KINEMATICS

1. The average speed

$$=\; \frac{\displaystyle \frac{2x}{5} + \displaystyle \frac{3x}{5}}{\displaystyle \frac{2x/5}{v_1} + \displaystyle \frac{3x/5}{v_2}} = \frac{5v_1v_2}{3v_1 + 2v_2} \; .$$

2. Average speed is given by

$$v_{av} = \frac{2v_0(v_1 + v_2)}{2v_0 + v_1 + v_2}$$
$$= \frac{2 \times 3(4.5 + 7.5)}{(2 \times 3) + 4.5 + 7.5} = 4 \text{ m/s}$$

 $V = \alpha \sqrt{x} \Rightarrow \frac{dx}{dt} = \alpha \sqrt{x} \Rightarrow \frac{dx}{\sqrt{x}} = \alpha dt$

Integrating, $\int_{0}^{x} x^{-1/2} dx = \int_{0}^{x} \alpha dt$

$$2\sqrt{x} = \alpha t \Rightarrow \sqrt{x} = \alpha t/2$$

Put this value of \sqrt{x} in the original given eqn.

$$V = \alpha \sqrt{x} = \alpha(\alpha.t/2) = \alpha^2 t/2$$

∴ V ∞ t

4.
$$\alpha = \frac{V_{\text{max}}}{t_1}$$
; $\beta = \frac{V_{\text{max}}}{t_2}$ $\therefore \frac{\beta}{\alpha} = \frac{\frac{V_{\text{max}}}{t_2}}{\frac{V_{\text{max}}}{t_1}} = \frac{t_1}{t_2}$ (i)

 $V_{\text{max}}^2 = 2 \alpha x_1 \qquad V_{\text{max}}^2 = 2\beta x_2$ $x_1 = \frac{V_{max}^2}{2\alpha}$ $x_2 = \frac{V_{max}^2}{2\beta}$ $\frac{x_1}{x_2} = \frac{\beta}{\alpha}$...(ii)

Now
$$\left[\frac{x_1}{x_2} = \frac{t_1}{t_2}\right]$$
 So $\left[\frac{x_1}{x_2} = \frac{t_1}{t_2} = \frac{\beta}{\alpha}\right]$

For uniformly accelerated motion, the x must be 5. quadratic in t. So,

$$t^2 = \frac{x - a}{b} \quad \text{or} \quad x = a + bt^2$$

 $s_1 = \frac{1}{2}a(10)^2$ and $s = \frac{1}{2}a(20)^2 = 4s_1$

 $\begin{array}{ll} \therefore & s_2 = s - s_1 = 3s_1 \\ \text{Velocity changes from } v_1 \text{ to } v_2 \text{ in time } t_1 + t_2, \text{ so} \end{array}$ 7.

$$a = \frac{v_2 - v_1}{t_1 + t_2} \, .$$

Similarly $a = \frac{v_3 - v_2}{t_0 + t_0}$

Thus
$$\left(\frac{v_2 - v_1}{t_1 + t_2}\right) = \left(\frac{v_3 - v_2}{t_2 + t_3}\right)$$

 $0 = u^2 - 2a s \Rightarrow s = \frac{u^2}{2a}$

 $\therefore \frac{s_1}{s_2} = \frac{u_1^2}{u_2^2} = \frac{u^2}{(\Delta_{11})^2} = \frac{1}{16}$

9.

$$1 = \frac{1}{2}gt_1^2, \qquad \qquad \therefore \qquad \qquad t_1 = \sqrt{\frac{2}{g}}$$

For 2m of fall,

$$2 = \frac{1}{2}gt^2, \qquad \therefore \qquad t = \sqrt{\frac{4}{g}},$$

$$\therefore \qquad t_2 = t - t_1 = \sqrt{\frac{4}{g}} - \sqrt{\frac{2}{g}} = (\sqrt{2} - 1)\sqrt{\frac{2}{g}}$$

For 3 m of fall,

$$3 = \frac{1}{2}gt^2$$
, \therefore $t = \sqrt{3}\sqrt{\frac{2}{g}}$,

$$\therefore \qquad \mathbf{t}_3 = \mathbf{t} - \mathbf{t}_2 = (\sqrt{3} - \sqrt{2}) \sqrt{\frac{2}{\sigma}} \ .$$

10. If h is the height of the building, then

$$v_A^2 = v^2 + 2gh$$

and $v_B^2 = (-v)^2 + 2gh$ Clearly $v_A = v_B$.

