

ELASTICITY

EXERCISE

5.0 Introduction

Q.1. Explain the effects of applied force on a rigid body.

Ans:

- i. When a body is subjected to a set of balanced forces then there is no motion of the body but the force may produce | change in its size, shape or both.
- ii. Due to applied force, some bodies undergo change in dimension easily, while some bodies are reluctant to undergo any change in dimension.
- iii. Some bodies show permanent change in their dimension after removal of applied force, whereas some bodies preserve their original dimension, i.e. size, shape or both after removal of applied force.

5.1 General explanation of elastic property and elasticity

***Q.2. Define and explain the concept of elasticity. Ans: Definition:**

The property by virtue of which material bodies regain their original dimensions (size, shape or both) after removal of deforming force is called elasticity.

Explanation for concept of elasticity:

- i. When a body is deformed by applying an external force, the molecules are displaced from their original positions of stable equilibrium.
- ii. The intermolecular distances change and restoring forces act on the molecules which bring them back to their original position.
- iii. This leads to the phenomenon of elasticity in material body.

***Q.3. What are elastic and inelastic (plastic) bodies? Give examples of each.**

Ans: Elastic bodies:

A body which possesses the property of elasticity i.e. the body which changes its size, shape or both when a deforming force is applied and comes to its original position as soon as deforming force is removed is called elastic body.

Example: Steel wire, rubber band, sponge ball etc.

Inelastic bodies:

Those bodies which do not regain their original shape or size after removal of deforming force are called inelastic bodies. Example: Plastic, glass, dough, putty etc.

Q.4. What are perfectly elastic and perfectly plastic bodies? OR

***Explain the term.**

- i. **Perfectly elastic body.**
- ii. **Perfectly plastic body.**

Ans: Perfectly elastic bodies:

- i. *Those bodies which regain their original shape, size or both completely after removal of deforming force are called perfectly elastic bodies.*
- ii. Perfectly elastic bodies do not exist in nature, however some bodies are considered to be perfectly elastic. Example: Quartz, phosphor, bronze etc.

Perfectly plastic bodies:

- i. *Those bodies which completely remain in their deformed state even after the removal of external force are called perfectly plastic bodies.*
- ii. Ideal perfectly plastic bodies do not exist in nature, however some bodies are considered to be perfectly plastic. Example: Wax, putty, clay etc.

Note:

1. Elastic properties of materials always lie between perfectly elastic and perfectly plastic bodies.
2. Elastic property is more in solid, less in liquid and least in gas.

5.2 Plasticity

Q.5. What is plasticity?

Ans:

- i. *Plasticity is the property of a material body to undergo a permanent deformation even after the removal of external deforming forces.*
- ii. Plastic bodies do not regain their original dimensions (size, shape or both) after removal of deforming forces.
- iii. These bodies can be deformed to a large extent by a small deforming force.
- iv. Perfect plasticity exhibited by certain material

is. its property to undergo irreversible deformation without any increase in load.

***Q.6. Distinguish between elasticity and plasticity.**

Ans:

No.	Elasticity	Plasticity
i.	Body regains its original shape or size after removal of external force.	Body does not regain its original shape or size after removal of external force.
ii.	External force changes the dimensions of the body temporarily.	External force changes the dimensions of the body permanently.
iii.	Internal restoring force is set up inside the body.	Internal restoring force is not set up inside the body.
iv.	Ratio of stress and strain remains constant.	Ratio of stress and strain does not remain constant.

5.3 Deformation

Q.7. Define the following terms.

- Deformation**
- Deforming force**

Ans:

i. Deformation:

The change in size, shape or both of a body due to applied external force is called deformation.

ii. Deforming force:

The force which produces deformation in a body is called deforming force. Deforming force or Deformation.

Q.8. How is deformation produced in a body?

OR

State the changes observed in deformed state of a body.

Ans:

- Deformation in a body is produced by applying deforming force. It is due to change in relative positions of the molecules within the body.
- In deformed state, the applied force is numerically equal to internal elastic restoring force within the body.

Direction of this force is opposite to the direction of internal restoring force.

- Deformation in a body produces change in length, change in volume, change in shape etc.
- For example,
 - When a rubber cord or a metal wire is stretched, its length increases.
 - When an inflated volley ball bladder is pressed by hands, there is change in its shape and volume.

Q.9. Define stress. State its unit and dimension.

Ans:

i. Definition:

The internal elastic restoring force per unit cross sectional area of a body is called stress.

OR

Stress is defined as applied force per unit cross sectional area of a body.

ii.
$$\text{Stress} = \frac{\text{Applied force}}{\text{Area of cross section}}$$

$$= \frac{\text{Elastic restoring force}}{\text{Area of cross section}} = \frac{F}{A}$$

- Unit : N/m² or Pa in SI system and dyne/cm² in CGS system.
- Dimensions : [M¹L⁻¹T⁻²]

***Q.10. What is longitudinal stress?**

Ans:

- If deforming force produces change in length of a body (wire, , beam etc.) the stress associated is called longitudinal stress or tensile stress.*
- Longitudinal stress

$$= \frac{\text{Applied force on wire}}{\text{Area of cross section}} = \frac{F}{A} = \frac{Mg}{\pi r^2}$$

where M = mass of the wire

r = radius of cross section of wire

g = acceleration due to gravity.

- This stress is produced in solid wire, rod, beam etc.

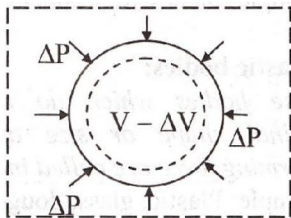
***Q.11. What is volume stress (bulk stress)?**

Ans:

- If a deforming force produces change in volume of a body, the stress associated with*

it is called volume stress.

- ii. This stress can be produced in solid, liquid and gases.
- iii. Let a gas balloon with original volume V be compressed by additional pressure ΔP on all sides.
- iv. Due to applied additional pressure ΔP , volume of gas balloon decreases by ΔV . Thus new volume of gas in balloon = $V - \Delta V$



- v. If original pressure is P and new pressure applied is $P + \Delta P$ then increase in pressure ΔP tries to restore the body back to its original dimensions.
- vi. ΔP generates an internal restoring force which acts on the walls of the balloon.
- vii. Internal restoring force = $\Delta P \times A$

$$\text{Volume stress} = \frac{\text{Applied force}}{\text{Area}}$$

$$\text{Volume stress} = \Delta P$$

#Q.12. Though dimensions of stress and pressure are alike/same, they are different. Justify.

Ans: At a point within a given fluid of constant density in hydrostatic equilibrium, the pressure is the same in all directions. Hence, pressure is a scalar quantity.

For a given deforming force acting on a body, the stress can have different magnitudes in different directions and of more than one type. Hence, stress is a tensor quantity (a quantity having different values in different directions).

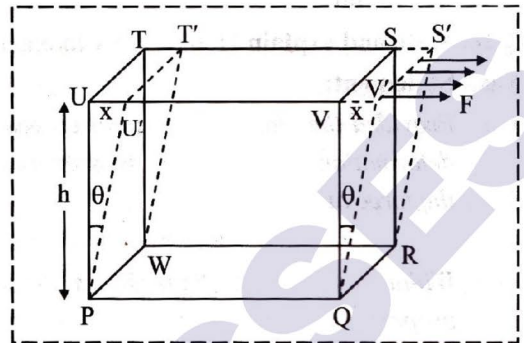
Q.13 What is shearing stress ?

Ans:

- i. If deforming force produces change in shape of a body, the stress associated with it is called shearing stress. OR The restoring

force per unit area developed due to applied tangential force is called shearing stress (tangential stress).

- ii. When a tangential force 'F' is applied on the surface QRSV of a cube as shown in the figure then position of surface changes at an angle θ without change in volume of the cube.



- iii. In the figure,
 $PQRSTUV$ = original shape of cube
 $PQRS'T'U'V'$ = new shape of cube in deformed state.
 F = tangential force
 $TT' = SS' = UU' = W' = x$ = lateral displacement of edge

$$\text{Shearing stress} = \frac{\text{Tangential force}}{\text{Area}} = \frac{F}{A}$$

Q.14. What is strain ?

Ans:

- i. The change in dimension per unit original dimension of a body is called strain.

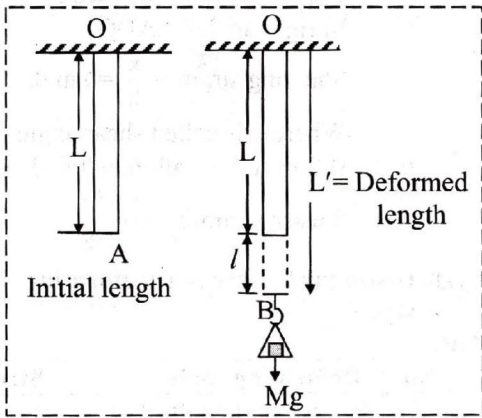
$$\text{Strain} = \frac{\text{change in dimension}}{\text{original dimension}}$$

- ii. Strain is the ratio of two similar physical quantities, hence it has no unit and dimension.
- iii. Also the ratio of change in length to the original length has a definite numeric value. Hence, strain is a pure number.
- iv. Strain is classified into three types:
 - a. Longitudinal strain (Tensile strain)
 - b. Volume strain (Bulk strain)
 - c. Shearing strain

***Q.15. What is longitudinal strain (tensile strain)?**

Ans:

- i. The ratio of the change in length (l) to the original length (L) is called longitudinal strain (Tensile strain).



- i. If deforming force produces change in shape of a body, the stress associated with it is called shearing stress. OR The restoring force per unit area developed due to applied tangential force is called shearing stress (tangential stress).
 - ii. In the figure,
 - L = original length of wire
 - $L' = L + l$
= deformed length due to load Mg
- ∴ Longitudinal strain = $\frac{L' - L}{L} = \frac{L + l - L}{L}$
- iii. It is observed in solid metal wires.

***Q.16. What is volume strain (bulk strain)?**

Ans:

- i. The ratio of change in volume to original volume is called volume strain.
- ii. If V = original volume of gas
 P = original pressure
 DP = increase in pressure
 DV = decrease in volume, then

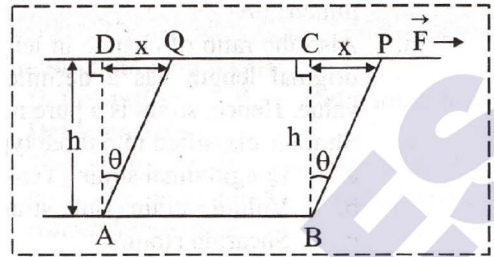
$$\text{Volume strain} = \frac{(V - DV) - V}{V} = - \frac{DV}{V}$$
- iii. Negative sign indicates that volume of gas decreases with increase in pressure.

***Q.17. What is shearing strain?**

Ans:

- i. The ratio of relative displacement of any layer to its perpendicular distance from fixed layer is called shearing strain.

$$\text{Shearing strain} = \frac{\text{Relative displacement}}{\text{Distance from fixed layer}}$$



- ii. In the figure, base of the block AB is fixed. A tangential force ' F ' is applied to the upper surface so that lateral displacement of the face is ' x '.
In right angled $DADQ$,
Shearing strain = $\frac{x}{h} = \tan q$.
Where q is called shear angle.
 - iv. If q is very small, then $\tan q \ll 0$
- ∴ Shearing strain = $q = \frac{x}{h}$

Q.18. Distinguish between deforming force and stress.

Ans:

No.	Deforming force	Stress
i.	It is externally applied force.	It is internal restoring force per unit area.
ii.	It tries to deform the body.	It opposes the deformation.
iii.	S.I unit is N.	S.I unit is N/m^2 .
iv.	Its dimension is $[M^1L^1T^{-2}]$.	Its dimension is $[M^1L^{-1}T^{-2}]$.
v.	It is a vector quantity.	It is a tensor quantity.

