Gravitation

EXERCISE

2.1 Introduction :

Q.1 What do you mean by gravitation and gravitational force?

Ans:

i. **Gravitation:**

Gravitation is a natural phenomenon by which material objects attract towards one another.

Existence of our solar system is due to gravitation.

ii. **Gravitational force:**

Force of attraction which keeps two bodies in the universe bound together is called gravitational force.

2.1 Newton's law of graviation :

***Q.2. State and explain Newton's law of gravitation.**

Ans: Statement:

Every particle of matter attracts every other particle of matter with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

OR

Gravitational force between two bodies in the universe is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

Explanation:

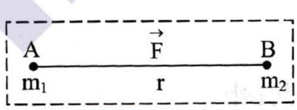
i. Let,

m_1 = mass of body A

m_2 = mass of body B

r = distance between A and B

$\vec{F}$ = gravitational force between **A** and B



ii. According to Newton's law of gravitation,

$F \mu m_1 m_2$ .....(i)

$F \mu \frac{1}{r^2}$ ..... (ii)

Combining equations (i) and (ii), we get,

$$F \mu \frac{m_1 m_2}{r^2}$$

\ $$F = G\frac{m_1 m_2}{r^2}$$

where G is a constant of proportionality known as universal gravitational constant.

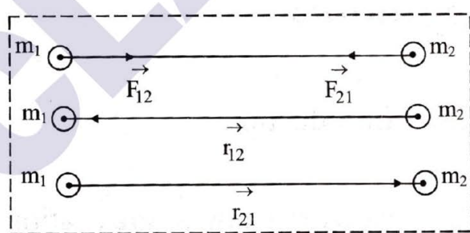
Q.3. Express Newton's law of gravitation in vector form.

Ans:

i. Let m_1 and m_2 be the two particles separated by a distance r.

$\vec{F}_{12}$ = gravitational force exerted on mass mj by mass m_2

$\vec{F}_{21}$ = gravitational force exerted on mass m_2 by mass m_1



$\vec{r}_{12}$ = position vector of mass m_2 with respect to mass m_1

$\vec{r}_{21}$ = position vector of mass mi with respect to mass m_2

ii. Force $\vec{F}_{12}$ on mass mi by mass m_2 is given by,

$$\vec{r}_{12} = G\frac{m_1 m_2}{r^2}(- \hat{r}_{12})$$

where $\vec{r}_{12} = \frac{\hat{r}_{12}}{r} =$ unit vector along $\vec{r}_{12}$.

Negative sign shows that direction of $\vec{F}_{12}$ is opposite to $\hat{r}_{12}$.

iii. Similarly, force $\vec{F}_{21}$ on mass m_2 by mass m_1 is given by,

$$\vec{F}_{21} = g\frac{m_1m_2}{r^2}(-\hat{r}_{21})$$

Where $\hat{r}_{21} = \frac{\vec{r}_{21}}{r}$ = unit vector along $\vec{r}_{21}$.

Negative sign shows that direction of $\vec{F}_{21}$ is opposite to $\vec{r}_{21}$.

iv. Since $\hat{r}_{12} = -\vec{r}_{21}$,

\ $\vec{F}_{12} = -\vec{F}_{21}$

Hence, $\vec{F}_{12}$ and $\vec{F}_{21}$ are equal in magnitude and opposite in direction which form action-reaction pair.

***Q.4. What is gravitational constant? State its unit and dimension.';**

Ans:

i. From Newton's law of gravitation,

$$F = G\frac{m_1m_2}{r^2}$$

where,G = constant called universal gravitational constant.

Its value is 6.67×10^{11} Nm^2/kg^2.

ii. $G = \frac{F_r^2}{m_1m_2}$

If $m_1 = m_2 = 1$ kg, r = 1 m then F = G.

Hence, the universal gravitational constant is the force of gravitation between two particles of unit mass separated by unit distance.

iii. Unit: Nm^2/kg^2 in SI system and dyne cm /g^2 in CGS system.

ii. Dimensions: $[M^{-1}L^3T^{-2}]$

Q.5. State Newton's law of gravitation. State the SI unit and dimensions of universal gravitation constant. [Mar 96]

***Ans:** Refer Q.2 and Q.4*

***Q.6. Obtain the dimensions of universal gravitational constant.**

Ans: According to Newton's law, of gravitation,

$$F = G\frac{m_1m_2}{r^2}$$

$$G = \frac{F_r^2}{m_1m_2}$$

Dimensions of G

$$\frac{\text{dimensions of force} \,'\, \text{dimensions of } r^2}{\text{dimensions of } m_1 \,'\, \text{dimensions of } m_2}$$

$$= \frac{[L^1M^1T^{-2}]\,'\,[L^2]}{[M^2]}$$

$[M^{-1}L^3T^{-2}]$

Dimensions of G = $[M^{-1}L^3T^{-2}]$

Q.7. Write down the main characteristics of gravitational forcc.

Ans: Characteristics of gravitational force:

i. Gravitational force is independent of medium between two bodies and is independent of presence of the other bodies.

ii. It is mutual force of attraction which forms action-reaction pair.

iii. It acts along the line joining two masses.

iv. This force is weakest of all types of forces.

iii. It is always attractive in nature.

iv. It is a long range force, which indicates that it is effective over a large distance.

v. It is a conservative force. (A force in which work done is independent of path or work done in moving a particle round a closed path is zero under gravitational force).

vi. It is a central force since it acts along the line joining the centres of the two particles or bodies.

***Q.8. Obtain the relation between universal gravitational constant and acceleration due to gravity on the surface of the earth.**

Ans: Relation between universal gravitational constant and acceleration due to gravity:

i. Let,

M = mass of the earth

R = radius of the earth

m = mass of object situated on the surface of earth

g = acceleration due to gravity at the earth's surface ,

ii. The weight of the object on the earth surface is equal to gravitational force acting on it.

\ Weight of the object (W)

= Gravitational force (F)

$$\therefore \quad mg = \frac{GMm}{R^2}$$

$$\therefore \quad g = \frac{GM}{R^2}$$

Q.9. State the relation between acceleration due to gravity on the surface of the earth and at a certain height.

OR

'g' is the acceleration due to gravity on the surface of the Earth and 'R' is the radius of I the Earth. Show that acceleration due to : gravity at height -h' above the surface of the Earth is $g_h = g\left(\frac{R}{R+h}\right)^2$

[Oct 2013]

Ans: Relation between acceleration due to gravity on the surface of the earth and at a certain height: ;

i. At the surface of the earth **acceleration** due lo gravity is given by,

$$g = \frac{GM}{R^2} \qquad \text{...... (i)}$$

ii. Let acceleration due to gravity at a height h above the surface of the earth be g_h.

iii. At height h,

Weight of the object = Gravitational force.

$$\therefore \quad mg_h = \frac{GMm}{(R+h)^2}$$

$$\therefore \quad g_h = \frac{GM}{(R+h)^2} \qquad \text{.... (ii)}$$

iv. Dividing equation (ii) by equation (i),

we have,

$$g_h = \frac{R^2}{(R+h)^2} = \left(\frac{R}{R+h}\right)^2$$

$$\therefore \quad g_h = g\left(\frac{R}{R+h}\right)^2$$

Note:

1. Centripetal acceleration on the moon's surface is 2.72×10^{-3} m/s^2.
2. Gravitational constant is a very small number so gravitational force is small unless masses of the two attractive bodies are large.
3. Laws of gravitation is obeyed by all material bodies in the universe irrespective of their sizes or the distance between them. It is true for minute particles as well as huge heavenly bodies.
4. Gravitational force is independent of the intervening medium.
5. Gravitational force is a conservative force.

2.2 Projection of satellite

Q.10. Why can't a single stage rocket be used for launching a satellite in a circular orbit round the earth?

Ans:

i. A satellite is placed at the tip of rocket.

ii. When first stage of the rocket is fired, it gives the vertical velocity to the satellite.

iii. If the vertical velocity of satellite is less than the escape velocity, the satellite will reach a certain maximum height and will come back to the earth's surface due to gravity.

iv. If vertical velocity of satellite is equal or greater than escape velocity, the satellite will escape from earth's gravitational field and will drift into space.

v. Hence, a satellite cannot be put in a circular orbit by using a single stage rocket

Note:

1. Critical velocity is the minimum horizontal velocity of projection for which a satellite performs circular motion round the earth.
2. Escape velocity is the minimum velocity with which a body should be projected from the surface of the earth, so that it escapes from the gravitational influence of the earth.

***Q.ll.Explain why is it necessary to use minimum two stage rocket to launch a satellite in a circular orbit round the earth.**

Ans:

i. When the first stage of the rocket is fired from the ground, it rises to a desired height above the earth.

ii. Now, by remote control, the first stage of the rocket is detached and the rocket is rotated through 90° so that it points in the horizontal direction.

iii. The second stage is then fired, with a particular value of horizontal velocity in the horizontal direction.

iv. Thus, satellite begins to revolve in a circular orbit.

v. The gravitational force acting on the satellite is exactly equal to the centripetal force necessary to keep the satellite moving in a circular orbit round the earth.

vi. Hence, minimum two stage rocket is used for launching a satellite.

Q.12. Why is it necessary to have a minimum two stage launching system to put a satellite into desired circular orbit? [Mar 97]

Ans: Refer Q.ll

***Q.13.Describe how an artificial satellite using two stage rocket is launched in an orbit around the earth. Explain the nature of all the possible orbits of a satellite with the help of suitable diagram. [Mar 09]**

Ans: Launching of satellite by using two stage rocket:

i. For projection of satellite, minimum two stage rocket is used. The satellite is kept at the tip of the rocket.

ii. Initially the fuel in the first stage of the rocket is ignited on the surface of earth, so that rocket rises to a desired height above the earth's surface.

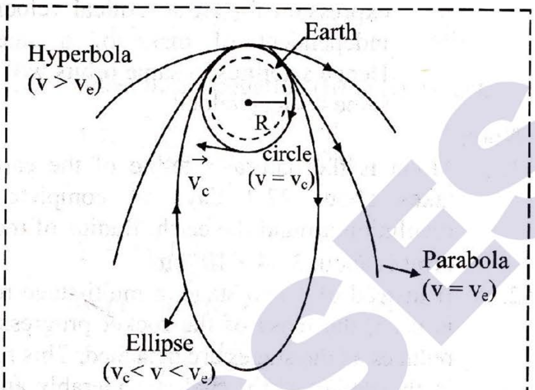
iii. Thus with the help of first stage, satellite attains desired height.

iv. Now by remote control, the empty first stage of the rocket is detached and the rocket is rotated through 90° so that it points in the horizontal direction.

v. Now, fuel in the second stage of the rocket is ignited so that satellite is projected in horizontal direction.

vi. When die fuel in the second stage is completely burnt, the empty second stage also gets detached from satellite. The resulting motion of the satellite depends upon the velocity of the projection given to it.

Nature of orbits of satellite



Path of satellite in various cases

i. If the velocity of the projection is equal to the critical velocity, i.e., $v = v_c$, then the satellite moves in circular orbit round the earth.

ii. If the velocity of the projection is greater than the critical velocity but less than the escape velocity, i.e., $v_c < v < v_e$, then the satellite moves in elliptical orbit.

iii. If the velocity of the projection is equal to the escape velocity, i.e., $v = v_e$, then the satellite moves in parabolic path.

iv. If the velocity of the projection is greater than the escape velocity, i.e., $v > v_e$, then the orbit will be hyperbolic and will escape the gravitational pull of the earth and continue to travel infinitely.

Q.14. What is a satellite? Why an artificial satellite is placed outside the earth's atmosphere?'

Ans:

i. *An object which revolves in an orbit around a planet is called as satellite. Example:*

a. Moon is a natural satellite of the earth.

b. INSAT-B is an artificial satellite of the earth.

ii. Artificial satellite is always placed outside the earth's atmosphere. If it is not so, the friction with atmosphere will produce so much heat that the satellite will get burnt.

Q.15. What is critical velocity? Derive an expression for critical velocity of a satellite orbiting at a certain height.

Also discuss the formula when satellite is very close to the earth's surface.

[Oct 06,08]

Ans:

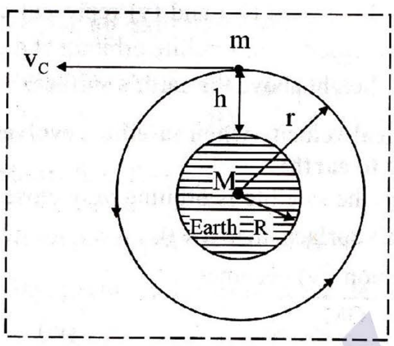
i. *Critical velocity is defined as the minimum horizontal velocity of projection for which a satellite performs circular motion round the earth.*

ii. Critical velocity is also called orbital velocity,

iii. Unit: m/s in SI system and cm/s in CGS system.

iv. Dimensions: $[M^0L^1T^{-1}]$

Expression for critical velocity:



i. Let,

M = mass of the earth

R = radius of the earth

h = height of the satellite from the earth's surface

m = mass of the satellite

v_c = critical velocity of the satellite in the given orbit

r = (R + h) = radius of the circular orbit.

ii. For the circular motion of the satellite, the necessary centripetal force is given as,

$$F_{CP} = \frac{mv_c^2}{r} \quad \text{...... (i)}$$

iii. The gravitational force of attraction between the earth and the satellite is given by,

$$F_G = \frac{GMm}{r^2} \quad \text{...... (ii)}$$

iv. Gravitational force provides the centripetal force necessary for the circular motion of the satellite.

\ $F_{CP} = F_G$

......[From equations (i) and (ii)]

\ $v_c^2 = \frac{GM}{r}$

\ $v_c = \sqrt{\frac{GM}{r}}$ ..... (iii)

v. But, r = R + h

\ $v_c = \sqrt{\frac{GM}{(R+h)}}$ ..... (iv)

Also, $GM = g_h (R + h)^2$

where, g_h is acceleration due to gravity at height 'h' above earth's surface.

$v_c = \sqrt{g_h (R+h)}$ ...... (v)

Equations (iv) and (v) represent critical velocities of satellite orbiting at a certain height above the earth's surface.