11.
$$\left(\frac{v}{2}\right)^2 = v^2 - 2g \times 3$$

$$\therefore$$
 $v = \sqrt{8g}$

 \therefore $v = \sqrt{8g} \; .$ If h is the further height, then

$$0 = \left(\frac{v}{2}\right)^2 - 2gh$$

$$h = \frac{v^2}{8g} = \frac{8g}{8g} = 1 \text{ m}.$$

12.
$$h = ut_1 - \frac{1}{2}gt_1^2$$

Also
$$h = ut_2 - \frac{1}{2}gt_2^2$$

After simplify above equations, we get

$$h = \frac{1}{2}gt_1t_2.$$

13. $0 + \frac{a_1}{2}(2 \times 5 - 1) = 0 + \frac{a_2}{2}[2 \times 3 - 1]$

$$\therefore \frac{a_1}{a_2} = \frac{5}{9}.$$

14.
$$h_1 = \frac{1}{2} \times 10 \times 5^2 = 125 \text{ m}$$

$$h_2 = \frac{1}{2} \times 10 \times 3^2 = 45 \text{ m}$$

$$h = h_1 - h_2 = 80 \text{ m}.$$

15.
$$\frac{dv}{dt} = at \qquad \text{or} \qquad \int_0^v dv = \int_0^t (at) dt$$

$$\therefore \qquad v = u + \frac{at^2}{2}$$

16.
$$\frac{v_A}{v_B} = \frac{\tan 30^\circ}{\tan 60^\circ} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3}$$

17. Effective acceleration in ascending lift =
$$(g + a)$$

$$t = \sqrt{\frac{2h}{g+a}} = \sqrt{\frac{2 \times 9.5}{32 + 6}}$$

$$t = \sqrt{\frac{2 \times 9.5}{38}} = \frac{1}{\sqrt{2}} \sec x$$

18. The maximum acceleration will occur in the duration 30 s to 40 s. So

$$a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{60 - 20}{40 - 30} = 4 \text{ m/s}^2.$$

- 19. It is the $\vec{s} - t$ graph of a body projected upward. It has uniform acceleration downward.
- The distance, $s = \frac{vT}{2}$ 20.

Also,
$$m = \frac{v}{T}$$
, $\therefore T = \frac{v}{m}$.

$$T = \frac{V}{V}$$

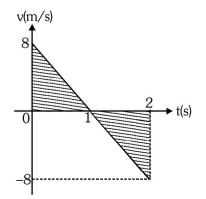
$$s = \frac{v^2}{2m}$$
.

21. Area of (v-t) curve = displacements

$$x = 2 + 8t - 4t^2$$

| V | 8 | 0 | -8 |
|---|---|---|----|
| t | 0 | 1 | 2 |

$$v = \frac{dx}{dt} = 8 - 8t$$



Distance travelled from t = 0 to t = 2 sec

$$=\frac{1}{2} \times 1 \times 8 + \frac{1}{2} \times 1 \times 8 = 4 + 4 = 8 \text{ m}$$

22. The height h is covered in time interval t = 1 s to t = 2s or t = 5s to t = 6s.

At maximum height t = 3.5 sec.

h between t₅ & t₆ is

$$h = \frac{1}{2}a(6-3.5)^2 - \frac{1}{2}a(5-3.5)^2$$

=
$$\frac{1}{2}$$
×7.5(6.25) $-\frac{1}{2}$ (7.5)(1.5)² = 15 m

23.
$$t_{asc} = \frac{u}{g}$$

time for upward journey from this particular point



$$= (t_{asc} - t_1) = \left[\frac{u}{g} - t_1 \right]$$

$$\therefore \quad \text{total time} = 2 \left[\frac{\mathbf{u}}{\mathbf{g}} - \mathbf{t}_1 \right]$$

24. Let downwards direction is +ve

$$h = -ut_1 + \frac{1}{2}gt_1^2$$
 (1

$$h = ut_2 + \frac{1}{2}gt_2^2$$
 (2)