Note:

- When deforming force is inclined to the surface of a body both the normal as well as tangential stress are developed in the body.
- If a metallic cylinder is compressed under the action of applied forces, the restoring force per unit area is called compressive stress.

3. If length decreases from its natural value then longitudinal strain is called compressive strain.
4. Shear strain is possible only for solid body. If the deforming force is applied parallel to the liquid surface it will begin to flow in the direction of applied force.
5. On twisting a wire or rod, shearing strain is produced in it.

***Q.19. Explain the term elastic limit?**

Ans:

- i. The maximum stress to which an elastic body can be subjected without causing permanent deformation is called as elastic limit.
- ii. The deformation produced in an elastic body depends upon the magnitude of deforming force. If the deforming force is small or large, deformation produced will be small or large. If the deforming force is removed, body regains its original shape and size.
- iii. However, if we go on increasing the deforming force, a stage is reached, where the body does not regain its original dimensions, even after the removal of deforming force. The body is said to acquire permanent deformation called "set".
- iv. When such a stage is reached we say that body is stretched beyond the elastic limit.

Q.20. State and explain Hooke's law in elasticity.

Ans: Statement:

Provided the elastic limit is not exceeded, the deformation of a material is proportional to the force applied to it.

OR

Within the elastic limit stress is directly proportional to strain.

Explanation:

- i. According to Hooke's law,
Stress \propto Strain

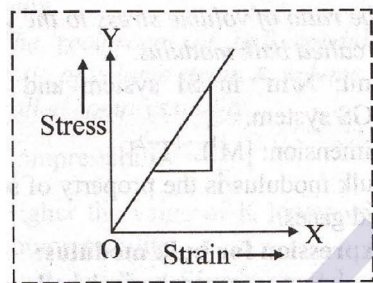
$$\frac{\text{Stress}}{\text{Strain}} = \text{constant (M)}$$

This constant of proportionality is called modulus of elasticity.

- ii. The graph of strain (on X-axis) and stress (on Y-axis) is a straight line passing through origin

as shown in the figure.

Thus, modulus of elasticity of a material is the slope of stress-strain curve in elastic deformation region



Q.21. What is modulus of elasticity?

Ans:

- i. The ratio of stress to, the corresponding strain for a material obeying Hooke's law is called modulus of elasticity.

OR

The modulus of elasticity of a material is defined as the slope of stress-strain curve in elastic deformation region.

$$\text{Modulus of elasticity, } M = \frac{\text{Stress}}{\text{Strain}}$$

- ii. Modulus of elasticity depends upon the nature of material body.
- iii. Unit: N/m² in SI system and dyne/cm in CGS system.
- iv. Dimension: [M¹L⁻¹T⁻²]

Note:

- 1 Modulus of elasticity of a material body depends on nature of material of the body and the manner in which it is deformed.
- 2 Hooke's law holds good only when a wire is loaded within its elastic limit.
3. Within elastic limit, if stress disappears, strain also disappears.

5.6 Elastic constants, Poisson's ratio

Q.22. Define Young's modulus of elasticity. Derive an expression for Young's modulus of elasticity.

OR

***Obtain an expression for Young's modulus of the material of wire. State SI unit and dimensions of 'Y'.**

Ans: Definition:

The ratio of longitudinal stress to longitudinal strain is called as Young's modulus of elasticity.

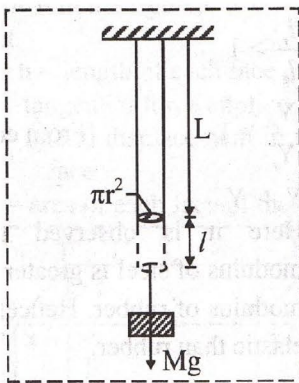
It is denoted by Y .

Unit: N/m^2 in SI system and dyne/cm^2 in CGS system.

Dimensions: $[M^1L^{-1}T^{-2}]$

Expression for Young's modulus:

- i. Let,
 L = original length of wire
 Mg = weight suspended to wire
 l = increase in length after stretching
 r = radius of the cross section of wire



- ii. Longitudinal stress :

$$= \frac{\text{Applied force}}{\text{Area of cross section}} = \frac{F}{A} = \frac{Mg}{\pi r^2}$$

- iii. Longitudinal strain = $\frac{\text{Increase in length}}{\text{Original length}}$

$$= \frac{l}{L}$$

- iv. From definition,
Young's modulus (Y)

$$= \frac{\text{longitudinal stress}}{\text{longitudinal strain}}$$

$$= \frac{Mg / \pi r^2}{l / L}$$

$$\therefore Y = \frac{Mg}{\pi r^2} \cdot \frac{L}{l}$$

Q.23. Why steel is more elastic than rubber?**Ans:**

- i. Consider two wires of same length and Same cross section.
- ii. Let, l_s = length of steel wire
 l_r = length of rubber wire
 A_s = area of cross section of steel wire
 A_r = area of cross section of rubber wire
 Y_s = Young's modulus of steel wire | Y_r = Young's modulus of rubber wire
- iii. If equal force F is applied to both the wires then it is observed that: change in length of rubber wire is greater than change in length of steel wire
i. e. $l_r > l_s$.

- iv. By definition, $Y_s = \frac{FL_s}{A_s l_s}$ and $Y_r = \frac{FL_r}{A_r l_r}$

$$\text{But, } A_s = A_r \text{ and } L_s = L_r$$

$$\therefore \frac{Y_s}{Y_r} = \frac{l_r}{l_s} \quad \dots\dots (i)$$

$$\text{As } l_r > l_s$$

$$\therefore \frac{l_r}{l_s} > 1 \quad [\text{From equation (1)}]$$

$$\therefore Y_s > Y_r$$

- v. Here it is observed that Young's modulus of steel is greater than Young's modulus of rubber. Hence, steel is more elastic than rubber.

Q.24. Within elastic limit, prove that Young's modulus of material of wire is the stress required to double the length of wire.*Ans:**

- i. Let, L = length of wire
 $2L$ = deforming length

$$\therefore \text{Increase in length} = l = 2L - L = L$$

- ii. Longitudinal strain of wire $\frac{l}{L} = \frac{L}{L} = 1$

- iii. Young's modulus of wire

$$= \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$$

$$\therefore Y = \frac{\text{Longitudinal stress}}{1}$$

v. $Y = \text{Longitudinal stress}$

iv. Hence, Young's modulus of material of wire is the stress required to double the length of wire.

#Q.25. Mountaineer generally use nylon rope for climbing mountains. Can nylon rope be replaced by cable made up of steel of equal strength?

Ans: Nylon rope cannot be replaced by cable made up of steel of equal strength because steel is more elastic than nylon. Nylon ropes can withstand more amount of tensile strain than steel cables.

Q.26. Define bulk modulus. Derive an expression for bulk modulus in elasticity.

OR

***Obtain an expression for bulk modulus. State SI unit and dimensions of 'K*.**

Ans: Definition:

The ratio of volume stress to the volume strain is called bulk modulus.

Unit: N/m^2 in SI system and dyne/cm^2 in CGS system.

Dimension: $[M^1L^{-1}T^{-2}]$

Bulk modulus is the property of solids, liquids and gases.

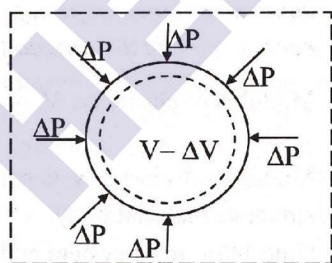
Expression for bulk modulus:

i. Suppose a spherical balloon of volume V is subjected to additional pressure DP to reduce its volume by DV .

ii. $V =$ original volume of gas in balloon

$DV =$ decrease in volume of gas in balloon

$V - DV =$ new Volume of gas in balloon



iii. Volume stress $= \frac{F}{A} = \frac{DP \cdot A}{A} = DP$

iv. Volume strain $= \frac{\text{Change in volume}}{\text{Original volume}}$

$$= \frac{(V - DV) - V}{V} = \frac{-DV}{V}$$

Negative sign shows that there is decrease in volume.

$$\text{Volume strain} = \frac{DV}{V} \text{ (numerically)}$$

v. From definition,

$$\text{Bulk modulus, } K = \frac{\text{Volume stress}}{\text{Volume strain}}$$

$$K = \frac{DP}{\frac{DV}{V}}$$

Note:

Under isothermal conditions, the bulk modulus of a gas is equal to its pressure P . Under adiabatic conditions, the bulk modulus is equal to gP .

Q.27. Define compressibility. State its unit and dimension.

Ans:

i. *The reciprocal of bulk modulus, i.e., ratio of volume strain to volume stress is called compressibility.*

$$\text{Compressibility} = \frac{1}{K}$$

Higher the value of K lower will be its compressibility.

ii. Unit: m^2/N in SI system and cm^2/dyne in CGS system.

iii. Dimensions: $[M^{-1}L^1T^2]$

#Q.28. Two spheres A and B having same volume are dropped from same height in ocean. Sphere A is made up of brass and sphere B is made up of steel. Will there be same change in volume of spheres at a certain depth inside water? What will be the ratio of change in volumes of the two spheres at this depth?

Ans:

i. Brass and copper have different elastic moduli. Hence, there won't be same change in the volume of spheres at a certain depth inside water.

ii. As two spheres are dropped from same height

and are at same depth in water pressure exerted on them remains same.

$$\sqrt{dP_A - dP_B = dP}$$

$$V_A = V_B = V$$

- iii. If dV_A and dV_B be the change in volume of two spheres A and B then,

$$dV_A = \frac{V \cdot dP}{K_A} \text{ and}$$

$$dV_B = \frac{V \cdot dP}{K_B}$$

- iv. Ratio of change in volume,

$$\frac{dV_A}{dV_B} = \frac{V \cdot dP}{K_A} / \frac{V \cdot dP}{K_B} = \frac{K_B}{K_A}$$

where K_A and K_B are bulk modulus of material of spheres A and B respectively.

Q.29. Define modulus of rigidity. Derive an expression for rigidity modulus

OR

***Obtain an expression for the modulus of rigidity. State SI unit and dimensions of 'h'.**

[Oct 11]

Ans: Definition:

The ratio of shearing stress to the shearing strain is called modulus of rigidity or rigidity modulus.

It is denoted by h .

Unit: N/m^2 in SI system and dyne/cm^2 in CGS system.

Dimensions: $[M^1L^{-1}T^{-2}]$

Expression for rigidity modulus:

- i. Consider a solid cube PQRWTSVU as shown in the figure,

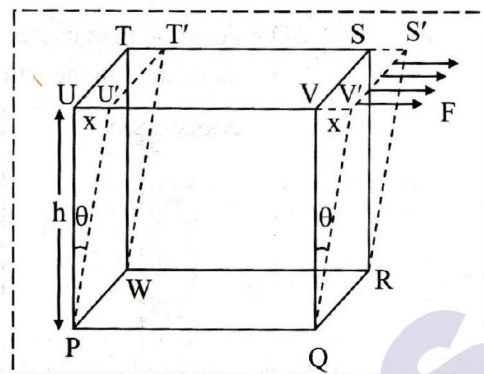
Let,

$L = h$ = length of each face of solid cube

F = tangential force applied to one face

x = lateral displacement in the length of face.

A = area of each face of the cube.



ii. Shearing stress = $\frac{\text{Tangential force}}{\text{Area}} = \frac{F}{A}$

iii. Shearing strain = $\frac{\text{Lateral displacement of any layer}}{\text{Distance from the fixed layer}}$

$$= \frac{x}{h} = \tan \theta$$

For small value of θ ,

$$\tan \theta \gg \theta$$

$$\sqrt{\frac{x}{h} = \theta}$$

- iv. Now, modulus of rigidity,

$$h = \frac{\text{Shearing stress}}{\text{Shearing strain}} = \frac{F/A}{\theta}$$

$$\sqrt{h = \frac{F}{A\theta} = \frac{Fh}{Ax}}$$

#Q.30. You are given a jelly cube. Which moduli of the elasticity are associated with it ?