Critical velocity when satellite revolves very close to earth:

When the satellite is orbiting very close to the earth's surface, thenh«0.

Equation (iv) becomes,

$v_c = \sqrt{\frac{GM}{R}}$ .... (vi)

But $GM = gR^2$

\ $v_c = \sqrt{gR}$

Equations (vi) and (vii) represent critical velocities of a satellite orbiting very close to the earth's surface.

***Q.16.Define critical velocity of a satellite and obtain the expression for it.**

Ans: *Refer Q.15*

Thus,

Q.17. Explain the factors which affect the critical velocity of orbiting satellite.

Ans: Factors affecting critical velocity:

Critical velocity is given by, $v_c = \sqrt{\frac{gm}{R+h}}$

Thus,

i. Critical velocity depends on mass of the planet around which satellite is orbiting.

It varies directly as $\sqrt{M}$.

i.e. $v_c \mu \sqrt{M}$

ii. Critical velocity depends on height of satellite 'h. It varies inversely as $\sqrt{R+h}$ .1

i.e. $v_c \mu \frac{1}{\sqrt{R+h}}$

where, R + h = r radius of orbit of satellite.

iii. Since there is no mass term in the expression for v_c, so critical velocity is independent of mass of a satellite. Hence satellites in same orbits will have same critical velocity.

Note:

1. Moon is the natural satellite of the earth. It takes about 27.3 days to complete one revolution around the earth. Radius of moon's orbit is about 3.84×10^s km.
2. If instead of a two stage, a multi-stage rocket is used, the mass of the rocket progressively! reduces as the stages are detached. This results in the saving of the fuel considerably and the desired heights can be easily reached.

2.3 Periodic time of a satellite

***Q.18. Define time period of a satellite.**

Obtain an expression for the period of a satellite in a circular orbit round the earth. Show that the square of the period of revolution of a satellite is directly proportional to the cube of the orbital radius.

Ans: Definition:

The time taken by the satellite to complete one revolution around the earth is called its time period.

Expression for time period:

i. Let,

M = mass of earth

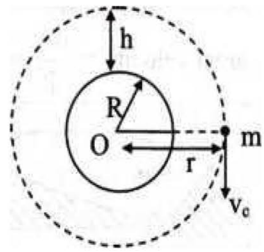
m = mass of satellite

R = radius of earth

h = altitude of satellite

r = R + h = radius of satellite orbit

v_c = critical velocity n



ii. In one revolution, distance covered by satellite is equal to c ircumference of its circulat orbit.

iii. If T is the time period of satellite, then

$$T = \frac{\text{Circumference of the orbit}}{\text{Critical velocity}}$$

\ $$T = \frac{2pr}{v_c} \qquad \text{...... (i)}$$

But, $$v_c = \sqrt{\frac{GM}{r}} \qquad \text{...... (ii)}$$

iv. Substituting equation (ii) in (i) we get,

$$T = \frac{2pr}{\sqrt{\frac{GM}{r}}} = 2p\sqrt{r^2 ´ \frac{r}{GM}}$$

\ $$T = 2p\sqrt{\frac{r^2}{GM}} \qquad \text{...... (iii)}$$

But r = R + h

\ $$T = 2p\sqrt{\frac{(R+h)^2}{GM}}$$

This is the required expression for period of satellite orbiting around the earth,

v. Squaring both side of equation (iii), we get,

$$T^2 = \frac{4p^2 r^3}{GM}$$

Since p, G and M are constant,

\ $T^2 \mu r^2$

Hence, square of the period of revolution of a satellite is directly proportional to the cube of the radius of its orbit.

Q.19. For an orbiting satellite very close to surface of the earth, show that

$$T = 2\pi\sqrt{\frac{R}{g}}.$$

Ans:

i. Time period of an orbiting satellite at certain height is given by,

$$T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$$

ii. If satellite is orbiting very close to the earth's surface, then $h \approx 0$

$$\therefore \quad T = 2\pi\sqrt{\frac{R^3}{GM}}$$

But $GM = gR^2$

$$\therefore \quad T = 2\pi\sqrt{\frac{R^3}{gR^2}}$$

$$\therefore \quad T = 2\pi\sqrt{\frac{R}{g}}$$

Note :

i. For a satellite orbiting at a height h,

Time period, $T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$

But, $GM = g_h\,(R+h)^2$

$$\therefore \quad T = 2\pi\sqrt{\frac{R+h}{g_h}}$$

$$\therefore \quad T = 2\pi\sqrt{\frac{r}{g_h}} \qquad [\because r = R + h]$$

ii. For a satellite which is orbiting very close to the surface of the earth, the period can be obtained by substituting $R = 6.4 \times 10^6$ m and g = 9.8 m/s in equation $T = 2\pi\sqrt{\frac{R}{g}}$

$$\therefore \quad T = 2\pi\sqrt{\frac{6.4 \times 10^6}{9.8}} = 84.6 \text{ minutes}$$

Q.20. Obtain an expression for time period of a satellite orbiting very close to earth's surface in terms of mean density.

Show that, $T = \sqrt{\frac{3\pi}{G\rho}}$,

where ρ = mean density of earth

Ans: Time period in terms of mean density:

i. We have, $T = 2\pi\sqrt{\frac{(R+h)^2}{GM}}$ ....(i)

ii. Mass of the earth, $M = \frac{4}{3}\pi R^3\rho$

where, ρ = mean density of the earth

iii. Substituting the value of M in the equation (i), we get,

$$T = 2\pi\sqrt{\frac{(R+h)^3}{G \times \frac{4}{3}\pi R^3\rho}} = \sqrt{\frac{4\pi^2(R+h)^3}{\frac{4}{3}\pi R^3\rho\, G}}$$

$$\therefore \quad T = \sqrt{\frac{3\pi R^3}{GR^3\rho}} \qquad \text{..... (iii)}$$

When satellite is orbiting very close to the earth, $h \approx 0$.

Equation (ii) becomes,

$$T = \sqrt{\frac{3\pi R^3}{GR^3\rho}}$$

$$T = \sqrt{\frac{3\pi}{G\rho}}$$

2.4 Kepler's laws of motion

Q.21. State and explain Kepler's laws of planetary motion.

Or

State Kepler's law of orbit and law of equal areas. [Feb 2013]

Ans: Kepler's first law (Law of orbit):

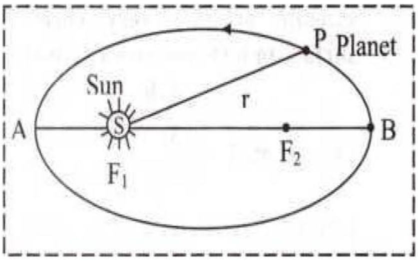
Every planet revolves around the sun in an elliptical orbit with the sun situated at one of the focii of the ellipse.

Explanation:

i. Suppose planet P revolves in an elliptical orbit with sun S at the focus F_1 as shown in the figure.

ii. When the planet is nearest to sun i.e at A, then speed of planet is maximum.

iii. When the planet is farthest from the sun i.e. at B, then speed of planet is minimum.



Kepler's second law (Law of equal areas): *The radius vector drawn from the sun to any planet sweeps out equal areas in equal intervals of time, i.e., areal velocity of the radius vector is constant.*

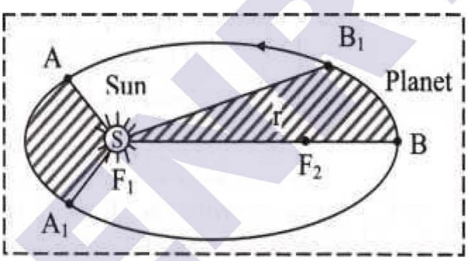
Explanation:

i. Suppose that at certain time, planet is at B and after time dt it moves to B_1. Then position vector of planet sweeps out area (BFiBi) in time dt.

ii. Again, in time dt, the planet moves from A to A_1 and sweeps out area (AF_1A_1).

iii. According to Kepler's second law, area (AF_1A_1) = area (BF_1B_1).

If dA is area swept out in time dt then areal vleocity $= \frac{dA}{dt} =$ constant



Kepler's third law (Law of periods) : *The square of the period of revolution of the planet round the sun is directly proportional to the cube of the semi-major axis of the elliptical orbit.*

This law is also called as harmonic law.

Explanation:

i. If T is the time period of the planet and r is the semi-major axis then, $T^2 \propto r^3$.

ii. If T_1 and T_2 be the periods of any two planets and r_1 and r_2 be their semi-major axes, then

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$$

iii. Kepler's third law is consistent with inverse square nature of the law of universal gravitation.

***Q.22.State Kepler's law of planetary motion.**

Ans: *Refer Q.21 (Only laws)*

Note:

On the basis of Kepler's law following facts are obtained:

1. Period of revolution of the earth around the sun in its elliptical path is 365 days 6 hours 48 minutes and 52 second, i.e., about $365\frac{1}{4}$ days. This quarter day becomes a full day after every four years which leads to be causing a leap year. In a leap year, month of February is of 29 days, i.e. 1 day more from its usual value.
2. Mercury is the nearest planet to the sun which takes only 88 days to complete one revolution around the sun.
3. The period of revolution of a planet around the sun is called the natural year of the planet

2.5 Binding energy and escape velocity of a sattelite

***Q.23. Define binding energy. State its unit and dimensions.**

Ans: i. Definition:

The minimum amount of energy required to remove (escape) a satellite from earth's gravitational influence is called its binding energy.

OR

The minimum amount of energy that must be provided to a satellite to escape from the gravitational influence is called binding energy of the satellite.

It is a scalar quantity.

i. Unit: joule in SI system and erg in CGS system.

ii. Dimensions: $[M^1L^2T^{-2}]$

Q.24. Obtain an expression for the binding energy of a satellite orbiting the earth at a certain height.

OR

***Obtain an expression for the binding energy of a satellite revolving in a circular orbit round the earth.**

Ans: Expression for binding energy of a satellite orbiting at a certain height:

i. Consider a satellite is revolving round the earth in a circular orbit.

Let,

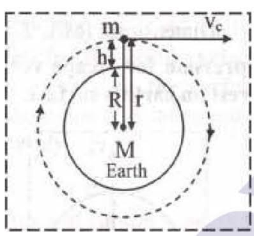
m = mass of the satellite

M = mass of the earth

R = radius of the earth

h = height of the satellite above the earth's surface

v_c = critical velocity of the satellite in the given orbit



ii. The satellite revolves in its orbit around the earth under the /effect of its gravitational pull. In this case we have, $F_{CP} = F_G$

But, $F_{CP} = \frac{mv_c^2}{(R+h)}$ and $F_G = \frac{GMm}{(R+h)^2}$

\ $\frac{mv_c^2}{R+h} = \frac{GMm}{(R+h)^2}$

\ $mv_c^2 = \frac{GMm}{(R+h)}$

iii. Kinetic energy of the satellite in its orbit is given by,

$\frac{1}{2}mv_c^2 = \frac{1}{2}\frac{GMm}{(R+h)}$

\ $K.E. = \frac{GMm}{2(R+h)}$

iv. Potential energy of satellite is,

P.E. = Gravitational potential x mass of the satellite

\ $P.E. = -\frac{GM}{r} ' m$

$= \frac{GMm}{(R+h)}$ $[\because r = R + h]$

v. Total energy of the satellite

T.E. = K.E. + P.E.

\ $T.E. = \frac{GMm}{2(R+h)} - \frac{GMm}{R+h}$

\ $T.E. = \frac{GMm}{2(R+h)}$

The negative sign shows that satellite is bound to the earth due to gravitational force of attraction,

vi. Since B.E. = – T.E.,

\ $B.E. = \frac{GMm}{2(R+h)}$

Q.25. Obtain an expression for binding energy of a satellite

i. when it is in position of rest at certain height and

ii. when it is in the position of rest on the earth's surface.

Ans:

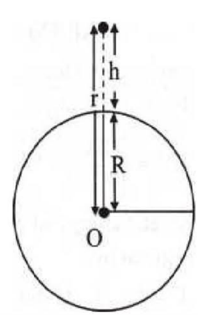
i. Expression for binding energy when satellite is in the position of rest at a certain height:

a. Let,

M = mass of the earth

m = mass of the satellite

h = height of satellite above the earth's surface



b. As the satellite is at rest, v = 0. Kinetic energy of satellite,

$$K.E. = \frac{1}{2}mv_c^2 = 0$$

c. Potential energy of satellite,

$$P.E. = -\frac{GMm}{(R+h)}$$

d. Total energy of satellite,

$$T.E. = K.E. + P.E. = 0 - \frac{GMm}{(R+h)}$$

e. Since, B.E. = – T.E.,

\ $$B.E. = \frac{GMm}{(R+h)} \quad \text{..... (i)}$$

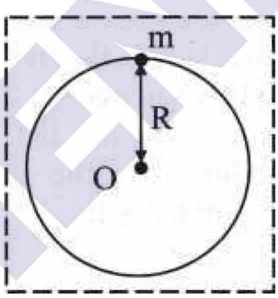
f. Also, $GM = g_h(R+h)^2$

\ $$B.E. = \frac{mg_h(R+h)^2}{(R+h)}$$

\ $$B.E. = mg_h(R+h) \quad \text{...... (ii)}$$

Equations (i) and (ii) represent binding energy of satellite at rest when it is at a certain height.

ii. **Expression for binding energy when satellite is in the position of rest on the earth's surface:**



a. Gravitational potential at the earth

$$\text{Surace} = \frac{GM}{R}$$

b. Potential energy of satellite = P.E
= Gravitational potential x mass of satellite

$$= \frac{GMm}{R}$$

c. Since the satellite is at rest on the earth,

\ v = 0

\ Kinetic energy of satellite,

$$K.E. = \frac{1}{2}mv^2 = 0$$

d. Total energy of satellite
= T.E = P.E + K.E

$$= \frac{GMm}{R} + 0$$

\ $$T.E. = -\frac{GMm}{R}$$

e. Since B.E = – T.E,

\ $$B.E. = \frac{GMm}{R}$$

***Q.26. Derive an expression for binding energy of a body at rest on the earth's surface.**

Ans: *Refer Q.25 (ii)*

***Q.27. Define escape velocity. Derive an expression for the escape velocity of an object from the surface of the earth. Discuss the factors which govern escape velocity of a body. Why escape velocity is same for all bodies projected from the surface of the Earth?**

[Feb 2013 old course]

Ans:

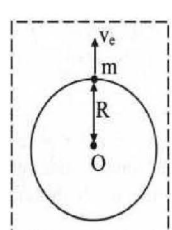
i. Escape velocity /I defified as the minimum velocity with which a body should be projected from the surface of the earth, so that it escapes from the gravitational influence of the earth.

ii. It is a vector quantity which is denoted by V_e

iii. Unit: m/s in SI system

iv. Dimensions: $[M^0L^1T^{-1}]$

Expression for escape velocity of a satellite at rest on earth's surface



i. The binding energy of the satellite of mass m at rest on the earth's surface is GMm given by, B.E. = $\frac{GMm}{R}$

ii. In order to escape the satellite from earth's gravitational influence, the kinetic energy of projection must be equal to its B.E.