Multiplying eqn. (1) by t_2 and eqn. (2) by t_1

$$ht_2 = -ut_1t_2 + \frac{1}{2}gt_1^2t_2$$
(3)

$$ht_1 = ut_2t_1 + \frac{1}{2}gt_2^2t_1$$
 (4)

adding eqn. (3) & (4)

$$ht_1 + ht_2 = \frac{1}{2}gt_1^2t_2 + \frac{1}{2}gt_2^2t_1$$

$$\Rightarrow$$
 h(t₁+t₂) = $\frac{1}{2}$ gt₁t₂(t₁+t₂)

$$\Rightarrow h = \frac{1}{2}gt_1t_2....(5)$$

for free fall when u = 0, then

$$h = \frac{1}{2}gt^2$$
..... (6)

From eqn. (5) & (6)

$$\frac{1}{2}gt^2 = \frac{1}{2}gt_1t_2 \Rightarrow t = \sqrt{t_1t_2}$$

25. $v = \beta x^{-4n}$

so
$$\frac{dv}{dx} = -4n\beta x^{-4n-1}$$

Now
$$a = v \frac{dv}{dx} = (\beta x^{-4n}) (-4n\beta x^{-4n-1})$$

$$\Rightarrow$$
 a = $-4n\beta^2 x^{-8n-1}$

26.
$$x = \frac{1}{t+10} \Rightarrow v = \frac{dx}{dt} = -\frac{1}{(t+10)^2}$$

Acceleration,
$$a = \frac{dv}{dt} = \frac{2}{(t+10)^3} \Rightarrow a \propto (velocity)^{3/2}$$

28.
$$x = 50 + 12 t - t^3$$

$$v = \frac{dx}{dt} = 0 + 12 - 3t^2$$

for
$$v = 0$$
 \Rightarrow $12 - 3 t^2 = 0$

$$\Rightarrow t^2 = 4 \Rightarrow t = 2 \sec^2 t$$

$$\Rightarrow$$
 $x_{t=0} = 50 \text{ m}$

$$X_{t=2S} = 50 + 12 \times 2 - (2)^3 = 66$$

distance travelled =
$$x_{t=2} - x_{t=0} = 66 - 50 = 16 \text{ m}$$

29. Instantaneous velocity =
$$\frac{ds}{dt}$$
 = slope of s-t curve.

30.
$$s_n = u + \frac{a}{2}(2n-1)$$

$$\frac{120}{100} = 0 + \frac{a}{2}(12 - 1)$$

$$a = \frac{6 \times 2}{5 \times 11}$$

$$a = 0.218 \text{ m/s}^2$$

32. Average velocity =
$$\frac{x_f - x_i}{t} = \frac{0 - 0}{t} = 0$$

$$=\frac{1}{2} \times 120 \times 1000 = 60,000$$
m $= 60$ km

34. Acceleration is always downward i.e. positive so slope of v-t curve will always be positive

Hence correct option (3)

36. Area under a-t graph gives the change in velocity during given time interval.

$$\therefore v_{\text{max}} = \frac{1}{2} \times 5 \times 6 = 15 \text{ m/s}$$

Since initial velocity = 0

.. Maximum speed of the particle = 15 m/s

37. If v_1 and v_2 are the velocities, then

$$(v_1 - v_2) \times 20 = (100 + 100)$$
(i)

and
$$(v_1 + v_2) \times 10 = (100 + 100)$$
(ii)

After solving above equations, we get

$$v_1 = 15 \text{ m/s} \text{ and } v_2 = 5 \text{ m/s}.$$

38.
$$v_A = u - gt$$
 and $v_B = gt$. $v_A - v_B = (u - gt) - gt = u - 2gt$.