Ans: A jelly cube possesses both modulus of rigidity and bulk modulus.

Q.31. Define Poisson's ratio. Derive an expression for Poisson's ratio.

Ans: Definition:

The ratio of lateral strain to the longitudinal strain is called Poisson's ratio.

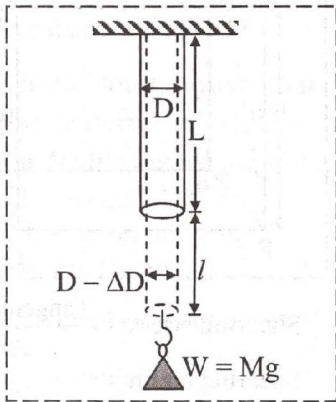
It is denoted by s .

Poisson's ratio is a pure number because it is the ratio of two strains.

$$\text{It is given by, } s = \frac{\text{Lateral strain}(b)}{\text{Longitudinal strain}(a)}$$

Expression for Poisson's ratio:

- i. Let a wire of length 'L' be stretched by a load (W) as shown in the figure.
- ii. When the wire is stretched, its length increases but its diameter decreases.
- iii. Let, DD = decrease in diameter
 l = elongation in the wire.



- iv. Longitudinal strain in the wire is given by,

$$a = \frac{l}{L}$$

- v. Lateral strain in the wire, $b = -\frac{DD}{D}$

Negative sign shows that diameter of wire decreases with increase in length.

- vi. From definition,

$$s = \frac{b}{a} = \frac{-DD/D}{l/L}$$

$$s = -\frac{DD}{D} \cdot \frac{L}{l} \quad \dots\dots (i)$$

- vii. Equation (i) can also be written as,

$$s = \frac{\frac{DD}{D} \cdot L}{\frac{D}{2} \cdot l} = \frac{-DrL}{rl}$$

where, r = radius of the cross section of the wire.

*Q.32. Explain Poisson's ratio. Discuss its limiting value.

Ans:

- i. When a force is applied to a body along any direction, there is
 - a. change in size along the direction of force.
 - b. change in size in perpendicular direction.
- ii. If a force is applied to the free end of a metal wire, it gets elongated. This is accompanied by decrease in the diameter of metal wire.
- iii. Thus longitudinal extension is accompanied by lateral contraction. This lateral contraction depends on the nature of the body and is proportional to its size, direction as well as stress. This proportional change is called lateral strain.
- iv. Within elastic limit, lateral strain (b) is proportional to longitudinal strain (a),
 i.e. $b \propto a$
 $b = a \times s$

Where s is proportional constant called Poisson's ratio.

$$\text{Poisson's ratio } (s) = \frac{b}{a}$$

- v. Limiting value of s :
 - a. For homogeneous isotropic material,
 $-1 < s < 0.5$
 - b. In actual practice s is always positive,
 $0.2 \leq s < 0.4$
- vi. Poisson's ratio is unitless and dimensionless quantity because it is the ratio of two similar physical quantities.

Q.33. Explain, why only solids possess all the three types of elastic moduli.

Ans:

- i. Young's modulus is related to a change in length. Bulk modulus is related to a change in volume. Modulus of rigidity is related to a change in shape.
- ii. Liquid and gases possess a definite volume. They do not have length or shape of their own.
- iii. Only solids possess a definite length, volume and shape.
- iv. Therefore, solids possess all the three elastic

moduliis

***Q.34. Distinguish between Young's modulus, bulk modulus and modulus of rigidity.**

	Young's modulus	Bulk modulus	Modulus of rigidity
i.	It is the ratio of longitudinal stress to longitudinal strain.	It is the ratio of volume stress to volume strain.	It is the ratio of shearing stress to shearing strain.
ii.	It is given by, $Y = \frac{MgL}{\pi r^2 l}$	It is given by, $K = \frac{V\Delta P}{\Delta V}$	It is given by, $\eta = \frac{F}{A\theta}$
iii.	It exists in solids.	It exists in solid, liquid and gases.	It exists in solids.
iv.	It relates to change in length of a body.	It relates to change in volume of a body.	It relates to change in shape of a body.

Note:

- Young's modulus of perfectly elastic body is infinity and that of perfectly plastic body is zero.
- Young's modulus of material decreases with rise in temperature.
- Young's modulus decides elastic nature of material.
- Only solids possess all the three elastic constants while liquids and gases possess only one elastic constant viz. bulk modulus.
- Liquids and gases do not have a definite shape

of their own, hence they do not possess modulus of rigidity.

- Modulus of elasticity for a perfectly rigid body is always infinity.
- If there is no change in the volume of a wire due to change in its length on stretching, then poisson's ratio of the material of the wire is – 0.5.
- Poisson's ratio σ is not the modulus of elasticity as it is the ratio of two strains and not of stress and strain.
- Water is more elastic than air because bulk modulus of elasticity is reciprocal of compressibility and air is more compressible than water.

10. Relation between Y, k, h and s:

i. $Y = 3k(1 - 2s)$

ii. $Y = 2h(1 + s)$

iii. $s = \frac{3k - 2h}{2h - 6k}$

iv. $\frac{9}{Y} = \frac{1}{k} + \frac{3}{h}$

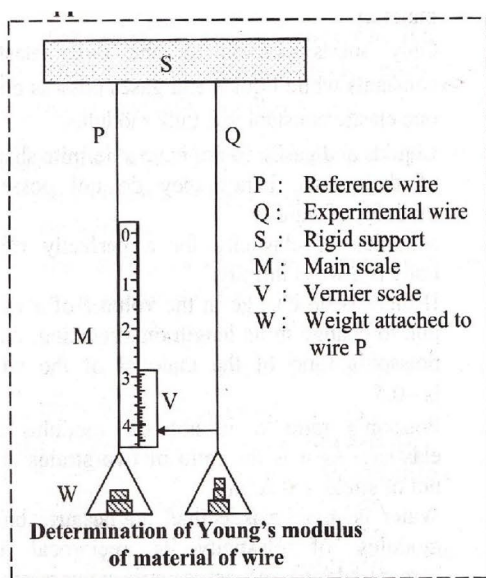
5.7 Determination of Young's modulus of material of wire :

***Q.35. Describe a method to determine the Young's modulus of a material of thin wire.**

Ans: Determination of Young's modulus of thin wire:

Apparatus:

	Young's Modulus (Y)	Bulk Modulus (k)	Modulus of Rigidity (η)	Poisson's ratio (σ)
i.	Corresponds to elasticity of length.	Corresponds to elasticity of volume	Corresponds to elasticity of shape	Is a constant of proportionality relating longitudinal extension to lateral contraction
ii.	S.I unit : N/m^2	S.I. unit: N/m^2	S.I. unit: N/m^2	–
iii.	Dimensions: $[M^1L^{-1}T^{-2}]$	Dimensions: $[M^1L^{-1}T^{-2}]$	Dimensions: $[M^1L^{-1}T^{-2}]$	–
iv.	$Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$	$K = \frac{\text{Volume stress}}{\text{Volume strain}}$	$\eta = \frac{\text{sheer stress}}{\text{shear strain}}$	$\sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}}$



- The apparatus is arranged as shown in the figure.
- Two long straight thin wires P and Q of same length, same radius and same material (usually steel) are suspended side by side from a rigid support S.
- The wire P, so called reference wire is kept taut by weight W attached to its lower end and carries a main scale M graduated in millimetres.
- The wire Q, so called experimental wire carries a pan in which known weights can be placed.
- Vernier scale V is attached at the bottom of experimental wire, whereas main scale M is fixed to reference wire.

Procedure:

- When weights are placed in the pan attached to experimental wire Q, load is increased which thereby increases the force F in wire Q.
- The reference wire is used in order to compensate change in length that may occur due to change in temperature i.e. the increase in length of reference wire and experimental wire due to change of room temperature is the same.
- Both reference wire and experimental wire are sufficiently loaded in order to make wire

straight and taut and vernier reading is noted.

- Now the experimental wire Q is gradually loaded in order to bring wire under tensile stress.
- Every time vernier reading is noted.
- The difference between two vernier readings gives elongation produced for a given load.

Calculation:

- Let r and L be the radius and length of experimental wire Q initially. The area of cross-section of wire Q is πr^2 . Let l be the elongation in length of wire Q.
- When mass M is attached to lower end of Q i.e. elongation produced in experimental wire Q is for applied force Mg, where g is the acceleration due to gravity.
- By definition

$$Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}} = \frac{\frac{Mg}{\pi r^2}}{\frac{l}{L}}$$

$$Y = \frac{MgL}{\pi r^2 l}$$

- Measuring extension l, radius r, original length L and load W, Young's modulus can be determined.

5.8 Behaviour of metal wire, under increasing

Q.36. Define the following terms:

- Yield point**
- Breaking stress**
- Breaking weight**
- Breaking point**
- Permanent deformation (set)**

Ans:

- Yield point:**

The point on stress-strain curve at which the strain begins to increase without any increase in the stress is called yield point.

A metal after yield point becomes ductile.

- Breaking stress (ultimate strength):**

The maximum stress which a wire can bear is called breaking stress.

- Breaking weight:**

Product of breaking stress and area of cross section is called breaking weight. Breaking weight = Breaking stress \times

Area of cross section

iv. Breaking point:

The point on stress-strain curve at which the wire breaks, is known as the breaking point.

v. Permanent deformation (set):

If the elastic limit is exceeded, the body does not preserve its original dimension after removal of the deforming force. This state is called permanent deformation (set).

***Q.37. Explain stress v/s strain graph with the help of a neat labelled diagram.**

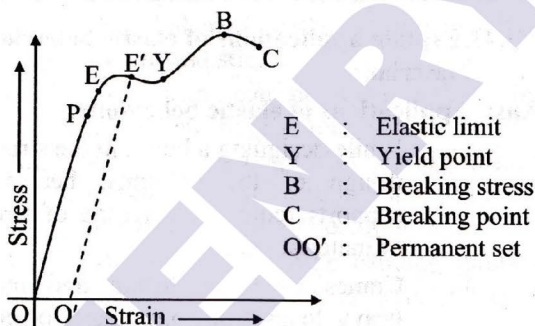
Ans: Stress v/s strain graph:

i. Elastic limit:

The initial portion OE of the graph is a straight line, which indicates that upto the point E stress is directly proportional to strain. Hence, Hooke's law is obeyed upto point E. In this region, wire is perfectly elastic and it completely regains its original length when the load is removed. Point E represents limit of proportionality between stress and strain.

ii. Permanent set:

If the load is increased so that stress becomes greater than that corresponding to the point E, the graph is no longer a straight line and the wire does not obey Hooke's law.



If the wire is strained upto E' beyond point E and then the load is removed, the wire does not regain its original length and there is a permanent increase in length. A small strain corresponding to OO' is set up permanently in the wire, called permanent set. However, the wire is still elastic and if loaded again, gives a linear relation shown by the dotted line OE'.

iii. Yield point:

As the stress is increased beyond the elastic

limit the graph is a curve and reaches a point Y where the tangent to the curve is parallel to the strain axis. This shows that for the stress corresponding to point Y the strain increases even without any increase in the stress. This is known as plastic flow.

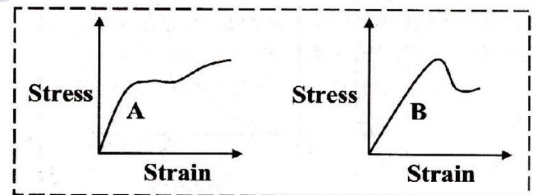
Point Y on the curve is called yield point. The value of stress corresponding to yield point is called yield stress.

iv. Breaking stress: When the wire begins to flow, its cross-section decreases uniformly and hence, the stress increases steadily. Later a neck or constriction begins to form at a weak point. The maximum stress corresponding to the point B is breaking stress.

v. Breaking point: Once the neck is formed, the wire goes on stretching even if the load is reduced, until the breaking point C is reached when the wire breaks.