\ K.E of satellite = B.E of satellite

\ $\frac{1}{2}mv_e^2 = \frac{GMm}{R}$

Where, v_e = escape velocity of the satellite.

\ $v_e^2 = \frac{2GM}{R}$

\ $v_e = \sqrt{\frac{2GM}{R}}$ ..... (i)

iii. But, $GM = gR^2$

\ $v_e = \sqrt{\frac{2gR^2}{R}}$

$= \sqrt{2gR}$ ......(ii)

Equations (i) and (ii) represent escape velocities from the surface of the earth,

iv. Escape velocity does not depend upon the mass of the body. Thus, escape velocity is same for all bodies projected from the surface of the Earth.

Factors affecting escape velocity:

a. Escape velocity depends upon mass of the planet. It increases with increase in mass of the planet.

b. Escape velocity depends upon radius of the orbit.

Q.28. Define escape velocity of the body. Obtain an expression for the escape velocity when the body is at rest on the surface of the earth and show that it is independent of the mass of the body. [Mar 99]

Ans: Refer Q. 27

Q.29. Obtain formula of escape velocity of a body stationary on the earth surface in terms of mean density of earth.

OR

Show that v_e = 2R

i. Let,

M = mass of the earth

m = mass of the body

r = mean density of the body

ii. Escape velocity on the earth's surface is given as,

$v_e = \sqrt{\frac{2GM}{R}}$

iii. But $M = \frac{4}{3} pR^3 r$

\ $v_e = \sqrt{\frac{2G\frac{4}{3}pR^3r}{R}}$

$= \sqrt{\frac{8pGR^3r}{3}}$

\ $v_e = 2R\sqrt{\frac{2pr G}{3}}$

Q.30. Obtain the relation between escape velocity and critical velocity when satellite is very close to earth and is in stationary position.

i. When satellite is very close to the earth and is in stationary position,

Escape velocity $v_e = \sqrt{\frac{2GM}{R}}$ .... (i)

ii. Critical velocity on the surface of the earth is given as,

$v_e = \sqrt{\frac{GM}{R}}$ ..... (ii)

iii. Dividing equation (i) by (ii), we get,

$\frac{v_e}{v_c} = \sqrt{\frac{\frac{2GM}{R}}{\frac{GM}{R}}} = \sqrt{2}$

\ $v_e = \sqrt{2}\, v_c$

Therefore, escape velocity is $\sqrt{2}$ times the critical velocity.

Q.31. Obtain relation between escape velocity and critical velocity of a satellite orbiting the earth at a certain height

Ans:

i. When the satellite of mass m is revolving in a circular orbit round the earth, binding energy is same as kinetic energy.

\ $$\frac{1}{2}mv_e^2 = \frac{1}{2}\frac{GMm}{R+h}$$

\ $$v_e = \sqrt{\frac{GM}{R+h}} \qquad ..... (i)$$

ii. Critical velocity is given by,

$$v_e = \sqrt{\frac{GM}{R+h}} \qquad ..... (ii)$$

iii. Dividing equation (i) by (ii),

$$\frac{v_e}{v_c} = \sqrt{\frac{\frac{GM}{R+h}}{\frac{GM}{R+h}}} = 1$$

Note:

1. The B.E of an orbiting satellite is equal to its K.E.
2. On earth's surface, $g = 9.8\ m/s^2$, $R = 6.4 \times 10^6$ m.
   Substituting these values we get,
   $$v_e = \sqrt{2\,'\,9.8\,'\,6.4\,'\,10^4} = 11200 \text{ m/s}$$
   or
   $v_e = 11.2$ km/s

2.6 Weightlessness condftitti In a satellite

Q.32. What do you mean by weight and weightlessness of a body?

Ans: Weight of a body:

i. Weight of a body is the gravitational force exerted on it by the earth.

ii. When a man stands on the floor, then the floor exerts normal reaction on the man equal to his weight i.e W = N = mg

Weightlessness of a body:

i. A body is said to be in the state of weightlessness if its apparent weight is zero.

ii. The sensation of weightlessness experienced by an astronaut is not the result of zero gravitational acceleration, but there being zero difference between the acceleration of the spacecraft and the acceleration of the astronaut.

***Q.33.Explain why an astronaut in an orbiting satellite has a feeling of weightlessness.**

Ans: Feeling of weightlessness of an astronaut in orbiting satellite:

i. Consider an astronaut of mass m in a satellite which is moving with constant speed along the orbit.

ii. At the time of orbiting, the satellite as well as the astronaut are attracted towards the centre of the earth with same centripetal acceleration.

iii. As astronaut is unable to exert weight on the floor of the satellite, in turn satellite does not provide normal reaction on the astronaut. Therefore, astronaut feels weightlessness.

iv. If 'a' is the centripetal acceleration of the satellite, then the force exerted by the wall on the astronaut = N = mg – ma. But, a = g. Hence, N = mg – mg = 0. Hence astronaut feels weightlessness.

Note:

1. The weightlessness condition is also known as zero gravity condition.
2. The sensation of weightlessness experienced by an astronaut is not the result of zero gravitational acceleration but being zero difference between the acceleration of the spacecraft and the acceleration of the astronaut.
3. The most common problem experienced by astronauts in the initial hours of the weightlessness is known as space adaptation syndrome (Space sickness).
4. Weightlessness in a satellite is independent of height of satellite from earth's surface.
5. In weightlessness condition, it is difficult to control movement of body as it tends to float freely.
6. In the case of weightlessness, it is not necessary that weight to be zero. Weight may be more or less than its actual value. This weight is called apparent weight.

2.7 Variation of g due to altitude, latitude depth

Q.34. How does acceleration due to gravity vary with change in altitude?

OR

Discuss the variation of acceleration due to gravity due to altitude.

Ans: Variation of g due to altitude:

i. Let,

M = mass of the earth

R = radius of the earth

h = height at which acceleration due to gravity is to be found,

g = acceleration due to gravity at the surface of the earth.

g_h = acceleration due to gravity at height h.

ii. On the surface of the earth,

$$g = \frac{GM}{R^2} \quad \text{...... (i)}$$

At height 'h' above the earth surface,

$$g_h = \frac{GM}{(R+h)^2} \quad \text{.... (ii)}$$

iii. Dividing equation (ii) by (i), we get,

$$\frac{g_h}{g} = \frac{GM/(R+h)^2}{GM/R^2}$$

$$= \frac{GM}{(R+h)} \times \frac{R^2}{GM}$$

$$\therefore \frac{g_h}{g} = \frac{R^2}{(R+h)^2} \quad \text{..... (iii)}$$

$$\therefore \frac{g_h}{g} = \frac{R^2}{R^2\left[1+\frac{h}{R}\right]^2}$$

$$\therefore \frac{g_h}{g} = \frac{1}{\left[1+\frac{h}{R}\right]^2}$$

$$\therefore \frac{g_h}{g} = \left[1+\frac{h}{R}\right]^{2} \quad \text{.... (iv)}$$

iv. Expanding equation (iv) by using binomial expansion[1], we have,

Above equation represents acceleration . due to gravity at altitude h.;' v,

v. Fom equation (v), til is concluded that the acceleration due to gravity decreases with increase in height (altitude)

Q.35. Discuss the variation of acceleration due to gravity with depth 'd' below the surface of the earth.

Ans: Variation of g with depth:

i. Let,

M = mass of the earth

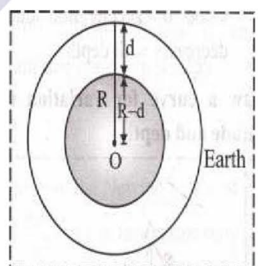
R = radius of the earth

d = depth of a point from the surface of earth where g is to be found,

g = acceleration due to gravity on the surface of the earth.

g_d = acceleration due to gravity at depth d.

ii. At the surface of the earth, $g = \frac{GM}{R^2}$



iii. If ρ is density of 'the material of earth, then,

$$M = \frac{4}{3}\pi R^3 \rho$$

$$\therefore g = \frac{G \times \frac{4}{3}\pi R^3 \rho}{R^2}$$

$$\therefore g = \frac{4}{3}\pi G R \rho \quad \text{..... (i)}$$

iv. Let gd be acceleration due to gravity at a depth d below the surface of the earth. A body at the depth d will experience force only due to the portion of the earth of radius (R – d).

v. The outer spherical shell, whose thickness is d, will not exert any force on body.

vi. If M' is mass of the portion of the earth, whosfe radius is R – d, then

$$g_d = \frac{GM'}{(R - d)^2}$$

But, $M' = \frac{4}{3}p(R - d)^3 r$

$$\backslash \quad g_d = \frac{G \times \frac{4}{3}p(R - d)^3 r}{(R - d)^2}$$

$$\backslash \quad g_d = \frac{4}{3}pG(R - d)r \qquad ..... (ii)$$

vii. Dividing the equation (ii) by (i) we have,

$$g_d = \frac{\frac{4}{3}pG(R - d)r}{\frac{4}{3}pGRr} = \frac{R - d}{R}$$

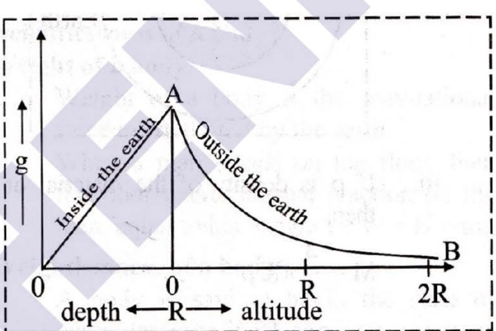
$$\backslash \quad g_d = g\left(1 - \frac{d}{R}\right) \qquad ...... (iii)$$

Equation (iii) represents acceleration due to gravity at certain depth d.

vii. From equation (iii), it is concluded that value of acceleration due to gravity decreases with depth.

Q.36. Draw a curve for variation of g due to altitude and depth.

Ans:



Q.37. Explain the variation of acceleration due to gravity due to rotational motion of the earth.

OR

Explain the effect of latitude on the value of acceleration due to gravity.

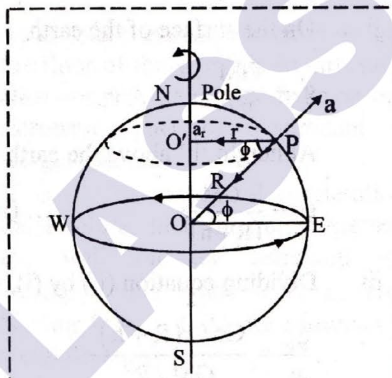
Ans: Variation of g due to latitude:

i. Consider a body placed at a point P on the surface of the earth making an angle f(latitude). The body placed at the point P moves along the circular path, whose centre is O' and radius is PO = r.

ÐEOP = f

\ ÐOPO' = f (∵ PO'||OE)

ii. Centripetal acceleration for the body at point P is $a_r = rw^2$.



Variation of g with latitude

i. In D OPO',

$$\cos f = \frac{PO'}{PO} = \frac{r}{R}$$

\ r = R cos q

\ a_r = Rcos f × w^2

\ a_r = Rw^2 cos f

iv. The radial component of centripetal acceleration,

a = a_r cos f

a = Rw^2 cos f cos f

\ a = $Rw^2 \cos^2$ \ ....(ii)

v. The effective acceleration due to gravity (g') at point P directed towards centre of earth is g' = g – a

where g is acceleration due to gravity in the absence of rotational motion of the earth.

g' = g – $Rw^2\cos^2 f$ ......(iii)

[From equation (ii)]

vi. As latitude f increases, cos f decreases. Therefore g' will increase. So the value of acceleration due to gravity increases as we move from equator to pole due to rotation of earth i.e. acceleration due to gravity increases with latitude \.

Important cases:

Case 1:

At equator $f = 0$

\ $\cos f = 1$

\ $g' = g - Rw^2$

\ $g_E = g' = g - Rw^2$

where, g_E is the acceleration due to gravity at equator. Thus, maximum reduction in acceleration due to gravity is at equator,

$g - g' = Rw^2 = \mathbf{0.03386}\ m/s^2$.

Case 2:

At poles, $f = 90°$

\ $\cos f = 0$

$g_P = 8' = g - 0 = g$

where, g_p is acceleration due to gravity at pole. Thus there is no reduction in acceleration due to gravity at poles.

***Q.38. Discuss the variation of g with**

i. altitude ii. depth

i. ***latitude***

Ans: *Refer Q.34, 35 and 37*

Note:

1. If earth suddenly stops to rotate ($w = 0$), then there will be no change in the acceleration due to gravity at the poles and that at equator i.e.
2. The acceleration due to gravity at the centre of earth is,

$$g_{centre} = g\left(1 - \frac{R}{R}\right) = 0$$

2.8 Communication satellite and uses of satellites

***Q.39.What is communication satellite?**

OR

What is geo-stationary satellite?

Ans:

i. *An artificial satellite which is orbiting the earth in same direction as direction of earth's spin and having the same period as the period of rotation of earth (i.e. 1 day = 24 hours) is called as geostationary or communication satellite.*

ii. Communication satellite appears to be stationary when observed from earth's surface, so known as geostationary satellite.

iii. Its motion is synchronous with rotational motion of earth, so it is also known as geosynchronous satellite or (synchronous satellite).

iv. The height of such satellite from surface of the earth is of about 36000 km.