39. If u is the velocity of projection, then $0 = u^2 - 2q(4h)$

$$\therefore$$
 $u = \sqrt{8gh}$

Now
$$y = \frac{1}{2}gt^2$$
 ...(i

and
$$h - y = ut - \frac{1}{2}gt^2$$
 ...(ii)

From above equations, we have

$$t = \frac{h}{u} = \frac{h}{\sqrt{8gh}} = \sqrt{\frac{h}{8g}}$$

40. For maximum velocity, a = 0 and so, 0 = b - cx or x = b/c.

and so,
$$0 = b - cx$$
 or $x = b/c$.
Now, $v \frac{dv}{dx} = b - cx$ $\Rightarrow \int_{a}^{b} v dv = \int_{a}^{x} (b - cx) dx$

$$\frac{v^2}{2} = bx - \frac{cx^2}{2} = b \times \frac{b}{c} - \frac{c(b/c)^2}{2}$$
 or $v = \frac{b}{\sqrt{c}}$

41.
$$s = \frac{1}{2}ft_1^2;$$
 \therefore $t_1 = \sqrt{\frac{2s}{f}}$

$$v_{\rm max} = ft_1 = f\sqrt{\frac{2s}{f}} = \sqrt{2 fs} \ .$$

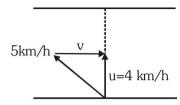
Thus
$$5s = \frac{1}{2} \left[\left[t + 3t_1 \right) + t \right] \times v_{max}$$

or
$$5s = \frac{1}{2}(2t + 3t_1) \times v_{max}$$

or
$$5s = \frac{1}{2} \left(2t + 3\sqrt{\frac{2s}{f}} \right) \times \sqrt{2 fs}$$

$$\therefore \qquad s = \frac{1}{2} ft^2$$

$$u = \frac{1}{1/4} = 4 \text{ km/h}$$



$$v = \sqrt{5^2 - 4^2} = 3 \text{ km/h}$$

43.
$$\vec{v}_{m} = 2\hat{i} + 3\hat{j} \text{ m/s}$$

$$\vec{v}_{m} = -4\hat{j} \text{ m/s}$$

$$\vec{\mathbf{v}}_{rm} = \vec{\mathbf{v}}_{r} - \vec{\mathbf{v}}_{m}$$

$$-4\hat{j} = \vec{v}_r - (2\hat{i} + 3\hat{j})$$

$$\vec{v}_r = 2\mathbf{i} - \hat{\mathbf{j}}$$

Now for downward motion

$$\vec{v}_{m} = \vec{v}_{r} - \vec{v}_{m} = 2\hat{i} - \hat{j} + (2\hat{i} + 3\hat{j}) = 2\hat{i} - \hat{j} + 2\hat{i} + 3\hat{j}$$

$$\vec{v}_{\rm m} = 4\hat{i} + 2\hat{j} \Rightarrow |\vec{v}_{\rm m}| = \sqrt{20} = 2\sqrt{5} \text{ m/s}$$

44.
$$v_c = 20 \text{ km/hr}$$

EAST

$$v_{TC} = 20\sqrt{3} \text{ km/hr}$$

NORTH

$$\vec{\mathbf{v}}_{\mathrm{TC}} = \vec{\mathbf{v}}_{\mathrm{T}} - \vec{\mathbf{v}}_{\mathrm{C}}$$

$$\vec{\mathbf{v}}_{\mathrm{T}} = \vec{\mathbf{v}}_{\mathrm{TC}} + \vec{\mathbf{v}}_{\mathrm{C}}$$

$$\vec{v}_{T} = 20\sqrt{3}\hat{i} + 20\hat{i}$$

$$|\vec{v}_T| = \sqrt{1200 + 400} = 40 \text{ m/s}$$

$$\tan\theta = \frac{20\sqrt{3}}{20}$$

$$\tan \theta = \sqrt{3}$$

$$\theta = 60^{\circ}$$

45.
$$t = \frac{d}{v + v \cos 90} = \frac{d}{v}$$

46.
$$t = \frac{d}{v - v \cos 120}$$

$$\frac{d}{v - \frac{v}{2}} = \frac{d}{\frac{v}{2}}$$

$$=\frac{2d}{v}$$

47. $\overrightarrow{v}_{rm} = \overrightarrow{v}_r - \overrightarrow{v}_m$

$$\Rightarrow \quad \stackrel{\rightarrow}{\mathbf{v}_{rm}} = -\mathbf{u}\hat{\mathbf{j}} - \mathbf{v}\hat{\mathbf{i}}$$



$$\Rightarrow$$
 $\tan\theta = \frac{v}{u}$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{\mathbf{v}}{\mathbf{u}}\right)$$