Q.38. Stress-strain curve for two materials A and B are shown in the figure. The graphs are drawn to the same scale.

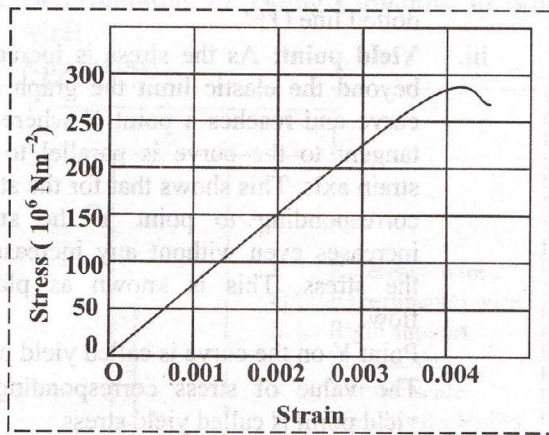
- Which material has greater Young's modulus?
- Which of the two is the stronger material? (NCERT)



Ans:

- Material B has greater value of Y. It is so because slope of the graph of materials B is larger than that of A. Thus for producing same strain more stress is required.
- Material A is stronger than B because A can bear greater stress before the breaking of the wire.

Q.39. Figure shows the stress-strain curve for a given material. What are (i) Young's modulus and (ii) approximate yield strength for this material? (NCERT)



Ans:

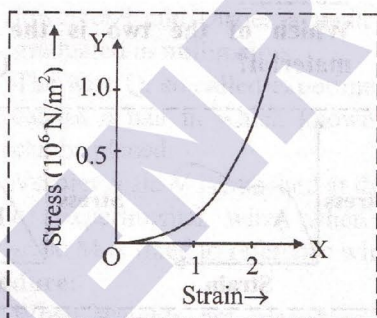
From the graph, it is found that the stress of $150 \times 10^6 \text{ Nm}^{-2}$ comes into play for a strain of 0.002. Therefore, Young's modulus is, stress,

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{150 \times 10^6}{0.002} = 7.5 \times 10^{10} \text{ Nm}^{-2}$$

- ii. The yield strength for the material is less than $300 \times 10^6 \text{ Nm}^{-2}$, i.e. $3 \times 10^8 \text{ Nm}^{-2}$ and greater than $2.5 \times 10^8 \text{ Nm}^{-2}$.

Q.40. Draw stress-strain curve in elastic tissue of aorta.

Ans: The stress-strain curve for aorta is as shown in the figure.



Q.42. Derive a relation between Young's modulus, coefficient of linear expansion and thermal stress.

Ans:

- i. Consider a metallic rod of length l_0 at 0°C
 ii. If the temperature of rod is increased by $\Delta\theta$, then final length of rod is given by,

$$l = l_0 (1 + \alpha \Delta\theta) \quad \dots(i)$$

Where, α is the coefficient of linear expansion

of material of the rod.

iii. Linear strain = $\frac{\text{Change in length}}{\text{Original length}}$

$$\text{Linear strain} = \frac{l - l_0}{l_0}$$

iv. From equation (i), $\frac{l - l_0}{l_0} = \alpha \Delta\theta$

v. Stress = $Y \times \text{Strain} = Y \alpha \Delta\theta$

v. Force exerted by rod due to heating = Thermal stress \times Area.

Note:

1. Malleability is the property of metal due to which it can be beaten into thin sheets.
2. Ductility is the property by virtue of which a metal can be drawn into wires.
3. Breaking stress is independent of the length or area of cross section of the wire. However it depends on the material of the wire.
4. If a wire is broken into two parts, the maximum load it can sustain remains same.
5. Load applied to the wire depends upon area not on length.

5.9 Application of elastic behaviour of materials

***Q.43. Explain applications of elastic behaviour of material.**

Ans: Applications of elastic behaviour:

- i. While designing a building the structural design of the columns, beams and supports require knowledge of strength of material used,
- ii. Cranes used for lifting and moving heavy loads from one place to another have a thick metal rope to which the load is attached. The thickness of such ropes in cranes is decided on the basis of elastic behaviour of material of the rope along with factor of safety.
- iii. Bridges are designed in such a way that they do not bend much or break under the load of heavy traffic, force of strongly blowing winds and also its own weight.

44. Explain the theory of bending of a beam.

Ans:

- i. When a beam is bent, the strain produced is longitudinal and so elastic modulus involved is

Young's modulus.

- ii. The bending moment is the algebraic sum of moments of all restoring forces developed in the filaments of the bent beam about a neutral axis.
- iii. When a rod of length ' l ' breadth ' b ' and depth ' d ' is loaded at the centre by a load W . then depression (sag) of the rod at centre is given

$$\text{by } d = \frac{Wl^3}{4Ybd^3}$$

Q.45. How can we prevent the bending of a beam ?

Ans:

- i. When a beam of length ' l ', breadth ' b ' and depth ' d ' is loaded at the centre by a load W then bending (depression),

$$d = \frac{Wl^3}{4Ybd^3}$$

- ii. To prevent the bending of beam:
 - a. l should be small
 - b. Y should be large
 - c. d should be large
- iii. The most effective method to reduce depression in the beam of given length and material is to make depth d of the beam large as compared to its breadth b as

$$d \propto \frac{1}{d^3}$$

Q.46. Why girders are made I shaped ?

Ans:

- i. The I-shaped beam minimizes both weight and stress.
- ii. An I-beam can be much lighter, but almost as strong as for bending.
- iii. I-beam has a large load bearing surface, which prevents buckling and is enough to reduce too much buckling.
- iv. I-beam provides high bending moment so lot of material is saved. Hence, girders are made I shaped.

Q.47. What is buckling? How is it prevented ?

Ans:

- i. Although strength of a beam is increased by increasing its depth as compared to its breadth.

- ii. Too much increase, in depth **also causes** bending of the beam as shown in fig. (a). This bending of beam is called buckling.

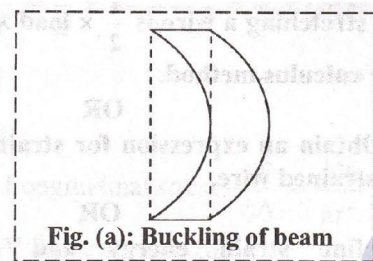


Fig. (a): Buckling of beam

- iii. To check the buckling, cross sectional shape of beam resemble I shaped as shown in fig.

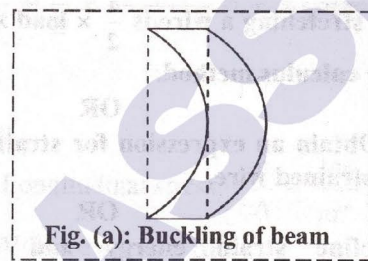


Fig. (a): Buckling of beam

Q.48. What is the basis of deciding the thickness of metallic ropes used in crane to lift the heavy weight ?

Ans:

- i. The thickness of the metallic ropes used, in cranes is decided on the basis of the elastic limit of the material of the rope and the factor of safety.
- ii. To lift a load upto 10^4 kg, the rope is made for a factor of safety of 10.
- iii. It should not break even when a load of 10^5 kgf i.e. $10^5 \times 9.8$ N is applied on it.
- iv. If ' r ' is the radius of the rope, then maximum

$$\text{stress} = \frac{10^5}{\pi r^2}$$

- v. The maximum stress (ultimate stress) should not exceed the breaking stress (5 to 20×10^8 N/m²) as well as elastic limit of steel Y_s (30×10^7 N/m²).

$$\frac{10^5}{\pi r^2} = 30 \times 10^7$$

$$\pi r^2 = 0.3333 \times 10^{-3} \text{ m}^2$$

$$A^3 \frac{W}{Y} = \frac{10^5}{30 \times 10^7}$$

$$\therefore A \geq 3.33 \times 10^{-4} \text{ m}^2$$

vi. In order to have flexibility, the rope is made up of large number of thin wires twisted together.

5.10 Elastic energy :

Q.49. Define strain energy. Prove that work

done in stretching a wire is $\frac{1}{2} \times \text{load} \times \text{extension}$, by calculus method. [Mar 09]

OR

***Obtain an expression for strain energy in a strained wire.**

OR

Define strain energy and derive an expression for it by using calculus method.

Ans: Definition:

Strain energy is defined as an elastic potential energy gained by a wire during elongation by stretching force.

Expression for strain energy:

i. Let,

L = length of wire

A = area of cross section of wire

r = radius of cross section of wire

l = elongation of the wire by applying load.

ii. If the wire is perfectly elastic then,

Young's modulus,

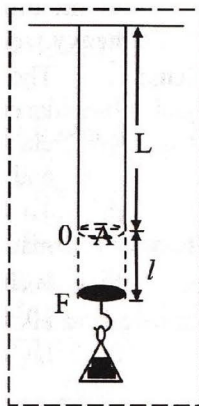
$$Y = \frac{F/A}{l/L}$$

$$= \frac{F}{A} \cdot \frac{L}{l}$$

$$\therefore F = \frac{FAL}{L}$$

..... (i)

iii. Let 'f' be the restoring force and 'x' be its corresponding extension at certain instant during the process of extension.



$$\therefore f = \frac{YAx}{L} \quad \text{..... (ii)}$$

iv. Let 'dW' be the work done for the further small extension 'dx'

$$\therefore dW = f dx$$

$$\therefore dW = \frac{YAx}{L} \cdot dx \quad \text{..... (iii)}$$

v. The total amount of work done in stretching the wire from 0 to l can be found out by integrating equation (iii).

$$W = \int_0^l dW = \int_0^l \frac{YAx}{L} dx = \frac{YA}{L} \int_0^l x dx$$

$$\therefore W = \frac{YA}{L} \left[\frac{x^2}{2} \right]_0^l$$

$$\therefore W = \frac{YAL^2}{2L}$$

$$\therefore W = \frac{YAL}{L} \cdot \frac{l}{2}$$

$$\text{But, } \frac{YAL}{L} = F$$

$$\therefore W = \frac{1}{2} \cdot F \cdot l \quad \text{..... (iv)}$$

Equation (iv) represents the work done by stretching a wire.

Work done in stretching a wire

$$W = \frac{1}{2} \times \text{load} \times \text{extension}$$

vi. Work done by stretching force is equal to strain energy gained by the wire.

$$\therefore U = W = \frac{1}{2} \cdot F \cdot l$$

$$\therefore U = \frac{1}{2} \cdot \text{load} \cdot \text{extension}$$

***Q.50. Show that strain energy per unit volume of a strained wire is (stress) (strain).**

OR

Show that strain energy per unit volume of a strained wire is half the product of stress and strain.

Ans:

Expression for strain energy per unit volume:

Refer Q.49

Strain energy in the stretched wire is given by,

$$U = \frac{1}{2} F l$$

If u is the strain energy per unit volume of stretched wire then,

$$u = \frac{U}{V} = \frac{1}{2} \frac{F l}{V} = \frac{1}{2} \frac{F l}{A L}$$

where A = area of cross section of wire

$$u = \frac{1}{2} \frac{F}{A} \frac{l}{L} = \frac{1}{2} \times \text{stress} \times \text{strain}$$

$$u = \frac{1}{2} \times \text{stress} \times \text{strain}$$

Other forms :

a. $u = \frac{1}{2} \text{ stress} \times \frac{\text{stress}}{Y}$

\ $u = \frac{1}{2} \frac{(\text{stress})^2}{Y}$

b. Also stress = $Y \times \text{strain}$

\ $u = \frac{1}{2} Y \text{ strain} \times \text{strain}$

\ $u = \frac{1}{2} Y \text{ strain}^2$

Summary:

- Elasticity is the property of material body due to which the body changes its shape, size or volume by applying an external force under elastic limit but whenever this force is removed, it regains its original shape.
- Among the existing bodies, some are elastic and some are inelastic (plastic) but no material bodies are perfectly elastic or perfectly plastic.
- For practical purposes, quartz and putty are assumed to be perfectly elastic and perfectly plastic bodies respectively.
- Stress is the force per unit area of elastic bodies,

- If stress is normal to the cross section then it is called longitudinal stress.
 - If stress is normal to change in the volume of elastic bodies then it is called volume stress. Volume stress is possessed for liquid or gases.
 - If stress is tangential to change in the shape of the body then it is called tangential stress or shear stress.
5. According to Hooke's law, stress oc strain, i.e., ratio between stress and strain is a constant known as elastic constant.

$$\frac{\text{Stress}}{\text{Strain}} = \text{Constant}$$

Hooke's law is obeyed within elastic limit but this law is not true beyond elastic limit.