***Q.40.State uses of geostationary (communication) satellite.**

Ans: Applications of communication satellite:

i. For the transmission of television and radiowave signals over large areas of earth's surface.

ii. For broadcasting telecommunication.

iii. For military purposes.

For weather forecasting and meteorological purposes.

i. For astronomical observations.

ii. For study of solar and cosmic radiation.

iii. To relay distress signals from ships.

iv. To transmit cyclone warnings to coastal villages.

Q.41. What is polar satellite?

Ans:

i. *Polar satellites are those satellites which circle the globe in a north-south orbit passing over the north arid south poles.*

Example: European SPOT, Indian Earth Resources Satellites (IERS).

i. Polar satellites are at low altitudes, i.e., 500 km to 800 km.

ii. Time period of this satellite is 100 minute. Hence it crosses any altitude many times a day.

Summary

1. According to the Newton's law of gravitation, $F = G\frac{m_1 m_2}{R^2}$.

   (The law is true only if the value of R is greater than 200 m.)

2. Artificial satellite is projected in space by using launching rocket. It provides critical velocity to park it in stable orbit.

Orbit of satellite is circular, elliptical or parabolic depending upon velocity of impart.

3. Critical velocity is the minimum horizontal velocity of projection for which a satellite performs circular motion round the earth.
4. Critical velocity of satellite depends upon the mass and radius of the Earth. If M and R are the mass and radius of earth and h is height of satellite from surface of earth then,

$$v_c = \sqrt{\frac{GM}{R+h}} = \sqrt{\frac{gR^2}{R+h}}$$

5. Square of the time period of satellite directly varies with cube of its orbital radius

$T^2 \propto (R+h)^3$.

6. J. Kepler proposed the basic Jaws of planetary motion which gives detailed picture of revolution of planets around the Sun.

According to his laws, the planet revolves in elliptical orbit and sweeps equal areas in equal interval of time. If T and R are the time period and radius of semi-major axis, then $T^2 \propto R^3$.

7. When the total energy of the satellite is zero, it will escape away from its orbit following a hyperbolic path.

8. When the height of a satellite is increased, its potential energy will increase and kinetic energy will decrease.
9. Binding energy is the minimum energy required to free a body from the influence of earth's gravitational field. For an orbiting satellite, $B.E = \frac{1}{2}\frac{GMm}{(R+h)}$
10. If a body is at rest on the earth's surface, B.E. is given by the formula, $B.E. = \frac{GMm}{R}$

and if it is stationary at height h then,

$$B.E. = \frac{GMm}{R+h}$$

11. Escape velocity is the minimum velocity with which a body should be projected from the surface of the earth, so that it escapes the gravitational influence of the earth. If M and R be the Mass and radius of the earth respectively, then escape velocity of the body is given by,

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$. It is $\sqrt{2}$ times the critical velocity. The value of escape velocity for the surface of earth is 11.2 km s^{-1}.

12. Value of g on the surface of earth is given by, $g = \frac{GM}{R^2}$
13. Value of g decreases with increase in altitude.

It is given by the formula, $g_h = g\left(\frac{R}{R+h}\right)^2$.

This formula can be applied as an approximation if $h << R$, as $g_h = g\left(1 - \frac{2h}{R}\right)$.

14. The value of g also decreases when we go inside the earth,

$g_d = g\left(1 - \frac{d}{R}\right)$. When d = R, then $g_4 = 0$.

This formula clearly shows that the value of g will be 0 at the centre of earth.

15. There is a change in value of g due to rotation of earth. It is given by,

$g' = g - R\omega^2\cos^2\phi$

where, ϕ is the angle with the horizontal.

Formlae :

1. Gravitational force between two bodies:

$$F = \frac{Gm_1m_2}{r^2}$$

2. Gravitational force between two equal masses: $F = F = \frac{Gm^2}{r^2}$

3. Gravitational constant: $G = F = \frac{Fr^2}{m_1m_2}$
4. Acceleration due to gravity:

i. On the earth surface, $g = \frac{GM}{R^2}$

ii. At a height h above earth,

$$g_h = \frac{GM}{(R+h)^2} = g\left(\frac{R}{r+h}\right)^2$$

iii. At a depth d below earth,

$$g_d = g\left(1 - \frac{d}{R}\right)$$

iv. At a latitude ϕ,

$g' = g - R\omega^2\cos^2\phi$

5. Critical velocity of satellite:

i. $v_c = \sqrt{\frac{GM}{R+h}} = \sqrt{g_h(R+h)}$

(at an altitude h)

ii. $v_c = \sqrt{\frac{GM}{R}} = \sqrt{gR}$

(close to the surface of earth)

6. period of satellite:

i. $T = 2\pi\sqrt{\frac{R}{g}}$ (Close to the earth)

ii. $T = 2n\sqrt{\frac{(R+h)^2}{GM}}$

(At height h from earth)

iii. $T\sqrt{\frac{3\pi}{\rho G}}$ (In terms of mean density)

7. Kepler's formula :

i. $T^2 \propto r^3$ i.e. $\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$

ii. Areal velocity, $\frac{dA}{dt}$ = constant

$$\frac{A_1}{t_1} = \frac{A_2}{t_2} \text{ or } \frac{A_1}{A_2} = \frac{t_1}{t_2}$$

8. Binding energy :

i. $B.E = \frac{GMm}{R}$

(Body is stationary on earth's surface)

ii. $B.E = \frac{GMm}{2(R+h)}$

(Body is revolving around the earth at certain height)

9. Potential energy of a body:

i. On the earth's surface,

$$P.E = \frac{-GMm}{R} = -mgR$$

ii. Revolving around the earth,

$$P.E = \frac{-GMm}{R+h}$$

10. Kinetic energy of a body:

i. Stationary on the earth, K.E = 0

ii. Revolving around the earth at certain height,

$$K.E = \frac{GMm}{2(R+h)}$$

11. Total energy of a body:

i. On the earth's surface, T.E. $= \frac{-GMm}{R}$

$= -mgR$

ii. Stationary at a height h from the earth's surface, T.E. $= \frac{-GMm}{(R+h)}$

iii. Revolving around the earth,

$$T.E = \frac{-GMm}{2(R+h)}$$

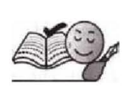
12. Escape velocity of a body:

i. On the earth's surface,

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

ii. $v_e = 2R\sqrt{\frac{2\pi\rho G}{3}}$

<u>Solved Prolems :</u>



Example 1

Calculate the force of attraction between two metal spheres each of mass 90 kg, if the distance between their centres is 40 cm. [G = 6.67 × 10 “ Nm¹/kg²]

Solution:

Given: $m_1 = 90$ kg, $m_2 = 90$ kg,

$r = 40$ cm $= 40 \times 10^{-2}$ m,

$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$

To find: Force of attraction (F)

Formula: $F = \dfrac{Gm_1m_2}{r^2}$

Calculation: From formula,

$$F = \frac{6.67 ´ 10^{-11} ´ 99 ´ 99}{(40 ´ 10^{-2})^2}$$

$$= 33.77 \times 10^{-4}$$

$$\mathbf{F = 3.377 \times 10^{-6}\ N}$$

Ans: The force of attraction between the two metal spheres is $\mathbf{3.377 \times 10^{-6}}$**.**

**Example 2*

Find the value of G from the following data:

[$M = 6 \times 10^{24}$ kg, R = 6400 km, $g = 9.774\ m/s^2$]

Given:

$M = 6 \times 10^{24}$ kg,

R = 6400 km = 64×10^5m**,**

$g = 9.774\ m/s^2$

To find : Gravitational constant (G)

Formula : $g = \dfrac{GM}{R^2}$

Calculation : From formula,

$$G = \frac{gR^2}{M}$$

$$G = \frac{9.7774 ´ (64 ´ 10^5)^2}{M}$$

$$= \frac{4.0034 ´ 10^{14}}{6 ´ 10^{24}}$$

\ $G = 6.672 \times 10^{-11}\ Nm^2/kg^2$

Ans: The value of gravitational constant is $\mathbf{6.672 \times 10^{-11}\ Nm^2/kg^2}$.

**** Example 3***

Two bodies of masses 5 kg and 6 × 10^{24} kg are placed with their centers 6.4 × 10^6 m apart. Calculate the force of attraction between the two masses. Also find the initial accelerations of the two masses. [Assume that no other forces act on them].

Solution:

Given: $m_1 = 5$ kg, $m_2 = 6 \times 10^{24}$ kg,

$r = 6.4 \times 10^6$m

To find: i. Force of attraction (F)

ii. Acceleration (a_1, a_2)

Formulae: i. $F = \dfrac{Gm_1m_2}{r^2}$ ii. F = ma

Calculation: From formula (i),

$$F = \frac{6.67 ´ 10^{-11} ´ 5 ´ 6 ´ 10^{24}}{(6.4 ´ 10^6)^2}$$

F = 48.85 N

From formula (ii),

$F = m_1a_1$

$$a_1 = \frac{F}{m_1} = \frac{48.85}{5}$$

\ $a_1 = 9.77\ m/s^2$

Also, $a_2 = \dfrac{F}{m_2} = \dfrac{48.85}{6 ´ 10^{24}}$

$a_2 = 8.142 \times 10^{-24}\ m/s^2$

i. The force of attraction between the two masses is **48.85 N.**

ii. The initial accelerations of the two masses are **9.77** m/s^2 and $\mathbf{8.142 \times 10^{24}\ m/s^2}$**.**

Example 4

Calculate the acceleration due to gravity at the surface of the earth from the given data.

[Mass of the earth = 6 x 10^{24} kg,

Radius of the earth = 6.4 × 10^6 m,

G = 6.67 × 10^{-11} Nm^2/kg^2]

Solution:

Given: $M = 6 \times 10^{24}$ kg,

$R = 6.4 \times 10^6$ m,

$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$

To find: acceleration due to gravity (g)

Formula: $g = \dfrac{GM}{R^2}$

Calculation : From formula,

$$g = \frac{6.67 ´ 10^{-11} ´ 6 ´ 10^{24}}{(6.4 ´ 10^6)^2}$$

$$= \frac{40.02 ´ 10^{13}}{40.96 ´ 10^{12}} = \frac{40.02}{40.96} ´ \ 10$$

g = **9.77 m/s²**

Ans: The acceleration due to gravity at the surface of the earth is **9.77 m/s².**

Example 5

Find the orbital speed of the satellite when satellite is revolving round the earth in circular orbit at a distance 9 × 10⁶ m from its centre.

[Given: Mass of earth = 6 × 10²⁴ kg,
G = 6.67 × 10⁻¹¹ Nm²/kg²]

Solution:

Given: $r = 9 \times 10^6$ m, $M = 6 \times 10^{24}$ kg,
$G = 6.67 \times 10^{-11}$ Nm²/kg²

To find: orbital speed (v_c)

Formula: $v_c = \sqrt{\frac{GM}{r}}$

Calculation: From formula,

$$v_c = \sqrt{\frac{6.67 ´ 10^{-11} ´ 6 ´ 10^{24}}{9 ´ 10^{6}}}$$

$$= \sqrt{\frac{40.02 ´ 10^{7}}{9}} = \sqrt{\frac{400.2}{9} ´ 10^{6}}$$

$$v_c = \sqrt{44.46} ´ 10^{3}$$

v_c = **6.668 × 10³ m/s**

Ans: The orbital speed of the satellite is **6.668 × 10³ m/s.**

Example 6

Taking radius of the earth as 6400 km and g at the earth's surface as 9.8 m/s², calculate the speed of revolution of a satellite orbiting close to the earth's surface.

Solution:

Given: $R = 6400$ km $= 6.4 \times 10^6$ m,
$g = 9.8$ n

To find: Critical velocity (v_c)

Formula: $v_c = \sqrt{gR}$

Calculation: From formula,

$$v_c = \sqrt{9.8 ´ 6.4 ´ 10^{6}}$$

$v_c = 7.92 \times 10^3$ m/s

Ans: The speed of revolution of the satellite orbiting close to the earth's surface is **7.92 × 103 m/s.**

****Example 7***

Assuming the earth to be a homogeneous sphere, determine the density of the earth from following data, [g = 9.8 m/s²,
G = 6.673 × 10⁻¹¹ Nm²/kg, R = 6400 km]

Solution:

Given: $g = 9.8$ m/s²,
$G = 6.672 \times 10^{-11}$ Nm²/kg²
$R = 6400$ km $= 6.4 \times 10^6$ m

To find: density (r)

Formula: $g = \frac{4}{3} pRr\,G$

Calculation: From formula,

$$r = \frac{3g}{4pRG}$$

$$r = \frac{3 ´ 9.8}{4p ´ 6.4 ´ 10^{6} ´ 6.673 ´ 10^{-11}}$$

r = **5478 kg/m**

Ans: The density of the earth is **5478 kg/m³.**

Example 8

The mean orbital radius of the earth around the sun is 1.5 × 10ˢ km. Estimate the mass of the sun. (NCERT)

Solution:

Given: $r = 1.5 \times 10^8 \times 10^3$m ,
$T = 365$ days $= 365 \times 24 \times 60 \times 60$ s

To find: mass (M)

Formula: $T = 2p\sqrt{\frac{r^2}{GM}}$

Calculation: From formula,

$$M = \frac{4p^2 r^3}{GT^2}$$

$$= \frac{4 ´ (3.14)^2 (1.5 ´ 10^{11})^3}{(6.67 ´ 10^{-11})(365 ´ 25 ´ 60 ´ 60)^2}$$

$$M = 2.01 \times 10^{30}\ \text{kg}$$

Ans: The mass of the sun is **2.01×10^{30} kg.**

[Trick: To 'weigh the sun', i.e., estimate its mass, one needs to know the period of one of its planets and the radius of the planetary orbit.]