48. The velocity upstream is (4 - 2) km/hr and downstream is (4 + 2) km/hr.

$$\therefore \text{ Total time taken} = \frac{2}{2} + \frac{2}{6} = \frac{4}{3} \text{ h}$$

49. Let v = speed of buses travelling between A and B

The relative speed of bus going from A to B w.r.t. cyclist = (v - 20) km hr⁻¹

and relative speed of bus goint from B to A w.r.t. cyclist = (v + 20) km hr⁻¹

in time T, distance covered by bus = vT

when bus and cyclist are in same direction then

$$\frac{vT}{v-20} = 18 \text{ min}$$
(1)

when bus and cyclist are in opposite direction then

$$\frac{vT}{v + 20} = 6 \text{ min}$$
(2)

dividing eq (1) by eq (2)
$$\frac{v+20}{v-20} = \frac{18}{6} = 3$$

$$\Rightarrow$$
 v = 40 km/h

putting value v in eq (1) $\frac{40 \times T}{40 - 20} = 18 \text{ min}$

$$\Rightarrow \frac{40}{20} \text{ T} = 18 \text{ min } \Rightarrow \text{ T} = 9 \text{ min}$$

At t = 0 let the man's position be the origin.

$$\therefore x_n = 0$$

The bus door is then at $x_{R} = 6.0 \text{ m}$.

The equation of motion for the man is

Distance covered by person in time t to access the door is $x_n = 4t$

Distance covered by bus in time t is

$$x_B = \frac{1}{2} \times 1.2t^2 = 0.6t^2$$

and also

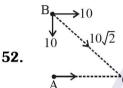
$$x_p - x_R = 6$$

$$x_p - x_B = 6$$

 $4t - 0.6t^2 = 6$

$$t = 2.3 \text{ s } & 4.4 \text{ sec.}$$

The man shall access the door at 2.3 sec.



$$|\vec{v}_{BA}| = \sqrt{10^2 + 10^2} = 10\sqrt{2} \, \text{kmph}$$

distance OB = $100 \cos 45^\circ = 50\sqrt{2} \text{ km}$

Time taken to reach the shortest distance between

A & B =
$$\frac{50\sqrt{2}}{|\vec{v}_{BA}|} = \frac{50\sqrt{2}}{10\sqrt{2}}$$

t = 5 hrs.

53. For two particles to collide, the direction of the relative velocity of one with respect to other should be directed towards the relative position of the other particle

i.e.
$$\frac{\vec{r}_1 - \vec{r}_2}{\left|\vec{r}_1 - \vec{r}_2\right|} \rightarrow$$
 direction of relative position of 1 w.r.t. 2.

&
$$\frac{\vec{v}_2 - \vec{v}_1}{|\vec{v}_2 - \vec{v}_1|} \rightarrow$$
 direction of velocity of 2 w.r.t. 1

so for collision of A & B

$$\frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|} = \frac{\vec{v}_2 - \vec{v}_1}{|\vec{v}_2 - \vec{v}_1|}$$

54.
$$x_{p}(t) = at + bt^{2}$$
 $x_{Q}(t) = ft + t^{2}$ $v_{Q} = f + 2t$ as $v_{P} = v_{Q}$

$$a + 2bt = f + 2t \Rightarrow t = \frac{f - a}{2(1 - b)}$$

55. $V_1 \rightarrow \text{velocity of Priya}$ $V_2 \rightarrow \text{velocity of escalator}$ $\ell \rightarrow distance$

$$t = \frac{\ell}{V_1 + V_2} \ = \frac{\ell}{\frac{\ell}{t_1} + \frac{\ell}{t_2}} \ = \frac{t_1 t_2}{t_1 + t_2}$$

57. At the highest point of trajectory, the acceleration is equal to g.

58.
$$R_1 = \frac{u^2 \sin(2 \times 15^\circ)}{g} = \frac{u^2}{2g}$$

and
$$R_2 = \frac{u^2 \sin(2 \times 45^\circ)}{g} = \frac{u^2}{g}$$

$$R_2 = 2R_1 = 2 \times 1.5 = 3 \text{ km}$$

59.
$$1960 = \frac{1}{2} \times 9.8 \times t^2$$

$$\therefore$$
 t = 20 s

Now AB = ut =
$$\left(600 \times \frac{5}{18}\right) \times 20 = 3333$$
 m.