- Poisson's ratio α is a pure number, because it is the ratio of two strains, i.e., ratio between lateral strain to longitudinal strain. It has no unit and dimension.
- Breaking stress is independent of the length or area of cross section of the wire. However, it depends on the material of the wire.
- If a wire is broken into two parts, the maximum load it can sustain remains same.
- Load applied to the wire depends upon area not on length.
- Strain energy is the potential energy stored in the body during extension. It is given by,

$$U = \frac{1}{2} F l$$

- Strain energy per unit volume is called energy density. It is given by,

$$u = \frac{U}{V} = \frac{1}{2} \frac{(\text{stress})^2}{Y} = \frac{1}{2} Y (\text{strain})^2$$

Formulae :

i. Longitudinal stress = $\frac{F}{A} = \frac{Mg}{\pi r^2}$

ii. Volume stress = DP

iii. Shearing stress = $\frac{F}{A}$

iv. Breaking force = $\frac{\text{Breaking force}}{\text{Area of cross section}}$

$$= \frac{F}{A}$$

2. Strain:

i. Longitudinal strain = $\frac{l}{L}$

ii. Volume strain = $\frac{DV}{V}$

iii. Shearing strain = $\frac{x}{h}$

3. Hooke's law:

$$M = \frac{\text{Stress}}{\text{Strain}}$$

4. Young's modulus:

$$Y = \frac{MgL}{pr^2l}$$

5. Bulk modulus:

$$K = \frac{\text{Bulk stress}}{\text{Bulk strain}} = V \cdot \frac{DP}{DV}$$

6. Modulus of rigidity for small angle:

$$h = \frac{F}{Aq} = \frac{Fh}{Ax}$$

7. Poisson's ratio:

$$h = \frac{LAD}{Dl} = \frac{LDr}{lr} \quad (\text{In magnitude})$$

8. Elongation in the wire due to load:

$$l = \frac{MgL}{pr^2Y}$$

9. Work done in stretching a wire:

$$W = \frac{1}{2} \cdot F \cdot l$$

10. Depression of c.g./bending of beam:

$$d = \frac{Wl^3}{4bd^3Y}$$

11. Elastic energy :

i. Elastic potential energy,

$$U = \frac{1}{2} \cdot F \cdot l = \frac{1}{2} \cdot \text{stress} \cdot \text{strain} \cdot \text{volume}$$

ii. Strain energy per unit volume,

$$u = \frac{U}{V} = \frac{1}{2} \cdot \frac{(\text{Stress})^2}{Y} = \frac{1}{2} (\text{Strain})^2 \cdot Y$$

Solved Problems

Example 1

A body of mass 10 kg is suspended from a wire of length 4 m and radius 3 mm, calculate the stress.

Solution:

Given : $M = 10 \text{ kg}$, $L = 4 \text{ m}$, $r = 3 \times 10^{-3} \text{ m}$

To find : Stress

Formula : Stress = $\frac{F}{A} = \frac{Mg}{pr^2}$

Calculation: From formula,

$$\text{Stress} = \frac{10 \cdot 9.8}{3.14 \cdot (3 \cdot 10^{-3})^2}$$

$$\text{Stress} = 3.46 \times 10^6 \text{ N/m}^2$$

Ans: The stress of the body is $3.46 \times 10^6 \text{ N/m}^2$.

Example 2

A piece of copper having a rectangular cross-section of 15.2 mm × 19.14 mm is pulled in tension with 44500 N force, producing only elastic deformation. Calculate the resulting strain. [Rigidity modulus of copper = $42 \times 10^9 \text{ Nm}^{-2}$]

Solution: (NCERT)

Given: $A = 15.2 \times 19.14 \times 10^{-6} \text{ m}^2$

$F = 44500 \text{ N}$, $h = 42 \times 10^9 \text{ N m}^{-2}$

To find : Strain (q)

Formula : $h = \frac{F}{Aq}$

Calculation: From formula,

$$q = \frac{F}{Ah}$$

$$q = 3.64 \times 10^{-3}$$

Ans: The strain produced in the piece of copper is 3.64×10^{-3} .

Example 3

When a load of 5 kg is attached to the free end of a suspended wire of length 4 m and diameter 2 mm, the elongation produced is 0.5 mm. Calculate the stress, strain and Young's modulus of the material of wire.

Solution:

Given: $M = 5 \text{ kg}$, $L = 4 \text{ m}$,
 $D = 2 \text{ mm}$, $r = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}$,
 $l = 0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ m}$

To find: i. Stress
 ii. Strain
 iii. Young's modulus (Y)

Formulae: i. $\text{Stress} = \frac{Mg}{\pi r^2}$

ii. $\text{Strain} = \frac{l}{L}$

$Y = \frac{\text{Stress}}{\text{Strain}}$

Calculation: From Formula

$$\text{Stress} = \frac{5 \times 9.8}{3.14 \times (10^{-3})^2}$$

$$\text{Stress} = 1.56 \times 10^7 \text{ N/m}^2$$

From formula (ii),

$$\text{Strain} = \frac{0.5 \times 10^{-3}}{4}$$

$$\text{Strain} = 1.25 \times 10^{-4}$$

From formula (iii),

$$Y = \frac{1.56 \times 10^7}{1.25 \times 10^{-4}}$$

$$Y = 1.248 \times 10^{11} \text{ N/m}^2$$

Ans:

- The stress of the material of wire is $1.56 \times 10^7 \text{ N/m}^2$.
- The strain of the material of wire is 1.25×10^{-4} .
- Young's modulus of the material of wire is $1.248 \times 10^{11} \text{ N/m}^2$.

Example 4

A wire of length 2 m and cross sectional area 10^{-4} m^2 is stretched by a load 102 kg. The wire is stretched by 0.1 cm. Calculate longitudinal stress, longitudinal strain, Young's modulus of wire.

Solution:

Given: $L = 2 \text{ m}$, $A = 10^{-4} \text{ m}^2$, $M = 102 \text{ kg}$,
 $g = 9.8 \text{ m/s}^2$, $l = 0.1 \text{ cm} = 0.1 \times 10^{-2} \text{ m}$

To find:

- Longitudinal stress = $\frac{Mg}{A}$
- Longitudinal strain = $\frac{l}{L}$
- $Y = \frac{\text{longitudinal stress}}{\text{longitudinal strain}}$

Calculation: From formula (i),

$$\text{i. Longitudinal stress} = \frac{102 \times 9.8}{10}$$

$$\text{ii. Longitudinal stress} = 9.996 \times 10^6 \text{ N/m}^2$$

$$\text{iii. Longitudinal stress} \gg 1 \times 10^7 \text{ N/m}^2$$

From formula (ii),

$$\text{Longitudinal strain} = \frac{0.1 \times 10^{-2}}{2}$$

$$\text{Longitudinal strain} = 5 \times 10^{-4}$$

From formula (iii),

$$Y = \frac{1 \times 10^7}{5 \times 10^{-4}} = 0.2 \times 10^{11} \text{ N/m}^2$$

$$Y = 20 \times 10^9 \text{ Nm}^2$$

Ans:

- The longitudinal stress of the wire is $9.996 \times 10^6 \text{ N/m}^2$.
- The longitudinal strain of the wire is 5×10^{-4} .
- Young's modulus of wire is $20 \times 10^9 \text{ Nm}^2$.

Example 5

Find the greatest length of a copper wire that can hang vertically without breaking.

[Breaking stress for copper = $5 \times 10^8 \text{ N/m}^2$ and density of copper = $9 \times 10^3 \text{ kg/m}^3$]

Solution:

Given: Breaking stress = $5 \times 10^8 \text{ N/m}^2$,
 $\rho = 9 \times 10^3 \text{ kg/m}^3$

To find: Length (L)

$$\text{Formula: Breaking stress} = \frac{F}{A} = \frac{Mg}{A}$$

Calculation: Mass of the wire, M

$$m = \rho V = \rho AL$$

From formula,

$$\text{Breaking stress} = \frac{r ALg}{A} = r Lg$$

$$L = \frac{\text{Breaking stress}}{r g}$$

$$= \frac{5 \times 10^8}{9 \times 10^3 \times 9.8} = \frac{5 \times 10^6}{88.2}$$

$$L = 5.669 \times 10^3 \text{ m}$$

Ans: The maximum length of the copper wire hung vertically without breaking is

$$5.669 \times 10^3 \text{ m.}$$

***Example 6**

The length of wire increases by 9 mm when weight of 2.5 kg is hung from the free end of wire. If all conditions are kept the same and the radius of wire is made thrice the original radius, find the increase in length.

Solution:

Given: $l_1 = 9 \text{ mm} = 9 \times 10^{-3} \text{ m}$, $M = 2.5 \text{ kg}$

$$r_2 = 3r_1, Y_1 = Y_2 = Y \text{ (material is same)}$$

To find: Increase in length (l_2)

Formula: $Y = \frac{FL}{Al}$

$$l_1 = \frac{FL}{YA_1} \text{ and } l_2 = \frac{FL}{YA_2}$$

$$\therefore \frac{l_1}{l_2} = \frac{A_2}{A_1} = \frac{\pi r_2^2}{\pi r_1^2}$$

$$\therefore \frac{l_1}{l_2} = \frac{\pi r_2^2}{\pi r_1^2} = \frac{\pi (3r_1)^2}{\pi r_1^2}$$

$$\therefore \frac{l_1}{l_2} = 9$$

$$l_2 = \frac{l_1}{9} = \frac{9 \times 10^{-3}}{9} = 10^{-3} \text{ m}$$

$$l_2 = 1 \text{ mm}$$

Ans: The increase in length of the wire is **1 mm**.

Example 7

A wire of diameter 0.5 mm and length 2 m is stretched by applying a force of 2 kg wt.

Calculate the increase in length of the wire.

[$g = 9.8 \text{ m/s}^2$, $Y = 9 \times 10^{10} \text{ N/m}^2$]

Solution:

Given: $L = 2 \text{ m}$, $F = 2 \text{ kg wt}$,

$$d = 0.5 \text{ mm} = 5 \times 10^{-4} \text{ m},$$

$$r = 2.5 \times 10^{-4} \text{ m},$$

$$Y = 9 \times 10^{10} \text{ N/m}^2$$

$$g = 9.8 \text{ m/s}^2$$

To find: Increase in length (l)

Formula: $Y = \frac{FL}{Al} = \frac{MgL}{\pi r^2 l}$

Calculation: From formula

$$l = \frac{1}{Y} \cdot \frac{MgL}{\pi r^2} = \frac{2 \times 9.8 \times 2}{9 \times 10^{10} \times 3.14 (2.5 \times 10^{-4})^2}$$

$$= \frac{19.6 \times 2}{9 \times 10^{10} \times 3.14 (2.5 \times 10^{-4})^2}$$

$$l = 2.219 \times 10^{-3} \text{ m}$$

Ans: The increase in length of the wire is

$$2.219 \times 10^{-3} \text{ m.}$$

Example 8

A steel wire of length 4 m has a mass of 25 g. It is elongated by 1.25 mm when stretched by a weight of 5 kg. Calculate the Young's modulus of steel.