Example 9

A saturn year is 29.5 times the earth's year. How far is the Saturn from the sun if the earth is 1. 50 × 10^8 km away from the sun? (NCERT)

Solution:

Given: $T_s = 29.5T_E$, $r_E = 1.50 \times 10^8$km

To find: semi-major axis (r_s)

Calculation: From formula,

Formula : $\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$

Calculation : From formula,

$$\left(\frac{T_S}{T_E}\right)^2 = \left(\frac{r_S}{r_E}\right)^3$$

$$\therefore \quad r_S = r_E\left(\frac{T_S}{T_E}\right)^{2/3} = (1.5 \times 10^8)\left(\frac{29.5}{1}\right)^{2/3}$$

Ans: Saturn is **14.32×10^8 km** away from the sun.

*** Example 10***

What should be the duration of the year if the distance between the earth and the sun gets doubled ?

Solution:

Given: $T_1 = 365$ days, $r_2 = 2r_1$

$$\therefore \quad \frac{r_2}{r_1} = 2$$

To find: Time period (T_2)

Formula: $\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{r_2}{r_1}\right)^3$

Calculation : From formula,

$$\therefore \quad \frac{T_2}{365} = \sqrt{8}$$

$$\therefore \quad T_2 = 2.828 \times 365$$

$$T_2 = \mathbf{1032\ days}$$

Ans: The duration of the year should be **1032 days.**

Example 11

A satellite orbits around the earth at a height equal to R of the earth. Find its period. [R = 6.4 × 10^6 m, g = 9.8 m/s^2]

Solution:

Given: $h = R = 6.4 \times 10^6$ m, $g = 9.8$ m/s^2

To find: Time period (T)

Formula: $T = 2\pi\sqrt{\frac{(R+h)^2}{GM}}$ ..... (i)

Calculation: Since $GM = gR^2$,

Formula (i) becomes,

$$T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}$$

$$= 2\pi\sqrt{\frac{(2R)^3}{gR^2}}\,2\pi\sqrt{\frac{8R^3}{gR^2}}$$

$$= 4 \times 3.14\sqrt{\frac{2 \times 6.4 \times 10^6}{9.8}}$$

$$= 12.56 \times 10^3\sqrt{\frac{12.8}{9.8}}$$

$$\therefore \quad T = 1.435 \times 10^4\ \text{s}$$

Ans: The time period of the satellite is **1.435×10^4 s.**

****Example 12***

The distances of two planets from the sun are 10^{13} m and 10^{12} m respectively. Find the ratio of time periods and orbital speeds of the two planets.

Solution:

Given: $r_1 = 10^{13}$m, $r_2 = 10^{12}$m

To find: i. Ratio of time period $\left(\frac{T_1}{T_2}\right)$

ii. Ratio of orbital speed $\left(\frac{V_{C_1}}{V_{C_2}}\right)$

Formulae'. i. $\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$ ii. $v_c = \sqrt{\frac{GM}{r}}$

Calculation: From formula (i),

$$\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2} = \left(\frac{10^{13}}{10^{12}}\right)^{3/2} = 10^{3/2}$$

$$\therefore \quad \frac{T_1}{T_2} = 31.62 : 1$$

From formula (ii),

$$v_{c_1} = \sqrt{\frac{GM}{r_1}}$$

and $v_{c_2} = \sqrt{\frac{GM}{r_2}}$

$$\therefore \quad \frac{v_{c_1}}{v_{c_2}} = \sqrt{\frac{r_2}{r_1}} = \sqrt{\frac{10^{12}}{10^{13}}} = \frac{1}{\sqrt{10}}$$

$$\therefore \quad \frac{v_{c_1}}{v_{c_2}} = 0.3162 : 1$$

Ans: The ratio of time periods and orbital speeds of the two planets are **31.62 : 1** and **0.3162 : 1** respectively.

Example 13

What would have been the duration of the year if the distance between the earth and the sun were half the present distance?

Solution:

Given: $T_1 = 365$ days, $r_2 = \frac{r_1}{2}$

$$\therefore \quad \frac{r_2}{r_1} = \frac{1}{2}$$

To find: Time period (T_2)

Formula: $\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{r_2}{r_1}\right)^3$

Calculation: From formula,

$$\left(\frac{T_2}{365}\right)^2 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

$$\therefore \quad T_2 = 365 \times \frac{1}{\sqrt{8}}$$

$$= 365 \times 0.3536$$

$$= 129.064$$

$\therefore$ T_2= 129 **days**

Ans: The duration of the year would be 129 **days.**

Example 14

The radii of orbits of two satellites revolving around the earth are in the ratio 3 : 8. Compare their

i. critical speeds and

ii. periods of revolution.

Solution:

Given: $\frac{r_1}{r_2} = \frac{3}{8}$

To find : i. Ratio of critical speed $\left(\frac{v_{c_1}}{v_{c_2}}\right)$

ii. Ratio of time period $\left(\frac{T_1}{T_2}\right)$

Formulae: i. $v_2 = \sqrt{\frac{GM}{r}}$

ii. $\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$

Calculation : From formula (i),

$$v_{c_1} = \sqrt{\frac{GM_1}{r_1}} \text{ and } v_{c_2} = \sqrt{\frac{GM_2}{r_2}}$$

$$\therefore \quad \frac{v_{c_1}}{v_{c_2}} = \frac{\sqrt{\frac{GM_1}{r_1}}}{\sqrt{\frac{GM_2}{r_2}}}$$

$$\therefore \quad \frac{v_{c_1}}{v_{c_2}} = \sqrt{\frac{M_1}{M_2} \times \frac{r_2}{r_1}}$$

$\therefore$ Both satellites orbit same planet i.e. earth

$\therefore$ $M_1 = M_2 = M$

$$\therefore \quad \frac{v_{c_1}}{v_{c_2}} = \sqrt{\frac{M_1}{M_2} \times \frac{r_2}{r_1}} = \sqrt{\frac{r_2}{r_1}} = \sqrt{\frac{8}{3}}$$

$\therefore \quad \frac{v_{c_1}}{v_{c_2}} = 1.633:1$

From formula (ii),

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{3}{8}\right)^3$$

$$\frac{T_1}{T_2} = \left(\frac{3}{8}\right)^{3/2}$$

$\therefore \quad T_1 : T_2 = 0.2296 : 1$

Ans: The ratio of critical speeds and time periods of the two satellites are **1.633 :1** and **0.2296 :1** respectively.

Example 15

The planet Mars has two moons, phobos and delmos.

i. Phobos has a period 7 hours 39 minutes and an orbital radius of 9.4 × 10³ km. Calculate the mass of mars.

ii. Assume that earth and mars move in circular orbits around the sun with the martian orbit being 1.52 times the orbital radius of the earth, What is the length of martian year in days? (NCERT)

Solution:

Given: T_P = 7 hrs 39 min. = 459 min.
= 459 × 60 = 2.754×10^4 s,
R = 9.4×10^3 km = 9.4×10^6 m,
$r_M = (1.52)r_E$, T_E = 365 days

To find: i. Mass of mars (M_m)
ii. Martian year (T_M)

Formulae: i. $T^2 = \frac{4\pi^2}{GM} r^3$

ii. $\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$

Calculation : i. From formula (i),

$$M_M = \frac{4\pi^2}{G}\frac{r^2}{T^2}$$

$$= \frac{4 \times (3.14)^2 \times (9.4 \times 10^6)^2}{6.67 \times 10^{-11} \times (2.754 \times 10^4)^2}$$

$$= \frac{4 \times (3.14)^2 \times (9.4)^2 \times 10^{18}}{6.67 \times (2.754)^2 \times 10^{-3}}$$

$\therefore \quad M_M = 6.48 \times 10^{23}$ kg

ii. From formula (ii),

$$\left(\frac{T_M}{T_E}\right) = \left(\frac{r_M}{r_E}\right)^{3/2}$$

$$\therefore \quad T_M = \left(\frac{1.52 r_E}{r_E}\right)^{3/2} T_E$$

$\therefore \quad T_M$ = **684 days**

Ans: i. The mass of mars in **6.48 × 10²³ kg**.

ii. The length of martian yera is **684 days**.

Example 16

The critical velocity of a satellite revolving around the earth is 10 km/s at a height where $g_h = 8$ m/s². Calculate the height of the satellite from the surface of the earth. [R = 6.4 x 10⁶ m]

Solution:

Given: v_c = 10 km/s = 10×10^3 m/s,
g_h = 8 m/s²,
R = 6.4×10^6 m

To find : Height of the satellite (h)

Formula: $v_c\sqrt{\frac{GM}{R+h}}$

Calculation: Since $gh = \frac{GM}{(R+h)^2}$

From formula,

$$v_c\sqrt{\frac{g_h(R+h)^2}{(R+h)}}$$

$\therefore \quad 10 \times 10^6 = \sqrt{8 \times (R+h)}$

Squaring both sides we get,

$100 \times 10^6 = 8(R + h)$

$\therefore \quad 8(R + h) = 100 \times 10^6$

$R + h = \frac{100}{8} \times 10^6$

$h = 12.5 \times 10^{-6} - R$

$= 12.5 \times 10^6 - 6.4 \times 10^6$

$= 6.1 \times 10^6$ m

h = **6100 km**

Ans: The height of the satellite from the surface of the earth is **6100 km.**

*Example 17

Show that the critical velocity of a body revolving very close to the surface of a planet of radius R and mean density p is 2R

Solution:

To show that: $v_c = 2R\sqrt{\dfrac{\pi\rho G}{3}}$

Proof : Since the body is revolving very close to the surface of a planet,

h << R

R = radius of planet

ρ = mean density of planet

Critical velocity of a body very close to earth is given by,

$$v_c = \sqrt{\frac{GM}{R}} \qquad \text{...... (i)}$$

$$\text{Also, } M = V_\rho = \frac{4\pi R^3 \rho}{3} \qquad \text{...... (ii)}$$

From equations (i) and (ii), we have,

$$v_c = \sqrt{\frac{\left(\dfrac{4\pi R^3\rho}{3}\right)}{R}}\sqrt{\frac{2^2R^2(\pi\rho G)}{3}}$$

$$\therefore \quad v_c = 2R\sqrt{\frac{\pi\rho G}{3}}$$

Example 18

You are given the following data:

g = 9.81 ms⁻², R_E = 6.37 × 10⁶ m, the distance to the moon R = 3.84 × 10⁸ m and the time period of the moon's revolution is 27.3 days. Obtain the mass of the earth M_E in two different ways.

[G = 6.67 × 10⁻¹¹ Nm²/kg²] (NCERT)

Solution:

Given: $g = 9.81\ ms^{-2}$,

$R_E = 6.37 \times 10^6$ m,

$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$

$R_M = 3.84 \times 10^8$m,

T_M = 27.3 days = 27.3 × 24 × 60 × 60

To find: Mass of earth (M_E)

formula : i. $g_E = \dfrac{GM_E}{R_E^2}$

ii. $T^2 = \dfrac{4\pi^2R^3}{GM_E}$

Calculation : From formula (i),

$$M_E = \frac{g_E R_E^2}{G}$$

$$M_E = \frac{9.81 \times (6.37 \times 10^6)^2}{6.67 \times 10^{-11}}$$

$M_E = 5.97 \times 10^{24}$ kg

From formula (ii),

$$M_E = \frac{4\pi^2R^3}{GT^2}$$

$$= \frac{4 \times (3.14)^2 \times (3.84)^3 \times 10^{24}}{6.67 \times 10^{-11} \times (27.3 \times 24 \times 60 \times 60)^2}$$

M_E = **6.02 x 10²⁴ kg**

Ans: The mass of the earth calculated from the two methods is **5.97 × 10²⁴ kg** and **6.02 × 10²⁴ kg.** (Both methods yield almost the same answer, the difference between them being less than 1**%.)**

**** Example 19***

A body weighs 4.0 kg wt on the surface of the Earth. What will be its weight on the surface of a plant whose mass is $\frac{1}{8}$ th of the mass of the Earth and radius half $\left(\frac{1}{2}\right)$ of that of the Earth ?

Solution:

Given: W_e = 4.0 kg wt.,

$$\frac{M_p}{M_e} = \frac{1}{8}, \quad \frac{R_p}{R_e} = \frac{1}{2}$$

To find : Weight (W_p)

Formula : $\dfrac{W_p}{W_e} = \dfrac{M_p}{M_e} \times \dfrac{R_e^2}{R_p^2}$

Calculation: From formula,

$$\frac{W_p}{4} = \frac{1}{8} \times \left(\frac{2}{1}\right)^2 = \frac{1}{2}$$

$$\therefore \quad W_p = \frac{1}{2} \times 4 = 2\text{kg - wt}$$

Ans: Weigh of the body on the surface of a planet will be **2 kg-wt.**

***Example 20**

Calculate the height of the communication satellite.

[Given: G = 6.67 × 10^{-11} Nm²/kg², M = 6 × 10^{24} kg, R = 6400 km] [Mar 05]

Solution:

Given: T = 24 × 60 × 60s

G = 6.67 × 10^{-11} Nm²/kg²,

M = 6 × 10^{24} kg,

To find: Height (h)

Formula: $T = 2\pi\sqrt{\frac{r^2}{GM}}$

Calculation: From formula,

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

$$\therefore \quad r^3 = \frac{T^2 GM}{4\pi^2}$$

$$\therefore \quad (R+h)^3 = \frac{T^2 GM}{4\pi^2} \qquad (\because\ r = R + h)$$

$$= \frac{(24 \times 60 \times 60)^2 \times (6.67 \times 10^{11}) \times (6 \times 10^{24})}{4 \times (3.14)^2}$$

$= 75.74 \times 10^{21}$

$(R + h) = \sqrt[3]{75.74 \times 10^{21}}$

$= 4.231 \times 10^7$m

$(R + h) = 42.31 \times 10^6$ m

$\therefore$ h = $42.31 \times 10^6 - R$

$\therefore$ h = $42.31 \times 10^6 - 6.4 \times 10^6$

$\therefore$ h = **35.91 × 10^6** m

$\therefore$ h = **35910 km**

Ans: The height of the communication satellite is **35910 km.**

Example 21

Calculate the B.E. of a satellite of mass 2000 kg moving in an orbit very close to the surface of the earth.

[G *m* 6.67 × 10^{11} Nm²/kg²]

Solution:

Given: m = 2 x 10^3 kg, R = 6.4 × 10^6 m,

R = **6.4** × **10^6** m, M = **6** × **10^{24}** kg,

G = **6.67** × **10^{-11}** Nm²/kg²

M = 6 × 10^{24} kg

To find: Binding Energy (B.E.)