60. The vertical components of the velocities must be equal so.

$$v_1 \sin 30^\circ = v_2 \text{ or } \frac{v_2}{v_1} = \frac{1}{2}$$

- 61. The boy velocity = horizontal velocity of the ball = $u \cos \theta$.
- Let u is the velocity of projection, then 62.

$$R_{max} = \frac{u^2}{g} = d$$
 or $u = \sqrt{gd}$

Let h is the height upto which ball rise, then

$$0 = u^2 - 2gh \text{ or } h = \frac{u^2}{2g} = \frac{gd}{2g} = \frac{d}{2}$$

63. $u_{ij} = u \sin \theta$

$$y = u_y t - \frac{1}{2}gt^2$$
 or $5 = (25 \sin \theta) \times 2 - \frac{1}{2} \times 10 \times 2^2$

$$\therefore \quad \sin \theta = \frac{1}{2} \quad \text{or} \quad \theta = 30^{\circ}.$$

64.
$$R = \frac{2u_x u_y}{q} = \frac{2 \times 10 \times 20}{10} = 40 \text{ m}$$

65. Time of motion.

$$t = \frac{x}{u_x} = \frac{80}{30} = \frac{8}{3}s$$

$$t = \frac{2u_y}{q}$$

$$u_y = \frac{8}{3} \times \frac{10}{2}$$

$$u_y = \frac{40}{3} \text{ m/s}$$

66.
$$H = \frac{u^2 \sin 45^\circ}{2g} = \frac{u^2}{4g} \implies R = \frac{u^2}{g} = 4H$$

67.
$$\Delta P = 2 \text{ mu sin } \theta$$

68.
$$H_1 = \frac{u^2 \sin^2 \theta}{2g}$$

and
$$H_2 = \frac{u^2 \sin^2(\pi/2 - \theta)}{2g} = \frac{u^2 \cos^2 \theta}{2g}$$

$$H_1H_2 = \frac{u^2 \sin^2 \theta}{2g} \times \frac{u^2 \cos^2 \theta}{2g}$$

$$=\frac{\left(u^22\sin\theta\cos\theta\right)^2}{16g^2}=\frac{R^2}{16}$$

$$\therefore \qquad \qquad R = 4\sqrt{H_1 H_2} \ .$$

69.
$$\vec{v} = a\hat{i} + b\hat{j} \implies u \cos \theta = a \text{ and } u \sin \theta = b$$

$$R = 2H_{max}$$

$$\tan \theta = \frac{b}{a}$$
 also $\frac{u^2 \sin 2\theta}{g} = \frac{2u^2 \sin^2 \theta}{2g}$

$$\Rightarrow$$
 $\tan\theta = 2$

$$\therefore \frac{b}{a} = 2 \Rightarrow b = 2a$$

70. For same range
$$\theta_1 + \theta_2 = 90^\circ$$

$$\theta_1 = \frac{\pi}{3} \text{ rad} = 60^{\circ}$$

$$\therefore \quad \theta_2 = 90^{\circ} - \theta_1 = 30^{\circ}$$

$$y_1 = \frac{u^2 \sin^2 60^\circ}{2q} = \frac{u^2 \times 3}{2q \times 4} \implies \frac{u^2}{8q} = \frac{y_1}{3}$$

$$y_2 = \frac{u^2 \sin^2 30^\circ}{2g} = \frac{u^2 \times 1}{2g \times 4} = \frac{y_1}{3}$$

71.
$$R = 2H$$
 \Rightarrow $\tan \theta = \frac{4H}{R} = \frac{4}{2} = 2$

$$R = \frac{2v^2}{g}\sin\theta\cos\theta$$

$$R = \frac{2v^2}{g} \times \frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{5}} = \frac{4v^2}{5g}$$



