2t\hat{i} + 3t^2\hat{j} \Rightarrow m\dfrac{d\vec{v}}{dt} = 2t\hat{i} + 3t^2\hat{j}$ {m = 1 kg}

$$\Rightarrow \int_0^{\vec{v}} d\vec{v} = \int_0^t (2t\hat{i} + 3t^2\hat{j})dt \Rightarrow \vec{v} = t^2\hat{i} + t^3\hat{j}$$

Power $= \vec{F}.\vec{v} = (2t^3 + 3t^5)$W