[Density of steel is $7.8 \times 10^3 \text{ kg/m}^3$]

Solution:

Given: $L = 4 \text{ m}$, $l = 1.25 \text{ mm} = 1.25 \times 10^{-3} \text{ m}$,

$$m = 25 \times 10^{-3} \text{ kg}, M = 5 \text{ kg},$$

$$r = 7.8 \times 10^3 \text{ kg/m}^3$$

To find: Young's modulus (Y)

Formulae: $Y = \frac{MgL}{\pi r^2 l} = \frac{MgL}{Al}$

Calculation: Since, $V = \frac{m}{r} = \frac{25 \times 10^{-3}}{7.8 \times 10^3}$

Also, $V = AL$

$$A = \frac{V}{L} = \frac{25 \times 10^{-3}}{7.8 \times 10^3 \times 4}$$

$$A = 0.8 \times 10^{-6} \text{ m}^2$$

From formula,

$$Y = \frac{5 \times 9.8 \times 4}{0.8 \times 10^{-6} \times 1.25 \times 10^{-3}}$$

$$Y = 19.6 \times 10^{10} \text{ N/m}^2$$

Ans: Young's modulus of steel is

$$19.6 \times 10^{10} \text{ N/m}^2.$$

Example 9

A steel wire of length 4.7 m and cross-section $3.0 \times 10^{-5} \text{ m}^2$ is stretched by the same amount as a copper wire of length 3.5 m and cross-section $4.0 \times 10^{-5} \text{ m}^2$ under a given load. What is the ratio of the Young's modulus of steel to that of copper?

Solution: $L_s = 4.7 \text{ m}, A_s = 3.0 \times 10^{-5} \text{ m}^2,$
 $L_c = 3.5 \text{ m},$
 $A_c = 4.0 \times 10^{-5} \text{ m}^2,$
 $F_s = F_c$ (same load applied)

To find : Ratio of Young's modulus $\frac{Y_s}{Y_c}$

$$\text{Formula: } Y = \frac{FL}{Al}$$

Calculation : From formula

$$Y_s = \frac{F_s L_s}{A_s l_s} = \frac{F_s \cdot 4.7}{3.0 \cdot 10^{-5} \cdot l_s}$$

$$Y_c = \frac{F_c L_c}{A_c l_c} = \frac{F_c \cdot 3.5}{4.0 \cdot 10^{-5} \cdot l_c}$$

$$\frac{Y_s}{Y_c} = \frac{F_s \cdot 4.7}{3.0 \cdot 10^{-5} \cdot l_s} = \frac{4.0 \cdot 10^{-5} \cdot l_c}{F_c \cdot 3.5}$$

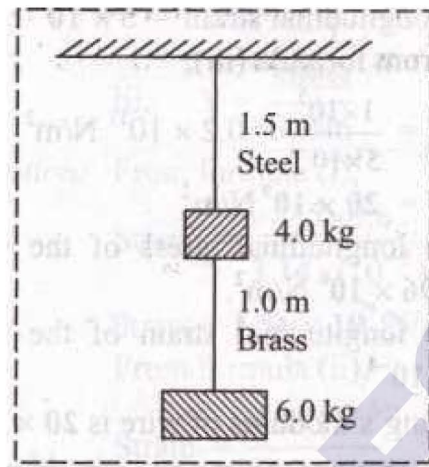
Ans: The ratio of the Young's modulus of steel to that of copper is **1.79**.

Example 10

Two wires of diameter 0.25 cm, one made of steel and the other made of brass are loaded as shown in the figure. The unloaded length of steel wire is 1.5 m and that of brass wire is 1.0 m. Compute the elongations of the steel and the brass wires.

$$[Y_s = 2.0 \times 10^{11} \text{ Nm}^{-2},$$

$$Y_B = 0.91 \times 10^{11} \text{ NnT}^2] \quad (\text{NCERT})$$



Solution :

Given: $D = 0.25 \text{ cm} = 0.25 \times 10^{-2} \text{ m},$

$$L_s = 1.5 \text{ m}, L_B = 1 \text{ m},$$

$$Y_s = 2.0 \times 10^{11} \text{ Nm}^{-2},$$

$$Y_B = 0.91 \times 10^{11} \text{ N m}^{-2}$$

Elongations of brass wire (l_B)

Elongations of steel wire (l_s)

Formula : $Y = \frac{FL}{Al}$

Calculation: Since, $A = \frac{\pi D^2}{4}$

$$A = \pi \cdot \frac{(0.25 \cdot 10^{-2})^2}{4}$$

$$A = 4.9 \times 10^{-6}$$

From formula,

$$l = \frac{FL}{AY}$$

For brass wire, load = 6 kg

$$l_B = \frac{MgL}{AY} = \frac{6 \cdot 9.8 \cdot 1}{4.9 \cdot 10^{-6} \cdot 0.91 \cdot 10^{11}}$$

$$l_B = 1.32 \times 10^{-4} \text{ m}$$

Free steel wire, load = 4 + 6 = 10 kg

$$l_s = \frac{F_s L}{AY_s} = \frac{MgL}{AY}$$

$$l_s = \frac{10 \cdot 9.8 \cdot 1.5}{4.9 \cdot 10^{-6} \cdot 10^{11}}$$

$$l_s = 1.5 \times 10^{-4} \text{ m}$$

Ans: The elongation of the steel wire is 1.5×10^{-4} m and that of brass wires is 1.32×10^{-4} m.

*** Example 11**

Two wires of equal cross section one made up of aluminium and other of brass are joined end to end. When the combination of wires is kept under tension the elongation in wires are found to be equal. Find the ratio of lengths of two wires.

[$Y_{AL} = 7 \times 10^{10}$ N/m² and $Y_{brASS} = 9.1 \times 10^{10}$ N/m²]

Solution:

Given : $A_{AL} = A_B, l_{AL} = l_B$

To find : Ratio of length $\frac{l_{AL}}{l_B}$

Formula : $Y = \frac{FL}{A\Delta l}$

$$Y_{AL} = \frac{FL_{AL}}{A\Delta l_{AL}} \text{ and } Y_B = \frac{FL_B}{A\Delta l_B}$$

$$\frac{Y_{AL}}{Y_B} = \frac{L_{AL}}{L_B} \cdot \frac{l_B}{l_{AL}}$$

Since, $l_B = l_{AL}$

$$\frac{Y_{AL}}{Y_B} = \frac{L_{AL}}{L_B}$$

$$\frac{L_{AL}}{L_B} = \frac{7 \times 10^{10}}{9.1 \times 10^{10}}$$

$$\frac{L_{AL}}{L_B} = \frac{0.7693}{1}$$

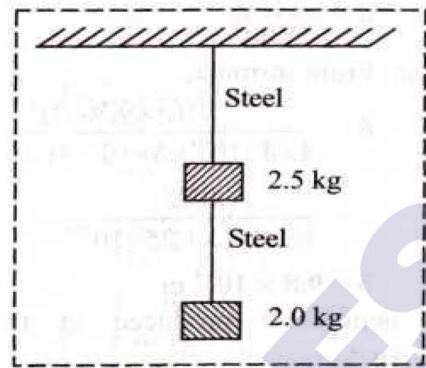
Ans: The ratio of length of two wires is $\frac{0.7693}{1}$

*** Example 12**

One end of steel wire is fixed to a ceiling and load of 2.5 kg is attached to the free end of the wire. Another identical wire is attached to the bottom of load and another load of 2.0 kg, is attached to the lower end of this wire. Compute the longitudinal strain produced in both the wires, if the cross-sectional area of wires is 10^{-4} m².

[$Y_{steel} = 20 \times 10^{10}$ N/m²]

Solution :



Given : $M_1 = 2.5$ kg, $M_2 = 2$ kg, $A = 10^{-4}$ m², $Y_{steel} = 20 \times 10^{10}$ N/m²

To find: Longitudinal strain of 1st wire (Strain₁), Longitudinal strain of 2nd wire (Strain₂)

Calculation: For 1st wire, $M = 2.5 + 2 = 4.5$ kg

$$Y = \frac{MgL}{A\Delta l}$$

$$\Delta l = \frac{MgL}{YA}$$

$$\Delta l = \frac{4.5 \times 9.8}{20 \times 10^{10} \times 10^{-4}}$$

$$\Delta l = 2.205 \times 10^{-6}$$

For 2nd wire, $M_2 = 2$ kg

$$\Delta l = \frac{l_2 M_2 g}{L YA}$$

$$= \frac{2.0 \times 9.8}{20 \times 10^{10} \times 10^{-4}}$$

$$\Delta l = 9.8 \times 10^{-7}$$

Ans: The longitudinal strain of 1st wire is 2.205×10^{-6} and that of 2nd wire is 9.8×10^{-7} .

Example 13

A bar of length 100 cm is supported at its two ends. The breadth and depth of bar are 5 cm and 0.5 cm respectively. A mass of 100 g is suspended at the centre of bar. Compute the depression produced in the bar. [$Y = 4 \times 10^{10}$ N/m²]

Solution:

Given: $l = 100 \text{ cm} = 1 \text{ m}$,
 $b = 5 \text{ cm} = 5 \times 10^{-2} \text{ m}$,
 $d = 0.5 \text{ cm} = 0.5 \times 10^{-2} \text{ m}$,
 $m = 100 \text{ g} = 0.1 \text{ kg}$,
 $Y = 4 \times 10^{10} \text{ N/m}^2$

To find: Depression (d)

$$\text{Formula: } d = \frac{Wl^3}{4Ybd^3}$$

Calculation: From formula

$$d = \frac{0.1 \times 9.8 \times (1)^3}{4 \times 4 \times 10^{10} \times 5 \times 10^{-2} \times (0.5 \times 10^{-2})^3}$$

$$= \frac{0.98}{80 \times 10^8 \times 1.25 \times 10^{-7}}$$

$$d = 9.8 \times 10^{-4} \text{ m}$$

Ans: The depression produced in the bar is $9.8 \times 10^{-4} \text{ m}$

Example 14

Calculate the decrease in the volume of 4 litre of water, when it is subjected to 11 atmospheric pressure.

[1 atm Pressure = $1.013 \times 10^6 \text{ dyne/cm}^2$
 and Bulk modulus of water = $2 \times 10^{10} \text{ dyne/cm}^2$]

Solution:

Given: $V = 4 \text{ litre} = 4000 \text{ cm}^3$,
 $DP = 11 - 1 = 10 \text{ atm}$
 $= 10 \times 1.013 \times 10^6 \text{ dyne/cm}^2$

To find: Decrease in volume (DV)

$$\text{Formula: } K = V \times \frac{DP}{DV}$$

Calculation: From formula,

$$DV = \frac{V \times DP}{K}$$

$$= \frac{10 \times 1.013 \times 10^6 \times 4000}{2 \times 10^{10}}$$

$$= \frac{1.013 \times 4}{2}$$

$$DV = 2.026 \text{ cm}^3$$

Ans: The decrease in the volume of water is 2.026 cm^3 .

*Example 15

Find the increase in pressure required to decrease volume of mercury by 0.001 %. [Bulk modulus of mercury – $2.8 \times 10^{10} \text{ N/m}^2$]

Solution:

Given: $DV = 0.001 \%$ of V

$$\frac{DV}{V} = \frac{0.001}{100} = 10^{-5}$$

$$K = 2.8 \times 10^{10} \text{ N/m}^2$$

To find: Increase in pressure (DP)

$$\text{Formula: } K = V \times \frac{DP}{DV} \text{ (Numerically)}$$

Calculation: From formula,

$$DP = K \times \frac{DV}{V}$$

$$DP = 2.8 \times 10^{10} \times 10^{-5}$$

$$DP = 2.8 \times 10^5 \text{ N/m}^2$$

Ans: The increase in pressure required decrease volume of mercury by 0.001 % is $2.8 \times 10^5 \text{ N/m}^2$.