Formula: For satellite very close to earth,

$$\text{B.E.} = \frac{1}{2} \times \frac{GMm}{R}$$

Calculation : From formula.

$$\text{B.E.} = \frac{1}{2} \times \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 2 \times 10^3}{6.4 \times 10^6}$$

$\therefore$ B.E. = **6.25 × 10^{10}joule**

Ans: The binding energy of the satellite is **6.25 × 10^{10} joule.**

*** Example 22***

Find the binding energy of a body of mass 50 kg at rest on the surface of the earth.

[Given: G = 6.67 × 10^{11} Nm²/kg², R = 6400 km, M = 6 × 10^{24} kg]

Solution:

Given: G = 6.67 × 10^{-11} Nm²/kg²,

R = 6400 km = 6.4 × 10^6m,

M = 6 × 10^{24} kg,

m = 50 kg

To find: Binding energy (B.E.)

Formula: $\text{B.E.} = \frac{GMm}{R}$

Calculation: From formula,

$$\text{B.E.} = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 50}{6.4 \times 10^6}$$

B.E. = **3.127 × 10^9J**

Ans: The binding energy of the body is **3.127 × 10^9 J.**

Example 23

Find the total energy and binding energy of an artificial satellite of mass 1000 kg orbiting at height of 1600 km above the earth's surface. [Given: G = 6.67 × 10^{-11} Nm^2/kg^2, R = 6400 km, M = 6 × 10^{24}kg]

Solution:

Given: h = 1600 km = 1.6 × 10^6 m,

G = 6.67 × 10^{24} Nm^2/kg^2,

R = 6400 km = 6.4 × 10^6 m,

m = 1000 kg,

M = 6 × 10^{24}kg

To find: i. Total Energy (T.E.)

ii. Binding Energy (B.E.)

Formulae: i $T.E. = \frac{GMm}{2(R+h)}$

ii. B.E. = –T.E.

Calculation: From formula (i),

$$T.E. = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 1000}{2(6.4+1.6) \times 10^6}$$

$$= \frac{40020 \times 10^7}{2 \times 8}$$

T.E. = –2.501 × 10^{10} J

From formula (ii),

B.E. = 2.501 × 10^{10} J

Ans: The total energy and binding energy of the artificial satellite are –2.501 × 10^{10} J and 2.501 × 10^{10} J respectively.

Example 24

Find the kinetic energy of an artificial satellite of mass 200 kg, orbiting the earth at a height of 1600 km above the earth's surface. Also calculate the potential energy, total energy and binding energy.

[Given: M_e = 6 × 10^{24} kg, R = 6.4 × 10^6 m, G = 6.67 × 10^{-11} Nm^2/kg^2]

Solution:

Given: m = 200 kg = 2 x 10^2 kg,

h = 1600 km,

R = 6400 km

R + h = 6400 + 1600

= 8000 km = 8 × 10^6 m

M = 6 × 10^{24}kg,

G = 6.67 × 10^{-11} Nm^2/kg^2

To find: i. Kinetic Energy (K.E.)

ii. Potential Energy (P.E.)

iii. Total Energy (T.E.)

iv. Binding Energy (B.E.)

Formulae: i. $K.E. = \frac{GMm}{2(R+h)}$

ii. P.E. = –2 K.E.

iii. T.E. = – K.E.

iv. B.E. = – T.E.

Calculation: From formula (i),

T.E. = –2.501 × 10^{10} J

From formula (ii),

B.E. = 2.501 x 10^{10} J

Ans: The total energy and binding energy of the artificial satellite are –2.501 × 10^{10} J and 2.501 × 10^{10} J respectively.

Example 24

Find the kinetic energy of an artificial satellite of mass 200 kg, orbiting the earth at a height of 1600 km above the earth's surface. Also calculate the potential energy, total energy and binding energy. [Given: M_e = 6 × 10^{24} kg, R = 6.4 × 10^6 m, G = 6.67 × 10^{-11} Nm^2/kg^2]

Solution:

Given: m = 200 kg = 2 × 10^2 kg, h = 1600 km,

R = 6400 km

R + h = 6400 + 1600

= 8000 km

= 8 × 10^6 m

M = 6 × 10^{24}kg,

G = 6.67 x 10"u Nm^2/kg^2

To find: i. Kinetic Energy (K.E.)

ii. Potential Energy (P.E.)

iii. Total Energy (T.E.)

iv. Binding Energy (B.E.)

Formulae:

i. $K.E. = \frac{GMm}{2(R+h)}$

ii. P.E. = –2 K.E.

iii. T.E. = – K.E.

iv. B.E. = – T.E.

From formula (i),

$$\text{K.E.} = \frac{6.67\,´\,10^{-11}\,´\,6\,´\,10^{24}\,´\,2\,´\,10^{2}}{2\,´\,8\,´\,10^{6}}$$

K.E. = 5.0025×10^9 J

From formula (ii),

P.E. = –2 (5.0025×10^9)

P.E. = –10.005×10^9 J

From formula (iii),

T.E. = – (5.0025×10^9)

T.E. = –5.0025×10^9 J From formula (iv),

B.E. = 5.0025×10^9 J

Ans:

i. The kinetic energy of the artificial satellite is 5.0025×10^9 J.

ii. The potential energy of the artificial satellite is –10.005×10^9 J.

iii. The total energy of the artificial satellite is –5.0025×10^9 J.

iv. The binding energy of the artificial satellite is 5.0025×10^9 J.

Example 25

If g = 9.8 m/s² on the surface of the earth, find its value at h = $\frac{R}{2}$ from the surface of the earth. 2

Solution :

Given: g = 9.8 m/s², h = $\frac{R}{2}$

To find: Acceleration due to gravity (g_h)

Formula: $\frac{g_h}{g} = \frac{R^2}{(R+h)^2}$

Calculation: From formula,

$$\frac{g_h}{g} = \frac{R^2}{\text{æ}R + \frac{R}{2}\text{ö}^2} = \frac{R^2}{\frac{9R^2}{4}} = \frac{4}{9}$$

\ $g_h = \frac{4}{9}\,´\,g = \frac{4}{9}\,´\,9.8$

g_h **= 4.35 m/s² R**

Ans: At h = $\frac{R}{2}$ from the surface of the earth, the value of g is 4.35 m/s².

****Example 26***

Calculate the escape velocity of a body from the surface of the earth. [Average density of earth = 5.5 x 10³ kg/m³, G = 6.67 × 10 ⁿ Nm²/kg², radius of earth R = 6.4 × 10⁶ m] [Oct 97]

Solution:

Given: r = 5.5×10^3 kg/m³,

R = 6.4×10^6 m,

G = 6.67×10^{-11} Nm²/kg²

To find: **Escape velocity (v_e)**

Formula: $v_e = 2R\sqrt{\frac{2pr\,G}{3}}$

Calculation: From formula.

$$v_c = 2\,´\,6.4\,´\,10^6\sqrt{\frac{2\,´\,3.142\,´\,10^3\,´\,6.67\,´\,10^{-11}}{3}}$$

$= 2 \times 6.4 \times 10^6 \times 8.759$

$v_e = 11.22 \times 10^3$ m/s

Ans: The escape velocity of a body is **11.22** × **10³** m/s.

Example 27

The escape velocity of a body from the surface of the earth is 11.2 km/s. Calculate the mean density of the earth.

[M = 6 × 10²⁴ kg, R = 6.4 × 10⁶ m, G = 6.67 × 10⁻¹¹ Nm²/kg²l

Solution :

Given : v_e = 11.2 km/s = 11.2×10^3 m/s,

M = 6×10^{24} kg, R = 6.4×10^6 m,

G = 6.67×10^{-11} Nm²/kg²

Find : Mean density (r)

Formula : $v_e = 2R\sqrt{\frac{2pr\,G}{3}}$

Calculation : From formula,

$$v_e^2 = \frac{8}{3}pR^2r\,G$$

\ $r = \frac{3v_e^2}{8\,´\,p\,´\,R^2\,´\,G}$

$$= \frac{3 \times (11.2 \times 10^3)^2}{8 \times 3.14 \times (6.4 \times 10^6)^2 \times 6.67 \times 10^{-11}}$$

$$= \frac{3 \times (11.2)^2 \times 10^6}{8 \times 3.14 \times (6.4)^2 \times 10^{12} \times 6.67 \times 10^{-11}}$$

$$= \frac{376.32 \times 10^6}{6862.87 \times 10} = 0.05483 \times 10^5$$

ρ = 5483 kg/m3

∴ The mean density of earth is **5483 kg/m³**.

Example 28

If the earth were made of wood, the mass of wooden earth is 10% as much as it is now, without the change in its diameter. Calculate the escape velocity of spaceshot from the surface of wooden earth.

Radius of earth: R = 6400 km.,

Mass of earth: M = 6 × 10^{24} kg.,

Constant of gravitation:

G = 6.67 x 10^{-11} Nm²/kg². [Mar12]

Solution:

Given: $M_w = 10\ \%\text{ of } M = \frac{M}{10}$,

$D_w = D$ or $R_w = R$,

$R = 6400 \times 10^3$ m, $M = 6 \times 10^{24}$ kg,

$G = 6.67 \times 10^{-11}$ Nm²/kg²

To find: Escape velocity (v_e)

Formula : $v_c = \sqrt{\frac{2GM}{R}}$

Calculation: From formula

$$\therefore \quad v_{e_w} = \sqrt{\frac{2GM}{R}}$$

$$\therefore \quad \frac{v_{e_w}}{v_e} = \sqrt{\frac{2GM_w}{R_w} \times \frac{R}{2GM}}$$

$$\therefore \quad \frac{v_{e_w}}{v_e} = \sqrt{\left(\frac{R}{R_w}\right) \times \left(\frac{M_W}{M}\right)} = \sqrt{1 \times \frac{1}{10}} = \sqrt{\frac{1}{10}}$$

$$\therefore \quad \frac{v_{e_w}}{v_e} = v_e \times \frac{1}{\sqrt{10}} = \sqrt{\frac{2GM}{10R}}$$

$$= \sqrt{\frac{2 \times 6.67 \times 10^{-10} \times 6 \times 10^{24}}{10 \times 6400 \times 10^3}}$$

$$= \sqrt{\frac{80.04 \times 10^{13}}{6400 \times 10^5}} = \sqrt{\frac{8004 \times 10^{11}}{6400 \times 10^5}}$$

$$= \sqrt{\frac{8004}{640}} \times 10^3$$

= 3.536 × 103 m/s ≈ 3.54 × 10^3 m/s

Ans: The escape velocity of the spaceshot from the surface of wooden earth is **3.54 km/s.**

Example 29

Find the value of g at a height of 400 km above the surface of earth.

[Given: R = 6400 km, g = 9.8 m/s²]

Solution:

Given: h = 400 km = 4 × 10^5 m,

R = 6400 km = 6.4 × 10^6 m,

g = 9.8 m/s²

To find : Acceleration due to gravity (g_h)

Formula: $g_h = g\left(\frac{R}{R+h}\right)^2$

Calculation: From formula,

$$g_h = 9.8\left(\frac{6.4 \times 10^6}{6.4 \times 10^6 + 4 \times 10^5}\right)^2$$

$$g_h = 9.8\left(\frac{6.4 \times 10^6}{6.8 \times 10^6}\right)^2 = 9.8 \times 0.886$$

∴ $\mathbf{g_h}$ **= 8.6828 m/s**

Ans: The value of g at the height of 400 **km** above the surface of earth is **8.6828 m/s².**

Example 30

At what distance above the earth's surface and at what distance below the earth's surface, is the acceleration due to gravity less by 10% of its value at the surface? [Given: Radius of the earth = 6400 km]

[Mar 10]

Solution:

Given: $g_h = 90\ \%\text{ of } g = \frac{9g}{10}$

$$g_d = 90 \text{ of } g = \frac{9g}{10}$$

To find: Height (h), Depth (d)

Formulae; i. $g_h = g\left(\frac{R}{R+h}\right)^2$

ii. $g_d = g\left(1 - \frac{d}{R}\right)$

Calculation: From formula (i),

$$\frac{9g}{10} = g\left(\frac{R}{R+h}\right)^2$$

$$\therefore \left(\frac{R}{R+h}\right)^2 = 0.9$$

$$\therefore \frac{R}{R+h} = 0.948$$

$$R = 0.948\,R + 0.948\,h$$

$$h = \frac{0.52R}{0.948} = \frac{0.052 \times 6400 \times 10^3}{0.948}$$

$$\therefore h = 351.055 \times 10^3 m$$

$\therefore$ h = **351.055 km**

From formula (ii),

$$\frac{g}{10} = g\left(1 - \frac{d}{R}\right)$$

$$\therefore 1 - \frac{d}{r} = 0.9$$

$$\therefore \frac{d}{r} = 0.1 \text{ or } d = 0.1\,R = 0.1 \times 6400 \text{ km}$$

d = 640 km

Ans: The accelertion due to gravity is less by 10 % of its value at the surface, at a distance of 351.055 km above the earth's surface and **640 km** below the earth's surface.

Example 31

A body weighs 63 N on the surface of the earth. What is the gravitational force on it due to the earth at a height equal to half the radius of theearth?

Solution:

Given : W = mg = 63 N, h = R/2

To find: Gravitational force (F_h)

Formula: $g_h = g\left[\frac{R}{R+h}\right]^2$

Calculation: Since $F_h = mg_h$,

From formula,

$$F_h = \frac{mgR^2}{(R+h)^2}$$

$$= \frac{63 \times R^2}{\left(R + \frac{R}{2}\right)^2} = \frac{63 \times R}{\frac{9R^2}{4}} = 28N$$

$\mathbf{F_h = 28\ N}$

Ans: The gravitational force acting on the body is **28 N.**

Example 32

Assuming the earth to be a sphere of uniform mass density, how much would a body weigh half way down to the centre of the earth if it weighed 250 N on the surface? (NCERT)

Solution:

Given: W = mg = 250 N, $d = \frac{R}{2}$

To find: Weight (W_d)

Formula: $g_d = g\left[1 - \frac{d}{R}\right]$

$$\therefore W_d = 250 \times \left[1 - \frac{\left(\frac{R}{2}\right)}{R}\right]$$

$$= 250 \times \frac{1}{2} = 125$$

$$\therefore W_d = 125 \text{ N}$$

Ans: The body would weigh **125** N half way down to the centre of the earth.