72. R is same at an angle θ and $(90^{\circ} - \theta)$

$$t_1 = \frac{2u\sin\theta}{g} \& t_2 = \frac{2u\sin(90 - \theta)}{g} = \frac{2u\cos\theta}{g}$$

$$t_1t_2 = \frac{2u\sin\theta}{g} \times \frac{2u\cos\theta}{g} = \ \frac{u^22\sin\theta\cos\theta}{g} \times \frac{2}{g}$$

$$\Rightarrow t_1 t_2 = \frac{2R}{g} \Rightarrow t_1 t_2 \propto R$$

73.
$$R_{max} = \frac{u^2}{g}$$



area =
$$\pi R_{max}^2 = \pi \left[\frac{u^2}{g} \right]^2$$

74.
$$t = \sqrt{\frac{2 \times 20}{g}} = 2 \sec x$$

$$x = v_x t \Rightarrow v_x = \frac{4}{2} = 2m/s$$

75.
$$v_x = 10\hat{i}$$

$$v_y = -10\hat{j}$$
 $t = 1.5$ sec. $a = -g\hat{j} = -10\hat{j}$

$$(\vec{\mathbf{u}}_{v}) = -10\hat{\mathbf{j}} - (-10\hat{\mathbf{j}}) \times (1.5)$$

$$\Rightarrow$$
 $\vec{u}_y = 5 \hat{j}$

$$\vec{u} = 10\hat{i} + 5\hat{j} \Rightarrow |\vec{u}| = 5\sqrt{5} \text{ m/s}$$

76.
$$u_{x_1}t + u_{x_2}t = x$$

$$\left[\frac{u}{\sqrt{3}}\cos 30^{\circ} + u\cos 60^{\circ}\right]t = x$$

$$\left[\frac{u}{\sqrt{3}} \times \frac{\sqrt{3}}{2} + \frac{u}{2}\right] t = x \qquad \Rightarrow \qquad t = \frac{x}{u}$$

77.
$$\vec{v} = \vec{u} + \vec{a}t = (3\hat{i} + 4\hat{j}) + (0.4\hat{i} + 0.3\hat{j}) \times 10$$

= $7\hat{i} + 7\hat{i}$

or
$$v = \sqrt{7^2 + 7^2} = 7\sqrt{2}$$
 unit

78.
$$x = \frac{1}{2} \times 6 \times 4^2 = 48 \text{ m}$$

and
$$y = \frac{1}{2} \times 8 \times 4^2 = 64 \text{ m}$$

$$\therefore \qquad s = \sqrt{x^2 + y^2} = \sqrt{48^2 + 64^2} = 80 \text{ m}$$

$$u = 0, \ \vec{a} = \text{const.} \quad \text{so path is straight line}$$

79.
$$\vec{R} = 2\sin(2\pi t) \hat{i} + 2\cos 2\pi t \hat{j}$$

$$\vec{v} = \frac{d\vec{R}}{dt} = 4\pi\cos 2\pi \,t\,\hat{i} - 4\pi\sin \,2\pi t\,\hat{j}$$

$$|\vec{v}| = 4\pi\sqrt{2}$$
 m/s

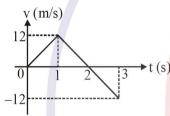
80.
$$\vec{r} = \cos \omega t \hat{x} + \sin \omega t \hat{y}$$

$$\Rightarrow \vec{v} = \frac{d\vec{r}}{dt} = -\omega \sin \omega t \ \hat{x} + \omega \cos \omega t \hat{y}$$

$$\Rightarrow \vec{a} = \frac{d\vec{v}}{dt} = -\omega^2 \cos \omega t \ \hat{x} - \omega^2 \sin \omega t \ \hat{y} = -\omega^2 \vec{r}$$

 \vec{a} is directed towards the origin.

Also
$$\vec{r} \cdot \vec{v} = 0$$
 hence $\vec{r} \perp \vec{v}$



Distance travelled in first second

$$S = \left(\frac{u+v}{2}\right)t = \left(\frac{0+12}{2}\right)(1) = 6m$$

Total distance = 9 m so average speed =
$$\frac{18m}{3s}$$
 = 6m/s

Displacement = 3m so average velocity =
$$\frac{6m}{3s}$$
 = 2 m/s

83.
$$v = At + Bt^2 \Rightarrow \frac{ds}{dt} = At + Bt^2$$

$$\Rightarrow \int_0^s ds = \int_2^3 (At + Bt^2) dt$$

$$\Rightarrow s = \frac{A}{2} (3^2 - 2^2) + \frac{B}{3} (3^3 - 2^3) = \frac{5A}{2} + \frac{19B}{3}$$