Example 16

Determine the volume contraction of solid copper cube, 10 cm on an edge, when subjected to a hydraulic pressure of $7.0 \times 10^6 \text{ Pa}$. [Bulk modulus of copper = $140 \times 10^9 \text{ Pa}$] (NCERT)

Solution:

Given: $L = 10 \text{ cm} = 0.1 \text{ m}$, $DP = 7 \times 10^6 \text{ Pa}$,

$$K = 140 \times 10^9 \text{ Pa}$$

To find: Volume contraction (DV)

$$\text{Formula: } K = V \times \frac{DP}{DV}$$

Calculation: From formula,

$$DV = L^3 \times \frac{DP}{K}$$

$$= (0.1)^3 \times \frac{7 \times 10^6}{140 \times 10^9}$$

$$DV = 5 \times 10^{-8} \text{ m}^3$$

Ans: The volume contraction of solid copper cube $5 \times 10^{-8} \text{ m}^3$.

Example 17

A solid brass sphere of volume 0.305 m^3 is dropped in ocean, where water pressure is $2 \times 10^7 \text{ N/m}^2$. If bulk modulus of liquid is $6.1 \times 10^{10} \text{ N/m}^2$, what is change in volume of sphere?

Solution:

Given: $V = 0.305 \text{ m}^3$ $DP = 2 \times 10^7 \text{ N/m}^2$,
 $K = 6.1 \times 10^{10} \text{ N/m}^2$

To find: Change in volume (DV)

Formula: $K = V \cdot \frac{DP}{DV}$

Calculation: From formula,

$$DV = \frac{V \cdot DP}{K}$$

$$DV = \frac{0.305 \cdot 2 \cdot 10^7}{6.1 \cdot 10^{10}}$$

$$DV = 10^{-4} \text{ m}^3$$

Ans: The change in volume of sphere is 10^{-4} m^3 .

Example 18

The Marina trench is located in the Pacific Ocean and at one place it is, nearly 11 km beneath the surface of water. The water pressure at the bottom of the trench is about $1.1 \times 10^8 \text{ Pa}$. A steel ball of initial volume 0.32 m^3 is dropped into the ocean and falls to the bottom of the trench. What is the change in the volume of the ball when it reaches the bottom?

[$K = 1.6 \times 10^{11} \text{ N/m}^2$]

Solution:

Given: $DP = 1.1 \times 10^8 \text{ Pa}$,

$V = 0.32$ $K = 1.6 \times 10^{11} \text{ N/m}^2$

To find: Change in volume (DV)

Formula: $K = V \cdot \frac{DP}{DV}$

Calculation: From formula

$$DV = \frac{V \cdot DP}{K} = \frac{(1.1 \cdot 10^8) \cdot 0.32}{1.6 \cdot 10^{11}}$$

$$DV = 2.2 \times 10^{-4} \text{ m}^3$$

Ans: The change in the volume of the ball when it reaches the bottom is $2.2 \times 10^{-4} \text{ m}^3$.

EXERCISE -1**Section - Pactice Problems**

- The radius of a wire is 0.2 mm. Find the force required to produce an elongation of 0.2% of the initial length of the wire.
[$Y = 1.7 \times 10^{11} \text{ N/m}^2$]
- Two wires of same material have radii and lengths in the ratio 1 : 2. If the extensions are equal, what is the ratio of loads applied?
- When a liquid of volume 4 litre is subjected to an additional pressure of $5 \times 10^7 \text{ Nm}^2$, the change in volume is found to be 4 ml. Calculate the bulk modulus of the liquid.
- A wire of length 2 m and diameter 1 mm when stretched by a load of 5 kg extends by 1 mm. Calculate Y.
- A tangential force of $2.5 \times 10^5 \text{ N}$ applied on a surface of area 4 cm displaces it sideways with respect to the opposite fixed layer so that the angle of shear is 2° . Find the modulus of rigidity of the material.
- The radius of a copper wire is 4 mm. What force is required to stretch the wire by 20 % of its length, assuming that the elastic limit is not exceeded? [$Y = 20 \times 10^8 \text{ N/m}^2$],
- The bulk modulus of a liquid is $1.25 \times 10^{10} \text{ N/m}^2$. Calculate the % decrease in volume of the liquid when the pressure is increased by 20 atmosphere.
[1 atmosphere = 10^5 N/m^2]
- A load of 2 kg wt. is applied to the end of a wire of length 2 m and diameter $2 \times 10^{-4} \text{ m}$, suspended from a rigid support. If Young's modulus of the material of the wire is $2 \times 10^{11} \text{ N/m}^2$, find the extension produced in the wire.
- A steel wire of diameter 1 mm and length 2 m is stretched by a force of 2 kg wt. Calculate
 - increase in length of wire
 - strain.
 [$g = 9.8 \text{ m/s}^2$, $Y = 2 \times 10^{11} \text{ N/m}^2$]
- A wire of length 5 m and radius 1 mm is stretched by a load of 10 N. If $Y = 2 \times 10^{11} \text{ N/m}^2$, find the work done/unit volume of wire.
- Strain energy of a stretched wire is $10 \times 10^{-3} \text{ J}$ and strain energy/unit volume of same wire is $6 \times 10^2 \text{ J/m}^3$.

Calculate volume of wire.

- A copper wire is stretched by 1% of its length. Calculate energy stored/ unit volume of wire.
[$Y = 1.1 \times 10^{11} \text{ N/m}^2$]
- How much should the pressure on a litre of water be changed to compress it by 0.10 %?
[Bulk modulus of elasticity of water
 $= 2.2 \times 10^9 \text{ N/m}^2$]

Section- B Theoretical Board Questions

- Define shearing strain and shearing stress. State their SI units. [Oct00] [Mar 98]
- Define**
 - Elasticity
 - Young's Modulus
 - Poisson's ratio. [Oct 01]
- What is Poisson's ratio? Why it does not have any unit? [Feb 02]
- Explain the behaviour of a wire under steadily increasing load with a neat graph. Also explain permanent set. [Oct 02]
- Define Young's modulus of elasticity. Write its SI unit and dimensions. Stress applied to a wire is equal to its Young's modulus. Find the change in length of the wire in terms of its original length. [Oct 03]
- State Hooke's law. With the help of a neat graph discuss the behaviour of a wire subjected to increasing load. [Feb 06]
- With the help of neat graph explain the behaviour of a wire under increasing load.
[Oct 08,10]
- Prove that strain energy per unit volume of a wire = $1/2 \times \text{stress} \times \text{strain}$. [Oct 09]
- Derive an expression for strain energy. Hence show that strain energy per unit volume is directly proportional to square of the stress.
[Mar 10]
- State Hooke's law. Give S.I. unit and dimensions of modulus of elasticity. [Oct 10]
- Distinguish between elastic and plastic body.
[Oct 11]
- Define three moduli of elasticity. State their formula. [Mar 12]
- Explain the behaviour of metal wire under increasing load. [Oct13]

Section C: Numerical Board Problems

1. A 3 cm long copper wire is stretched to increase in length by 0.3 cm. Find the lateral strain produced in the wire, if the Poisson's ratio for copper is 0.26. [Mar 96]
2. The elastic limit of copper is $1.5 \times 10^4 \text{ N/m}^2$. Find the minimum radius a copper wire must have, if its elastic limit is not to be exceeded under a load of 10 kg. ($g = 9.8 \text{ m/s}^2$) [Oct 97]
3. Find the longitudinal stress to be applied to a wire to decrease its diameter uniformly by 10%. [Poisson's ratio = 0.25, Young's modulus = $2 \times 10^{10} \text{ N/m}^2$]
4. What would be the greatest length of a steel wire which when fixed at one end can hang freely without breaking?
[Density of steel = 7800 kg/m^3 , Breaking stress for steel = $7.8 \times 10^8 \text{ N/m}^2$] [Oct 98]
5. Two wires have lengths in the ratio 4 : 3, diameters in the ratio 2 : 1 and Young's moduli in the ratio 1 : 2. Calculate the ratio of the elongations produced in the wires when they are subjected to the same stretching force.
[Oct 02]
6. A metal plate has an area of a face $l \times l \text{ m}$ and thickness of 1 cm. One face of larger area is fixed and a tangential force is applied to the opposite face. The displacement of the edge produced thereby is 0.005 cm. Find the shearing stress, strain and magnitude of tangential force applied. [Modulus of rigidity of the metal is, $h = 8.4 \times 10^{10} \text{ N/m}^2$]
[Feb 04]
7. A 3m long copper wire is stretched to produce an extension of 0.3 cm. If Poisson's ratio for copper is 0.26, what is the lateral strain produced in the wire. [Mar 08]
8. When the pressure applied to one litre of a liquid is increased by $2 \times 10^6 \text{ N/m}^2$, its volume decreases by 10^{-6} m^3 . Find the bulk modulus of the liquid. [Oct 08]
9. What pressure is required to reduce the volume of a lead block by 1%? [Given = Bulk modulus of lead = $6 \times 10^9 \text{ S.I. unit}$]

SECTION - D MULTIPLE CHOICE QUESTION

1. Which one of the following substance possess the highest elasticity?
(A) Rubber (B) Glass
(C) Steel (D) Copper
2. Solids which break or rupture before the elastic limits are called
(A) brittle (B) ductile
(C) malleable (D) elastic
3. The length of a wire increases by 1% by a load of 2 kg wt. The linear strain produced in the wire will be
(A) 0.02 (B) 0.001
(C) 0.01 (D) 0.002
4. S.I unit of stress is
(A) Newton/ metre
(B) Newton/ metre²
(C) Newton /metre
(D) Newton/metre³
5. A wire of length 'L', radius 'r' when stretched with a force 'F' changes in length by 7'. What will be the change in length of a wire of same material having length '2L', radius '2r' and stretched by a force '2F'?
(A) $l/2$ (B) l
(C) $2l$ (D) $4l$
6. Poisson's ratio is the ratio of lateral strain to [Mar 09]
(A) Volume stress
(B) Shearing strain
(C) Longitudinal stress
(D) Longitudinal strain
7. A wire is stretched to double of its length the strain is
(A) 2 (B) 1
(C) zero (D) 0.5
8. At a yield point, tangent to the tensile stress versus tensile strain curve, becomes parallel to strain axis. This implies
(A) Hooke's law is obeyed.
(B) wire is permanently deformed.
(C) extension begins to increase even without any increase in stress.