****Example 33***

The mass of body on surface of the earth is 100 kg. What will be its

i. mass and

ii. weight at an altitude of 1000 km?
(R = 6400 km, g = 9.8 m/s²)

Solution:

Given: m = 100 kg, R = 6400 km = 6.4×10^6m
$g = 9.8\ m/s^2$, h = 1000 km = 10^6m

To find: i. Mass (m_h) ii. Weight (W_h)

Formula: $g_h = g\left(1 - \frac{2h}{R}\right)$

Calculation:

i. Mass of the body does not depend on height,

$\therefore$ m_h = 100 kg

ii. From formula,

$$g_h = g\left(1 - \frac{2 \times 10^6}{6.4 \times 10^6}\right)$$

g_h = 9.8 (1 – 0.3125)

g_h = 6.738

Since, $W_h = mg_h$

= 100 × 6.738

W_h = **673.8** N

Ans: The mass and weight of the body at an altitude of 1000 km are **100 kg** and **673.8** N respectively.

Example 34

The acceleration due to gravity at a depth 'd' reduces to 25% from the value at the surface. Calculate the value of 'd' if radius of earth is 6400 km.

Solution :

Given : $g_d = 25\ \%\ \text{of}\ g = \frac{25}{100} \times g = \frac{g}{4}$,

R = 6400 km = 6.4×10^6 m

To find : Depth (d)

Formula : $g_d = g\left(1 - \frac{d}{R}\right)$

Calculation : From formula,

$$\frac{g}{4} = g\left(1 - \frac{d}{R}\right)$$

$$\frac{1}{4} = 1 - \frac{d}{R}$$

$\therefore$ $\frac{d}{R} = 1 - \frac{1}{4} = \frac{3}{4}$

$\therefore$ $d = \frac{3}{4} \times R$

$\therefore$ $d = \frac{3}{4} \times 6.4 \times 10^6$

$\therefore$ $d = 3 \times 1.6 \times 10^6$

$= 4.8 \times 10^6$ m

d = **4800** km

Ans: At a depth of **4800** km, the acceleration due to gravity reduces to 25% from the value at the surface.

Example 35

If the earth were a perfect sphere of radius 6.4×10^6 m rotating about its axis with the period of one day (8.64×10^4 s), what is the difference in acceleration due to gravity from poles to equator?

Solution:

Given: R = 6.4×10^6 m, T = 8.64×10^4 s

To find: Difference in acceleration due to gravity ($g_P - g_E$)

Formula: $g' = g - R\omega^2 \cos^2\phi$

Calculation: Since $\omega = \frac{2\pi}{T}$,

$$\omega = \frac{2 \times 3.14}{8.64 \times 10^4} = \frac{6.28}{8.64 \times 10^4}$$

$= 0.7268 \times 10^{-4}$

$\omega = 7.268 \times 10^{-5}$ rad/s

At poles, $\phi = 90°$

From formula,

$g_p = g - R\omega^2\cos^2 90°$

$= g - 0$ [$\because \cos 90° = 0$]

$g_p = g$

At equator, $\theta = 0°$,

$\therefore$ $g_E = g - R\omega^2\cos^2\theta$

$g_E = g - R\omega^2$ ....(ii)

Subtracting equation (ii) from equation (i), we have,

$g_p - g_E = g - (g - Rw^2)$

$g_p - g_E = Rw^2$

$= 6.4 \times 10^6 \times (7.268 \times 10^{-5})^2$

$= 6.4 \times 10^6 \times 52.82 \times 10^{-10}$

$= 338 \times 10^{-4}$

\ $g_p - g_E = \mathbf{3.38 \times 10^{\,2}\ m/s^2}$

Ans: The difference in acceleration due to gravity from poles to equator is **3.38×10^{-2} m/s .**

Example 36

The Earth is rotating with angular velocity co about its own axis. R is the radius of the Earth.

If Rod² = 0.03386m/s², calculate the weight of a body of mass 100 gram at lattitude 25°. (g = 9.8 m/s²) [Feb 2013]

Solution:

Given: $Rw^2 = 0.03386 m/s^2$, $f = 25°_v$

$m = 0.1$ Kg, $g = 9.8\ m/s^2$

To find: Weight (W)

Formula: i. $g' = g - Rw^2 \cos^2 w$

ii. $W = mg$

Calculation: From formula (i),

$g' = 9.8 - [0.03386 \times (0.9063)^2]$

$g' = 9.8 - 0.02781$

$g' = 9.772\ m/s^2$

From formula (ii),

$W = 0.1 \times 9.772$

W = **0.9772** N

Ans: The weight of a body at lattitude 25° is 0.9772 N.

EXERCISE

Section A: Practice Problems

1. Distance of a planet from the earth is 2.5×10^7 km and the gravitational force between them is 3.82×10^8 N. Mass of the planet is equal to that of earth, each being 5.98×10^{24} kg. Find universal constant of gravitation.

2. Calculate the critical velocity of a satellite orbiting around the earth at *a* height equal to radius of the earth. [$G = 6.67 \times 10^{-11}$ Nm²/kg², mass of earth, $M = 6 \times 10^{24}$ kg, radius of earth, R = 6400 km]

3. What will be the duration of the year if the distance between the sun and the earth becomes one third?

4. Calculate the escape velocity of a body from the surface of a planet. [Radius of the planet = 1100 km, acceleration due to gravity on the surface of the planet =1.6 m/s²]

5. At what height from surface of earth the value of acceleration due to gravity will fall to half that on surface of earth?

6. At a certain height above the earth's surface, the acceleration due to gravity is 10% of its value at the earth surface. Determine the height. [R = 6400 km]

7. A satellite is orbiting in a circular orbit around the earth with a speed equal to half the magnitude of escape velocity from the earth. Find the height of the satellite above earth's surface.

8. A satellite is revolving in a circular orbit round a planet with a velocity 8 km/s at a height where value of acceleration due to gravity is 8 m/s². How high is the satellite from the planet's surface? [Radius of planet = 6400 km]

9. Two satellites x and y are orbiting in circular orbit of radii r and 2r respectively around the same planet. What is the ratio of their critical velocities?

10. Calculate the time period of a planet jupiter round the sun given that the ratio of radii of the jupiter's orbit and the earth's orbit is 5 : 2.

11. The escape velocity of a body from the earth's surface is 11.2 km/sec. If the mass of the moon is 1 /40th of the earth's mass and the radius of the moon is 1/8*11 of the radius of the earth. Find the escape velocity from moon's surface.

12. Calculate the period of revolution of neptune around the sun, given that diameter of its orbit is 30 times of the diameter of earth's orbit around the sun, both orbits being assumed to be circular.

13. Venus is orbiting round the Sun in 225 days. Calculate the orbital radius and speed of planet. [Mass of Sun – 2×10^{30} kg, $G = 6.67 \times 10^{-11}$ SI unit]

14. Determine

i. kinetic energy

ii. potential energy

iii. total energy and

iv. binding energy of a satellite of mass 50 kg revolving in a circular orbit around the earth at a height of 600 km above the earth's surface.

[Radius of earth R = 6400 km, mass of earth $M = 6 \times 10^{24}$ kg, $G = 6.67 \times 10^{-11}$ SI units]

15. Two satellites orbiting earth have their critical speeds in the ratio 4:5. Compare their radii of orbit.

Section B : Theoretical Board Questions

1. Obtain an expression for critical velocity of a satellite revolving around the earth. [Oct 96, Feb 03]

2. State Newton's law of gravitation and express it in the vector form. **[Oct 97]**

3. Obtain an expression for total energy of a satellite orbiting round the planet. **[Mar 98]**

4. Obtain an expression for critical velocity when the satellite is at a height h above the surface of earth. Hence deduce an expression for period of the satellite. **[Oct 98]**

5. Define critical velocity of a satellite and obtain an expression for it. On what factors critical velocity depends? **[Mar 2000, Oct 09]**

6. Define escape velocity. Why is it same for all bodies projected from the surface of the earth? **[Oct 00]**

7. Define binding energy of satellite, hence obtain an expression for binding energy of satellite revolving around the earth at certain height. State the factors affecting it. **[Feb 01,04, Oct 01]**

8. What is meant by geostationary (communication) satellite? Give its any two uses. **[Feb 02, Feb 03, Oct 06]**

9. Derive an expression for B.E of a satellite at rest on the earth's surface. Hence obtain the expression for escape velocity of a satellite at rest on the surface of the earth. **[Oct 02]**

10. Define critical Velocity of a satellite. Obtain an expression for critical velocity of the satellite. State under what condition it describes an elliptical orbit. **[Oct 03]**

11. State the conditions under which a satellite follows:

i. a parabolic path

ii. an elliptical path. **[Oct 04]**

12. Define period of an orbiting satellite and obtain its expression. Hence show

i. $T^2 \mu r^3$

ii. $T = 2p\sqrt{\frac{r}{g_h}}$

r is radius of orbit of satellite. **[Oct 05]**

13. Define communication satellite. State any two applications of communication satellite. **[Oct 06]**

14. What are various possibilities when a satellite is projected in a horizontal direction from a certain height? **[Feb 06]**

15. What is critical velocity? Obtain an expression for critical velocity of an orbiting satellite. On what factors does it depend? **[Oct 08]**

16. Define binding energy and hence its expression for a satellite revolving at certain height above the Earth's surface. **[Mar 11]**

16. Define binding energy and obtain an expression for binding energy of a satellite revolving in a circular orbit round the earth. **[Oct 11]**

Section C: Numerical Board Problems

1. At a certain height above the earth's surface the gravitational acceleration is 4% of its value at the surface of the earth. Determine the height. [Radius of the earth *m* 6400 km] [Mar 96]

2. Find the altitude at which, the acceleration due to gravity is 25% of that at the surface of the earth. [Radius of the earth = 6400 km]

[Oct 96]

3. Calculate escape velocity of a body from the surface of a planet of diameter 2200 km. Acceleration due to gravity on the surface of the planet =1600 cm/sec^2. **[Mar 98]**

4. A body weighs 3.5 kg-wt. on the surface of the earth. What will be its weight on the surface of a planet whose mass is $\left(\frac{1}{7}\right)^{th}$ of the mass of the earth and radius half of that of the earth? **[Oct 00]**

5. What will be the binding energy of a satellite of mass 1500 kg, moving in circular orbit close to surface of earth?
[$G = 6.67 \times 10^{-11}$ Nm^2/kg^2, radius of earth = 6400 km, mass of earth = 6×10^{24} kg]
[Feb 02]

6. A body weighs 4.5 kg wt. on the surface of earth. How much will it weigh on the surface of the planet whose mass is $\left(\frac{1}{9}\right)^{th}$ mass of the earth and radius half that of the earth.
[Oct 04]

7. A satellite is revolving round the earth in a circular orbit with the critical velocity 7 km/s. Find the radius of the orbit of the satellite and period of its revolution.
[$G = 6.67 \times 10^{-11}$ N/m^2/kg^2,
$M = 5.98 \times 10^{24}$ kg.] **[Oct 04]**

8 The escape velocity of a body from the earth's surface is 11.2 km/sec. The mass of the moon is 1/80 of the earth's mass and the radius of the moon is 1/4th of the radius of the earth. Find the escape velocity from moon's surface. **[Feb 06]**

9. Find the kinetic energy, potential energy, total energy and binding energy of an artificial satellite orbiting at a height of 3600 km above the surface of the earth. [Mass of earth = 6×10^{24} kg, Radius of earth = 6400 km, mass of satellite = 10^2 kg and $G = 6.67 \times 10^{-n}$ S.I. unit] **[Oct 10]**

1. **1. If the distance between the sun and the earth is increased by three times, then attraction between the two will**
(A) remain constant
(B) decrease by 63%
(C) decrease by 83%
(D) decrease 89%

2. Which of the following statements about the gravitational constant is true?
(A) It has no units.
(B) It has same value in all systems of units.
(C) It is a force.
(D) It does not depend upon the nature of medium in which the bodies lie.

3 The weight of body is maximum ....
[Mar 10]
(A) at poles of the earth
(B) at equator of the earth
(C) below the surface of the earth
(D) above the surface of the earth

4. The gravitational force between two bodies is ________ **[Feb 2013 old course]**
(A) attractive at large distance only
(B) attractive at small distance only
(C) repulsive at small distance only
(D) attractive at all distances large or small

5. The tidal waves in the sea are primarily due to gravitational effect of
(A) earth on the sea
(B) sun on the earth
(C) earth on the moon
(D) moon on the earth

6. The acceleration due to gravity on earth depends upon
(A) size of the body .
(B) gravitational mass of the body.
(C) gravitational mass of the earth.
(D) all of these

7. The ratio between masses of two planets is 1 : 3 and ratio between their radii is 3 : 2. The ratio between acceleration due to gravity on these two planets is
(A) 4 : 9 (B) 8 : 27
(C) 9:4 (D) 27:8

8. The gravitational force of earth on a ball of mass one kilogram is 9.8 N. The attraction of ball on the earth is

(A) 9.8 N
(B) negligible
(C) more than 9.8 N
(D) slightly less than 9.8 N

9. The gravitational potential due to the earth is minimum at **[Mar 08]**
(A) the centre
(B) the surface
(C) a distance equal to 100 times the radius of 4 earth
(D) Infinite distance

10. If the earth would have been of iron of density 8×10^3 kg/m^3, then the value of acceleration due to gravity would be
(A) zero (B) equal to g
(C) less than g (D) greater than g

11. Radius of the earth is 6400 km and g = 10 m/s^2. In order that a body of 5 kg weighs zero at the equator, the angular speed of the earth is
(A) $\frac{1}{80}$ radian /s (B) $\frac{1}{400}$ radian/ s
(C) $\frac{1}{800}$ radian/s (D) $\frac{1}{1600}$ radian/s

12. Mass of a particle at the centre of the earth is
(A) infinite
(B) zero
(C) same as at other places
(D) greater than at the poles

6. If the gravitational mass of a body on the moon be denoted by M_m and that on the earth by M_e, then
(A) $M_m = \frac{1}{6} M_e$ (B) $M_m = M_e$
(C) $M_m = \sqrt{M_e}$ (D) $M_m = 6M_e$

14. The time period 'T' of the artificial satellite of earth depends on average density r of the earth as **[Oct 08]**
(A) $T \mu r$ (B) $T \mu \sqrt{r}$
(C) $T \mu \frac{1}{\sqrt{r}}$ (D) $T \mu \frac{1}{r}$

15. Persons sitting in an artificial satellite circling around the earth have
(A) zero mass
(B) zero weight
(C) infinite weight
(D) infinite mass

16. The unit of quantity G/g where symbols have usual meaning is
(A) kg/m^3 (B) kg/m
(C) m^2/kg (D) kgm^2

17. $[M^{-1}L^3 T^{-2}]$ are the dimensions of. [Mar 12]
(A) acceleration due to gravity
(B) gravitational constant
(C) gravitational force
(D) gravitational potential energy

18. Weight of a body at the centre of the earth will
(A) be greater than that at earth's surface
(B) be equal to zero
(C) be less than that at earth's surface
(D) become infinite.

