SOLUTIONS

UNITS, DIMENSION & MEASUREMENTS

1. Time period, $T = kS^a r^b \rho^c$

$M^0L^0T^1 = [MT^{-2}]^a [L]^b [ML^{-3}]^c$

$= [M]^{a+c} [L]^{b-3c} [T]^{-2a}$

$\therefore$ $-2a = 1$ or $a = -1/2$

Also $a + c = 0$ or $c = 1/2$

and $b - 3c = 0$ or $b = 3c = 3/2$

Thus $T = k\sqrt{\dfrac{\rho r^3}{S}}$

2. SI units of A ≡ units of $(Ex^3) = \dfrac{N-m^3}{C}$

SI units of B ≡ units of $(E/y) \equiv \left(\dfrac{N}{C}\right)\dfrac{1}{m} = \left(\dfrac{V}{m}\right)\dfrac{1}{m} = \dfrac{V}{m^2}$

SI units of C ≡ units of $(E/z^2) \equiv \left(\dfrac{N}{C}\right)\dfrac{1}{m^2} = \dfrac{N}{m^2-C}$

3. $\left[\dfrac{A}{B}\right] = \left[\dfrac{F/\sqrt{x}}{F/x^2}\right] = \left[x^{3/2}\right] = L^{3/2}$

4. $\left[\dfrac{x}{\gamma}\right]$ is dimensionless

but $[\alpha x] = [\beta]$ so $[x] = \left[\dfrac{\beta}{\alpha}\right]$

Therefore $\left[\dfrac{\beta}{\alpha\gamma}\right]$ or $\left[\dfrac{\alpha\gamma}{\beta}\right]$ is a dimensionless combined

5. $\dfrac{1}{\sqrt{\mu_0 \in_0}} = c \Rightarrow \left[\dfrac{1}{\sqrt{\mu_0 \in_0}}\right] = LT^{-1}$

6. Planck's constant $h = \dfrac{\text{Energy}}{\text{Frequency}}$

$h = \dfrac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}]$

$F = \dfrac{Gm_1m_2}{r^2}$

$\therefore$ Gravitational constant $G = \dfrac{Fr^2}{m_1m_2}$

$= \dfrac{[M^1L^1T^{-2}][L^2]}{[M^2]} = [M^{-1}L^3T^{-2}]$

7. Applying dimensional analysis

$S \propto E^a V^b T^c$

$[M^1L^0T^{-2}] = k\,[M^1L^2T^{-2}]^a\,[L^1T^{-1}]^b\,[T^1]^c$

$[M^1L^0T^{-2}] = k\,[M^aL^{2a}T^{-2a}]\,[L^bT^{-b}]\,[T^c]$

$[M^1L^0T^{-2}] = k[M]^a[L]^{2a+b}[T]^{-2a-b+c}$

Comparision

$a = 1$ $\quad 2a + b = 0 \quad -2 = -2a - b + c$

$b = -2 \quad -2 = -2(1) + 2 + c$

$c = -2$

So the dimensional formula for surface tension will be $[E^1V^{-2}T^{-2}]$

Alternate solution :

Surface Tension = $\dfrac{\text{Surface energy}}{\text{Area}}$

[Surface tension] = $\dfrac{[E]}{[V\cdot T]^2} = [E\,V^{-2}\,T^{-2}]$

8. $\nu_c \propto [\eta^x\,\rho^y\,r^z]$

$[L^1T^{-1}] \propto [M^1L^{-1}T^{-1}]^x\,[M^1L^{-3}]^y\,[L^1]^z$

$[L^1T^{-1}] \propto [M^{x+y}]\,[L^{-x-3y+z}]\,[T^{-x}]$

taking comparision on both size

$x + y = 0,\ -x - 3y + z = 1,\ -x = -1$

$\Rightarrow \quad x = 1,\ y = -1,\ z = -1$

9. $[L] = [c]^a [G]^b \left[\dfrac{e^2}{4\pi\varepsilon_0}\right]^c$

$[L] = [LT^{-1}]^a [M^{-1}L^3T^{-2}]^b \left[ML^3T^{-2}\right]^c$

$[L] = L^{a+3b+3c}\,M^{-b+c}\,T^{-a-2b-2c}$

$a + 3b + 3c = 1$

$-b + c = 0$

$a + 2b + 2c = 0$

On solving,

$a = -2,\ b = \dfrac{1}{2},\ c = \dfrac{1}{2}$

$\therefore\ L = \dfrac{1}{c^2}\left[G.\dfrac{e^2}{4\pi\varepsilon_0}\right]^{\frac{1}{2}}$

10. The dimensions of Plank constant = $[ML^2T^{-1}]$

Energy = $[ML^2T^{-2}]$

11. The work done = force × displacement

$\therefore$ unit $u_1 = Fs$

and $u_2 = 4F \times 4s = 16u_1$

12. $n_1u_1 = n_2u_2$

$$\therefore n_2 = n_1\frac{u_1}{u_2} = 8\left[\frac{M_1}{M_2}\right]\left[\frac{L_2}{L_1}\right]^3 = 8\left[\frac{1}{20}\right]\left[\frac{5}{1}\right]^3 = 50$$

13. $n_2 = n_1$

$$\left[\frac{M_1}{M_2}\right]^1\left[\frac{L_1}{L_2}\right]^2\left[\frac{T_1}{T_2}\right]^{-2} = 1\times\frac{1}{\alpha}\times\left(\frac{1}{\beta}\right)^2\times\left(\frac{1}{\gamma}\right)^{-2}$$

$$= \gamma^2/\alpha\beta^2$$

14. According to the rules of significant figures

$0.007\ m^2$ has one significant figures.