- (D) the maximum stress value a wire can take has reached.
9. Which one of the following quantities does not have the unit of force per unit area?
- (A) Stress
(B) Strain
(C) Young's modulus of elasticity
(D) Pressure
10. The change in the shape of a regular body is due to
- (A) bulk strain
(B) shearing strain
(C) longitudinal strain
(D) metallic strain
11. The shear modulus of a liquid is
- (A) zero
(B) infinite
(C) unity
(D) some other finite value
12. From stress against strain graph, behavior of wire between elastic limit and yield point is _____ **[Mar 12, Feb 2013 old course]**
- (A) perfectly elastic
(B) perfectly plastic
(C) formation of neck
(D) elastic but with permanent deformation
13. An external force of 10 N acts normally on a square area of each side 50 cm. The stress produced in equilibrium state is
- (A) 10 N/m^2 (B) 20 N/m^2
(C) 40 N/m^2 (D) 50 N/m^2
14. The Young's modulus for steel $15 \times 10^{10} \text{ N/m}^2$. What is its value in CGS system? is
- (A) $15 \times 10^9 \text{ dyne/cm}^2$
(B) $15 \times 10^{10} \text{ dyne/cm}^2$
(C) $15 \times 10^{11} \text{ dyne/cm}^2$
(D) $15 \times 10^8 \text{ dyne/cm}^2$
15. The force required to stretch a steel wire 2 cm^2 in cross-section to double its length is [Young's modulus for steel, $Y = 2 \times 10^{11} \text{ N/m}^2$]
- (A) $2 \times 10^7 \text{ N}$ (B) $3 \times 10^7 \text{ N}$
(C) $4 \times 10^7 \text{ N}$ (D) $5 \times 10^7 \text{ N}$
16. When a solid rubber ball is subjected to a uniform stress of $1.5 \times 10^4 \text{ N/m}^2$ its volume is reduced by 15%. The bulk modulus of rubber is
- (A) 10^2 N/m^2 (B) 10^3 N/m^2
 10^4 N/m^2 (D) 10^5 N/m^2
17. For which of the following is the modulus of rigidity highest?
- (A) Aluminium (B) Quartz
(C) Rubber (D) Water
18. A force of 100 N produces a change of 0.1% in a length of wire of area of cross section 1 mm. Young's modulus of the wire is
- [Mar 10]**
- (A) 10^5 N/m^2 (B) 10^9 N/m^2
(C) 10^{11} N/m^2 (D) 10^{12} N/m^2
19. A spherical ball is compressed by 0.01% when a pressure of 100 atmosphere is applied on it. Its bulk modulus of elasticity in dyne/cm^2 will be approximately
- (A) 10^{12} (B) 10^{14}
(C) 10^6 (D) 10^2
20. and a steel wire ($Y = 1 \times 10^{11} \text{ N/m}^2$) of length 4 m each of cross-section 10^{-5} m^2 are fastened end to end and stretched by a tension of 100 N. The elongation produced in the copper wire is
- (A) 0.2 mm (B) 0.4 mm
(C) 0.6 mm (D) 0.8 mm
21. A copper solid cube of 60 cm side is subjected to a pressure of $2.5 \times 10^4 \text{ Pa}$. If the bulk modulus of copper is $1.25 \times 10^{11} \text{ N/m}^2$, then change in the volume of cube is
- (A) -43.2 m^3 (B) -43.2 m^2
(C) -43.2 m^2 (D) -43.2 mm^3
22. Modulus of rigidity of a liquid is
- (A) non zero constant
(B) ∞
(C) zero
(D) cannot be predicted
23. The strain energy per unit volume of the wire under increasing load is [Oct 10]
- (A) $\frac{1}{2} \times (\text{stress})^2 \times \text{strain}$
(B) $\frac{1}{2} \times \text{stress} \times (\text{strain})^2$

- (C) $0.5 \times \text{stress} \times \text{strain}$
- (D) $0.5 \times (\text{strain})^2 \times \frac{1}{2}$
24. When a force is applied to the free end of a metal wire, metal wire undergoes
- (A) longitudinal and lateral extension.
 (B) longitudinal extension and lateral contraction.
 (C) longitudinal contraction and lateral extension.
 (D) longitudinal and lateral contraction.
25. For homogeneous isotropic material the poisson's ratio lies between
- (A) -1 and -0.5 (B) -1 and 0
 (C) -1 and $+0.5$ (D) -1 and $+1$
26. An elongation of 0.2% in a wire of cross-section 10^{-4} m^2 causes a tension of 1000 N . Then its Young's modulus is
- (A) $6 \times 10 \text{ N/m}$
 (B) $5 \times 10^9 \text{ N/m}$
 (C) 10^8 N/m^2
 (D) 10^7 N/m^2
27. When a mass of 4 kg is suspended from a string its length is l_1 . If a mass of 6 kg is suspended its length is l_2 . What will be its length when a mass of 10 kg is suspended from it? [Take $g = 10 \text{ ms}^{-2}$]
- (A) $l_2 + l_2$ (B) $3l_2 - 2l_1$
 (C) $2l_2 - 3l_1$ (D) $2l_1 + l_2$
28. Within the elastic limit, the slope of graph between stress against strain gives .
 [Oct 11]
- (A) compressibility
 (B) Poisson's ratio
 (C) modulus of elasticity
 (D) extension
29. A force of 1 N doubles the length of a cord having cross-sectional area 1 mm^2 . The Young's modulus of the material of cord is
- (A) 1 Nm^{-2} (B) $0.5 \times 10^6 \text{ Nm}^2$
 (C) 10^6 Nm^2 (D) $2 \times 10^6 \text{ Nm}^2$
30. Young's modulus of a wire is Y , strain energy per unit volume is E , then its strain is given by
- (A) $\sqrt{\frac{Y}{2E}}$ (B) $\sqrt{E/Y}$
 (C) $\sqrt{\frac{2E}{Y}}$ (D) $2EY$
31. A beam of metal supported at the two ends is loaded at the centre. The depression at the centre is proportional to
- (A) Y (B) Y^2
 (C) $1/Y$ (D) $1/Y^2$
32. The Young's modulus of a material is 10^{11} Nm^2 and its Poisson's ratio is 0.2 . The modulus of rigidity of the material is
- (A) $0.42 \times 10^{11} \text{ N/m}^2$
 (B) $0.42 \times 10^{14} \text{ N/m}^2$
 (C) $0.42 \times 10^{16} \text{ N/m}^2$
 (D) $0.42 \times 10^{18} \text{ N/m}^2$
33. How does the isothermal bulk modulus K_i depend on pressure P of the gases?
- (A) $K_i = P$
 (B) $K_i > P$
 (C) $K_i < P$
 (D) No such relation is possible
34. In equilibrium the tensile stress to which a wire of radius r is subjected by attaching a mass 'm' is _____ [Mar 11]
- (A) $\frac{mg}{pr}$ (B) $\frac{mg}{2pr}$
 (C) $\frac{mg}{pr^2}$ (D) $\frac{mg}{2pr^2}$
35. When load is applied to a wire, the extension is 3 mm , the extension in the wire of same length but half the radius by the same load is ;
- (A) 0.75 mm (B) 6 mm
 (C) 1.5 mm (D) 12.0 mm
36. Two pieces of wire A and B of the same material have their lengths in the ratio $1 : 2$, and their diameters are in the ratio $2 : 1$. If they are stretched by the same force, their elongations will be in the ratio
- (A) $2 : 1$ (B) $1 : 4$
 (C) $1 : 8$ (D) $8 : 1$

37. The compressibility of water is 4×10^{-5} per unit atmospheric pressure! The decrease in volume of 100 cm of water under a pressure of 100 atmosphere will be
 (A) 0.004 cm (B) 0.025 cm
 (C) 0.4 cm³ (D) 4×10^{-5} cm³
38. When the pressure applied to one litre of a liquids increased by 2×10^6 N/m². Its Volume decreases by 1 cm³. The bulk modulus of the liquid is
 (A) 2×10 N/m (B) 2×10 dyne/cm
 (C) 2×10^3 N/m² (D) 0.2×10^9 N/m²
39. A cube of aluminum of each side 0.1 m is subjected to a shearing force of 100 N, The top face of the cube is displaced through 0.02 cm with respect to the bottom face. The shearing strain would be
 (A) 0.02 (B) 0.1
 (C) 0.005 (D) 0.002
40. Which of the following statement is correct?
 (A) Y, K, h and s have same units and dimensions.
 (B) Y, K, h and s have same units and dimensions.
 (C) Y, s and h have same units and dimensions.
 (D) Y, K. and h have same units and dimensions.
41. A metal rod having coefficient of linear expansion (α) and Young's modulus (Y) is heated to raise the temperature by $\Delta\theta$. The stress exerted by the rod is: [Oct 13]
 (A) $\frac{Y\alpha}{Dq}$ (B) $\frac{YDq}{\alpha}$
 (C) $Y\alpha Dq$ (D) $\frac{YDq}{Y}$
42. A wire has Poisson's ratio of 0.5. It is stretched by an external force to produce a longitudinal strain of 2×10^{-3} . If the original diameter was 2 mm, the final diameter after stretching is V
 (A) 2.002 mm (B) 1.998 mm
 (C) 1.98 mm, (D) 1.999 mm
43. Two wires of different material having same cross-sectional area and length are stretched by the stretching forces of same magnitude.
 (A) 1 : 3 (B) 1 : 3
 (C) 1 : 1 (D) 1 : 4
44. The compressibility of a substance is the reciprocal of % . [Oct 09]
 (A) Young's modulus .
 (B) Bulk modulus
 (C) Modulus of rigidity
 (D) Poisson's ratio
45. Depression in beam of length l, breadth b and depth d subjected to a load W at its middle point is given by,
 (A) $d = Wl^2 + 4Ybd^3$
 (B) $d = Wb^3 + 4Ybl^3$
 (C) $d = Wl^3 + 4Ydb^3$
 (D) $d = Wl^3 + 3bd^3$
46. If work done in stretching a wire by 1 J, the work necessary for stretching another wire of same material, but with double the radius and half the length by 1 mm in joule is
 (A) 1/4 (B) 4
 (C) 8 (D) 16
47. When the load on a wire is slowly increased from 3 to 5 kg wt, the elongation increases from 0.61 to 1.02 mm. The work done during the extension of wire is
 (A) 0.16 J (B) 0.016 J
 (C) 1.6 J (D) 16 J.
48. Strain energy per unit volume is given by [Oct 08]
 (A) $\frac{1}{2} \cdot \frac{(\text{stress})^2}{Y}$ (B) $\frac{1}{2} \cdot (\text{stress})^2 Y$
 (C) $\frac{1}{2} \cdot \frac{\text{strain}}{\text{stress}}$ (D) None of these
49. A wire of Young's modulus 1.5×10^{12} N/m² is stretched by a force so as to produce a strain of 2×10^{-4} . The energy density in the wire is
 (A) 6×10^3 J/m³ (B) 3×10^4 J/m³
 (C) 1.5×10^3 J/m³ (D) 3×10^8 J/m³
50. When a 4 kg wt. load is applied to a wire its length is increased by 0.8 mm. Subsequently when the load is changed to 6 kg wt. the elongation increases from 0.8 to 1.2 mm. The work done during the extension is

- (A) $1.96 \times 10^{-2} \text{ J}$ (B) $4 \times 10^{-2} \text{ J}$
 (C) $3 \times 10^{-2} \text{ J}$ (D) $4 \times 10^{-2} \text{ J}$
51. The buckling of a beam is found to be more if
 [Feb2013]
- (A) the breadth of the beam is large.
 (B) the beam material has large value of Young's modulus.
 (C) the length of the beam is small.
 (D) the depth of the beam is small.
52. The energy stored per unit volume of a strained wire is

(A) $\frac{1}{2} \text{ (load) } \times \text{ (extension)}$

(B) $\frac{1}{2} \frac{Y}{(\text{strain})^2}$

(C) $\frac{1}{2} Y(\text{strain})^2$

(D) Stress \times strain

Section : A

- 42.7 N
- 1 : 2
- $5 \times 10^{10} \text{ N/m}^2$
- $1.248 \times 10^{11} \text{ N/m}^2$
- $1.791 \times 10^{10} \text{ N/m}^2$
- $2.015 \times 10^7 \text{ N}$
- $16 \times 10^5 \%$
- 6.24 mm
- i. $2.496 \times 10^{14} \text{ mii. } 1.248 \times 10^4$
- 25.36 J/m^3
- i. $3 - m$ 6
- $5.5 \times 10^6 \text{ J/m}^3$
- $2.2 \times 10^8 \text{ Pa}$

Section C

- 2.6×10^{12}
- 0.04561 m
- $8 \times 10^{10} \text{ N/m}^2$
- $1.02 \times 10^4 \text{ m}$
- 2:3
- $4.2 \times 10^8 \text{ m}^2, 5 \times 10^3, 4.2 \times 10^8 \text{ N}$
- 2.6×10^{-4}

8. $2 \times 10^9 \text{ N/m}^2$

9. $6 \times 10^7 \text{ N/m}^2$

Section D

10. (C) 11. (K) 12. (M) 13. (B)
14. (B) 15. (P) 16. (B) 17. (B) 18. (C)
19. (B) 20. (B) 21. (A) 22. (m)
23. (C) 24. (C) 25. (C) 26. (D)
27. (B) 28. (C) 29. (A) 30. (n)
31. (D) 32. (C) 33. (C) 34. (B)
35. (C) 36. (B) 37. (D) 38. (D)
39. (A) 40. (D) 41. (C) 42. (B)
43. (D) 44. (B) 45. (A) 46. (D)
47. (B) 48. (A) 49. (B) 50. (A)
51. (M) 52. (C)