19. The mass of the moon is $\frac{1}{81}$ of the earth but the gravitational pull is $\frac{1}{6}$ of the earth. It is due to the fact that
(A) radius of the moon is $\frac{81}{6}$ of the earth
(B) radius of the earth is $\frac{9}{\sqrt{6}}$ of the moon
(C) moon is the satellite of the earth
(D) radius of moon is $\frac{9}{\sqrt{6}}$ of the earth.

20. If v_e and v_0 represent the escape velocity and orbital velocity of a satellite corresponding to a circular orbit of radius R respectively, then
(A) $v_e = v_0$

(B) $\sqrt{2}\,v_o = v_e$

(C) $v_e = \frac{1}{\sqrt{2}} v_0$

(D) v_e and v_0 are not related

21. If a satellite is orbiting the earth very close to its surface' then the orbital velocity mainly depends on

(A) mass of the satellite only

(B) radius of the earth only

(C) orbital radius only

(D) mass of the earth only

22. A man weighs 60 kg at earth surface. At what height above the earth's surface will his weight become 30 kg?

(Given: Radius of earth is 6400 km.)

(A) 2650 km (B) 3000 km

(C) 2020 m (D) 5298 km

23. A satellite revolves around the earth in an elliptical orbit. Its speed

(A) is the same at all points in the orbit.

(B) is greatest when it is closest to the earth.

(C) is greatest when it is farthest from the earth.

(D) goes on increasing or decreasing continuously depending upon the mass of the satellite.

24. Acceleration due to gravity above the earth's surface at a height equal to the radius of the earth is. **[Oct 10]**

(A) 2.5 m/s^2 (B) 5 m/s^2

(C) 9.8 m/s^2 (D) 10 m/s^2

25. A clock S is based on oscillation of a spring and a clock P is based on pendulum motion. Both clocks run at the same rate on earth. On a planet having the same density as earth but twice the radius

(A) S will run faster than P.

(B) P will run faster than S.

(C) They will both run at the same rate as on the earth.

(D) None of these

26. A body weighs 700 g-wt. on the surface of the earth. How much will it weigh on the surface of a planet whose mass is 1/7 and radius 1/2 of the earth?

(A) 50 g-wt. (B) 200 g-wt.

(C) 300 g-wt. (D) 400 g-wt.

27. g_e and g_p denote the respective accelerations due to gravity on the surface of the earth and another planet whose mass and radius are twice as that of earth. Then

(A) gp = ge (B) $g_p = g_e/2$

(C) g_p = 2ge (D) gp = $g_e/\sqrt{2}$

28. A satellite is orbiting the earth in a circular orbit of radius r. Its period of revolution varies as

(A) $\sqrt{r}$ (B) r

(C) $r^{3/2}$ (D) r^2

29. If gravitational force of earth disappears, what will happen to the satellite revolving round the earth?

(A) Satellite will come back to earth

(B) Satellite will continue to revolve

(C) Satellite escapes in tangential path

(D) Satellite escapes towards centre

30. If satellite is stopped suddenly and allowed to fall freely under gravity, then velocity with which it hits the ground is

(A) 7.92 km/s (B) 8.92 km/s

(C) 11.2 km/s (D) 5.6 km/s

31. A body weights 0.700 kg wt on the surface of the earth. How much will it weigh on the surface of a planet whose mass is $\frac{1}{7}$ of the mass of the earth and radius is 1/2 of that of the earth? **[Oct 11]**

(A) 0.050 kg wt (B) 0.200 kg wt

(C) 0.300 kg wt (D) 0.400 kg wt

32. Time period of revolution of a satellite around a planet of radius R is T. Period of revolution around another planet whose radius is 3R is

(A) T (B) 9T

(C) 3T (D) $3\sqrt{3}\,T$

33. The velocity of a satellite orbiting near the earth's surface is

(A) $\sqrt{GR}$ (B) $\sqrt{gR}$

(C) $\sqrt{\dfrac{GM}{R^2}}$ (D) $\sqrt{2gR}$

34. The ratio of kinetic energy of a body orbiting near the earth's surface and the kinetic energy of the same body escaping the earth's gravitational field is

(A) 1 (B) 2

(C) $\sqrt{2}$ (D) 0.5

35. A geostationary satellite

(A) revolves about the polar axis.

(B) has a time period less than that of the near earth satellite.

(C) moves faster than a near earth satellite.

(D) is stationary in the space.

36. Consider a satellite going round the earth in a circular orbit. Which of the following statements is wrong?

(A) It is a freely falling body.

(B) It is acted upon by a force directed away from the centre of the earth which counter balances the gravitational pull.

(C) It is moving with a constant speed.

(D) Its angular momentum remains constant.

37. A satellite of earth can move only in those orbits whose plane coincides with

(A) the plane of great circle of earth.

(B) the plane passing through the poles of earth.

(C) the plane of a circle at any latitude of earth.

(D) none of these.

38. The energy required to move an earth's satellite of mass m from a circular orbit of radius 2R to a radius 3R is (R is radius of the earth)

(A) $\dfrac{GMm}{R}$ (B) $\dfrac{GMm}{2R}$

(C) $\dfrac{GMm}{12R}$ (D) $\dfrac{GMm}{4R}$

39. A satellite appears to be at rest when seen from the equator. Its height from the earth's surface is nearly

(A) 35800 km (B) 12800 km

(C) 6400 km (D) 24000 km

40. A missile is launched with a velocity less than the escape velocity. The sum of kinetic energy and potential energy will be

(A) positive

(B) negative

(C) negative or positive, uncertain

(D) zero

41. A planet is revolving around a star in a circular orbit of radius R with a period T. If the gravitational force between the planet and –3' the star is proportional to R^2 , then [Oct 13]

(A) T^2 a $R^{5/2}$ (B) T^2 a $R^{-7/2}$

(C) T^2 a $R^{3/2}$ (D) T^2 a R^4

42. If the radius of earth is to decrease by 4% and its density to remain the same, then its escape velocity will

(A) remain same

(B) increase by 4%

(C) decrease by 4%

(D) increase by 2%

43. An earth satellite S has an orbital radius which is 4 times that of a communication satellite C. The period of revolution of S is

(A) 4 days (B) 16 days

(C) 8 days (D) 32 days

44. As we go from the equator towards the pole of the earth, the value of acceleration due to gravity. [Mar 11]

(A) remains constant

(B) decreases

(C) increases

(D) decreases up to latitude of 45°

45. Kinetic energy needed to project a body of mass m from the surface of the earth of radius

(R) to infinity is

(A) mgR/2 (B) 2mgR

(C) mgR (D) mgR/4

46. The period of a satellite in a circular orbit around a planet is independent of

(A) the mass of satellite

(B) the mass of planet

(C) the radius of planet

(D) all of these

47. If B.E. of a satellite of mass 1000 kg is 10^6 J, then B.E. of another satellite of mass 10^4 kg, at the same height from the earth will be

(A) 10^{10} J (B) 10^7J

(C) 10^5 J (D) 10^2J

48. If the kinetic energy of a satellite is 2×10^4 J, then its potential energy will be

(A) -2×10^4 J (B) 4×10^4 J

(C) -4×10^4 J (D) -10^4 J

49. Potential energy of a body in the gravitational field of planet is zero. The body must be

(A) at centre of planet.

(B) on the surface of planet.

(C) at infinity.

(D) at distance equal to radius of earth.

50. What is the ratio of potential energy to the kinetic energy of the moon orbiting around the earth?

(A) 1 : 4 (B) 1 : 2

(C) 4 : 1 (D) 2 : 1

51. A man inside an artificial satellite feels weightlessness because the force of attraction due to earth is

(A) zero at the place.

(B) equal to centripetal force.

(C) balanced by the force of attraction due to moon.

(D) non-effective due to particular design of satellite.

52. Escape velocity of the air molecules from the surface of the earth is

(A) 3.1 kms^{-1} (B) 7.9kms^{-1}

(C) 11.2 kms^{-1} (D) 36 kms^{-1}

53. The escape velocity from the earth does not depend upon

(A) mass of the earth

(B) mass of the body

(C) radius of the earth

(D) acceleration due to gravity

54. Kepler's second law regarding constancy of areal velocity of a planet is a consequence of the law of conservation of

(A) energy

(B) linear momentum

(C) angular momentum

55. According to Kepler's law, the areal velocity of a planet around the sun always .

[Mar 09]

(A) increases

(B) decreases

(C) remains constant

(D) first increases and then decreases

56. If the earth stops rotating, the value of 'g' at the equator will

(A) increase (B) decrease

(C) remain same (D) zero

57. When a body of mass 'M' is taken from the surface of the earth to a height equal to radius of the earth (R) then the change in its P.E. is

[Oct 08]

(A) $\frac{1}{4}$ mgR (B) $\frac{1}{2}$ mgR

(C) mgR (D) 2 mgR

58. A geostationary satellite is orbiting the earth at a height of 6R above the surface of the earth; R being the radius of the earth. What will be the time period of another satellite at a height 2.5 R from the surface of the earth?

(A) $6\sqrt{2}$ hour (B) $6\sqrt{2.5}$

(C) $6\sqrt{3}$ hour (D) 12 hour

59. The radii of two planets are respectively R_1 and R_2 and their densities are respectively r_1 and r_2. The ratio of the accelerations due to gravity at their surfaces is

(A) $g_1 : g_2 = \frac{r_1}{R_1^2} : \frac{r_2}{R_2^2}$

(B) $g_1 : g_2 = R_1R_2 : r_1r_2$

(C) $g_1 : g_2 = R_1r_2 : R_2r_1$

(D) $g_1 : g_2 = R_1r_1 : R_2r_2$

60. Escape velocity on a planet is v_e. If radius of the planet remains same and mass becomes 4 times, the escape velocity becomes

(A) $4v_e$ (B) $2v_e$

(C) V_e (D) 0.5 Ve

61. If the escape velocity of a body on earth is 11.2 km/s, the escape velocity of the body thrown at an angle 45° with the horizontal will be

(A) 11.2 km/s (B) 22.4 km/s

(C) $\frac{11.2}{\sqrt{2}}$ km/s (D) $11.2\sqrt{2}$ km/s

62. The acceleration due to gravity on the surfiB of earth is 10 m/s² and the radius of earth I 6400 km. With what minimum velocity mua I body be thrown from the surface of the eat I so that it reaches a height of 6400 km?

(A) 8 km/s (B) 64 km/s

(C) 1 km/s (D) 32 km/s

63. The escape velocity of a body project vertically upwards from the surface of B earth is v. If the body is projected ini direction making an angle 0 with the vertiJ the escape velocity would be

(A) v (B) v cos q

(C) v sin q (D) v tan q

64. The acceleration due to gravity of that plaa whose mass and radius are half those of earth will be (g is acceleration due to gravity % earth's surface)

(A) 2g (B) g

(C) g/2 (D) g/4

65. An iron ball and a wooden ball of the saa radius are released from a height h in vacuu The times taken by both of these to reach a ground are

(A) unequal (B) exactly equal

(C) roughly equal (D) zero

[ANSWERS]

Section A

1. 6.676×10^{-11} Nm²/kg²
2. 5.592 km/s
3. 70 days
4. 1.876 km/s
5. 2650 km
6. 13840 km
7. h = R
8. 1600 km
9. 1.41411
10. 11.86 years
11. 5.008 km/s
12. 164.32 years
13. 1.085×10^{11} m, 35.06×10^3 m/s
14. i. 1.4295×10^9 J
    ii. -2.859×10^9 J
    iii. -1.4295×10^9 J
    iv. 1.4295×10^9 J
15. 25 : 16

Section : C

2.56×10^4 km

6400 km 1.876×10^3 m/s

2 kg-wt.

4.689×10^{10} J 19.6 N

i. 8140 km ii. 2.028 h

2.504 km/s

ii. 2.001×10^{10} J ii. -4.002×10^{10} J

iii. -2.001×10^{10} J iv. 2.001×10^{10}

Section: D

1.	(D)	2.	(D)	3.	(A)	4.	(D)
5.	(D)	6.	(C)	7.	(B)	8.	(A)
9.	(B)	10.	(D)	11.	(C)	12.	(C)
13.	(B)	14.	(C)	15.	(B)	16.	(C)
17.	(B)	18.	(B)	19.	(B)	20.	(B)
21.	(B)	22.	(A)	23.	(B)	24.	(C)
25.	(B)	26.	(D)	27.	(B)	28.	(C)
29.	(C)	30.	(A)	31.	(D)	32.	(D)
33.	(B)	34.	(D)	35.	(A)	36.	(B)
77.	(A)	38.	(C)	39.	(A)	40.	(B)

41. (A) 42. (C) 43. (C) 44. (C)
5. (C) 46. (A) 47. (B) 48. (C)
49. (C) 50. (D) 51. (B) 52. (C)
53. (B) 54. (C) 55. (C) 56. (A)
57. (B) 58. (A) 59. (D) 60. (B)
61. (A) 62. (A) 63. (A) 64. (A)
65. (B)