2.64×10^{24} kg has three significant figures.

$0.0006032\ m^2$ has four significant figures.

6.3200 J has five significant figures.

15. Density of material

$$= 4\frac{g}{cm^3} = \frac{4\times1000}{100}\ \frac{100g}{(10cm)^3}$$

$$= 40\ \frac{(100g)}{(10cm)^3}$$

16. The percentage error = $\frac{1}{5}\times\frac{100}{25} = 0.8\%$

17. The mean value of refractive index,

$$\mu = \frac{1.34+1.38+1.32+1.36}{4} = 1.35$$

and

$$\Delta\mu = \frac{|(1.35-1.34)|+|(1.35-1.38)|+|(1.35-1.32)|+|(1.35-1.36)|}{4}$$

$= 0.02$

Thus $\frac{\Delta\mu}{\mu}\times100 = \frac{0.02}{1.35}\times100 = 1.48\%$

19. $h = \frac{1}{2}gt^2$ (for free fall)

$$g = \frac{2h}{t^2} \Rightarrow \frac{\Delta g}{g} = \frac{\Delta h}{h}+\frac{2\Delta t}{t} = e_1+2e_2$$

20. Large number of readings will reduce the random errror.

21. $\frac{\Delta X}{X} = a\frac{\Delta M}{M}+b\frac{\Delta L}{L}+c\frac{\Delta T}{T} = a\alpha\% + b\beta\% + c\gamma\%$

$$\frac{\Delta X}{X}\times100 = (a\alpha + b\beta + c\gamma)\%$$

22. Mean value $\bar{a} = \frac{2.63+2.56+2.42+2.71+2.80}{5}$

$\bar{a} = 2.624$

absolute error in various readings are

$\Delta a_1 = 2.624 - 2.63 = -.006$

$\Delta a_2 = 2.624 - 2.56 = .064$

$\Delta a_3 = 2.624 - 2.42 = .204$

$\Delta a_4 = 2.624 - 2.71 = -.086$

$\Delta a_5 = 2.624 - 2.80 = -.176$

$$\overline{\Delta a} = \frac{|\Delta a_1|+|\Delta a_2|+|\Delta a_3|+|\Delta a_4|+|\Delta a_5|}{5} = 0.11\ sec$$

23. $\because V = \frac{\pi D^2\ell}{4}$

$$\therefore \frac{\Delta V}{V} = \frac{2\Delta D}{D}+\frac{\Delta\ell}{\ell} = \left(\frac{2(0.01)}{4}+\frac{0.1}{5}\right)\times100$$

$= 2.5\%$ (Approx)

24. $\varepsilon = \frac{xy^2}{10z^{1/3}} \Rightarrow \varepsilon \propto x^1y^2z^{-1/3}$

$$\Rightarrow \frac{\Delta\varepsilon}{\varepsilon}\times100 = \left(\frac{\Delta x}{x}+2\frac{\Delta y}{y}+\frac{1}{3}\frac{\Delta z}{z}\right)\times100$$

$= 2\% + (2\times1\%) + (\frac{1}{3}\times3\%)$

$= 2\% + 2\% + 1\% = 5\%$

25. $\rho = \frac{M}{V} = \frac{M}{\pi r^2\ell}$

$\frac{\Delta\rho}{\rho} = \frac{\Delta M}{M}+\frac{2\Delta r}{r}+\frac{\Delta\ell}{\ell}$ (For maximum error)

$\frac{\Delta M}{M}\times100 = \frac{0.003}{0.3}\times100 = 1\%$,

$\frac{\Delta r}{r}\times100 = \frac{.005}{0.5}\times100 = 1\%$

$\frac{\Delta\ell}{\ell}\times100 = \frac{0.06}{6}\times100 = 1\%$

$\frac{\Delta\rho}{\rho}\times100 = 4\%$

26. $R = \frac{V}{I} = \frac{10}{2} = 5\Omega$

Also, $\frac{\Delta R}{R}\times 100 = \frac{\Delta V}{V}\times 100 + \frac{\Delta I}{I}\times 100$

$= \frac{0.5}{10}\times 100 + \frac{0.2}{2}\times 100 = 15\%$

Thus $R = 5 \pm 15\%\ \Omega$

27. $T = 2\pi\sqrt{\frac{\ell}{g}}\quad \therefore\ g = \frac{4\pi^2\ell}{T^2}$

and $\frac{\Delta g}{g}\times 100 = \left[\frac{\Delta \ell}{\ell} + 2\frac{\Delta T}{T}\right]\times 100$

$= \left[\frac{0.1}{100} + 2\frac{0.1}{2\times 100}\right]\times 100$

$= 0.2\ \%$

28. One main scale division, 1 M.S.D. = x cm

One vernier scale division, 1 V.S.D. = $\frac{(n-1)x}{n}$

Least count = 1 M.S.D. – 1 V.S.D.

$= \frac{nx - nx + x}{n} = \frac{x}{n}$ cm.

29. Reading of screw gauge

= MSR + VSR × LC + zero error

= 0.5 cm + 25 × 0.001 cm + 0.004 cm

= 0.529 cm