## 1.0 Introduction

**Q.l. Define circular motion.**
**Give its examples.**

**Ans: Definition:**

Motion of a particle along the circumference of a circle is called circular motion.

**Examples:**

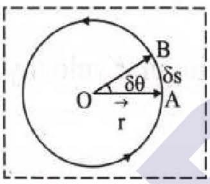
i. The motion of a cyclist along a circular path.
ii. Motion of the moon around the earth.
iii. Motion of the earth around the sun.
iv. Motion of the tip of hands of a clock.
v. Motion of electrons around the nucleus in an atom.

## 1.1 Angular displacement

**Q.2. What is radius vector?**

**Ans:**

i) *A vector drawn from the centre of a circle to position of a particle on circumference of circle is called as 'radius vector'.*




ii) It is given by, $|\vec{r}| = \frac{ds}{dq}$

where, ds = small linear distance
dq = small angular displacement.

iii) It is directed radially outwards.

iv) **Unit:** metre (m) in SI system and centimeter (cm) in CGS system,

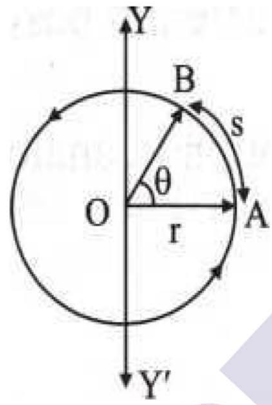
v) Dimensions: $[M^{\circ}L^{1}T^{\circ}]$

**Q.3. What is angular displacement? OR**
***Define angular displacement ?**

**Ans:**

i) Angle traced by a radius vector in a given time, at the centre of the circular path is called as angular displacement.

ii) Consider a particle performing circular motion in anticlockwise sense as shown in the figure.




Let, A = initial position of particle at t = 0
B = final position of particle after time t
q = angular displacement in time t
r = radius of the circle
s = length of arc AB

iii) Angular displacement is given by,

$$q = \frac{\text{Length}}{\text{Radius of circle}}$$

$$\backslash \quad q = \frac{s}{r}$$

iv) Unit: radian

v) Direction of angular displacement is given by right hand thumb rule or right handed screw rule.

Note:

1. If a particle performing circular motion describes an arc of length 8s, in short time interval dt then angular displacement is given by $dq = \frac{ds}{r}$.

$$\backslash \quad ds = dq.r$$

In the vector form, $\vec{ds} = \vec{dq} + \vec{r}$

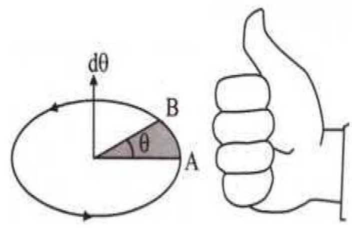
2. If a particle performing circular motion completes one revolution then angular displacement is given by 0 = 360° = $2p^{c}$ where $p^{c}$ represents angular displacement in radians.

3. One radian is the angle subtended at the centre of a circle by an arc of length equal to radius of the circle.

**Q.4. State right hand thumb rule to find the direction of angular displacement.**

**Ans: Right hand thumb rule:**

*Imagine the axis of rotation to be held in right hand with the fingers curled around it and thumb out-stretched. If the curled fingers give the direction of motion of a particle performing circular motion then the direction of out-stretched thumb gives the direction of angular displacement vector.*

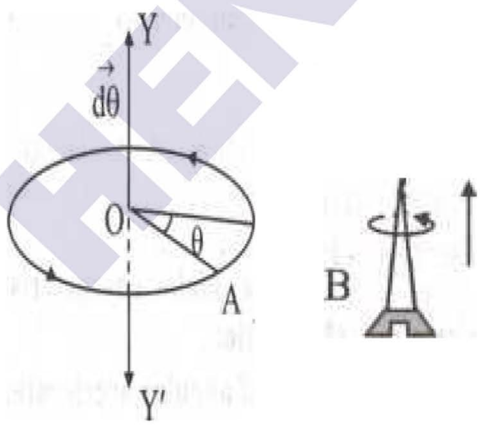



**Direction of angular displacement**

**Q.5. Explain right handed screw rule to find the direction of angular displacement.**

**Ans:** i. *Imagine the right handed screw to be held in the place in which particle is performing circular motion. If the right handed screw is rotated in the direction of particle performing circular motion then the direction in which screw tip advances, gives the direction of angular displacement.*

ii. The tip of the screw advances in downward direction, if sense of rotation of the object is anticlockwise whereas the tip of the screw advances in upward direction, if sense of rotation of the object is clockwise as shown in the figure.




Right handed screw rule Tip of screw advancing in upward direction

**Q.6. Write down the four characteristics of angular displacement.**

**Ans:** Characteristics of angular displacement:

i. Instantaneous angular displacement is a vector quantity (true vector), so it obeys commutative and associative laws of vector addition.

ii. Finite angular displacement is a pseudo vector.

iii. Direction of infinitesimal angular displacement is given by right hand thumb rule or right handed screw rule.

iv. For anticlockwise sense, angular displacement is taken as positive while in clockwise sense, angular displacement is taken negative.

**#Q.7. Are the following motions same or different?**

i. Motion of tip of second hand of a clock.

ii. Motion of entire second hand of aclock.

**Ans:** Both the motions are different.

The tip of the second hand of a clock performs uniform circular motion while the entire hand performs rotational motion with the second hand as a rigid body.

## 1.2 Angular velocity and angular acceleration

**Q.8. What is angular velocity? State its unit and dimension.**

**OR**

*** Define angular velocity?**

**Ans:**

i. *Angular velocity of a particle performing circular motion is defined as the time rate of change of limiting angular displacement*

OR

*The ratio of angular displacement to time is called angular velocity.*

ii. Instantaneous angular velocity is given by,

$$\vec{\omega} = \lim_{dt \to 0} \frac{d\vec{\theta}}{dt} = \frac{d\vec{\theta}}{dt}$$

Finite angular velocity is given by,

$$\omega = \frac{\theta}{t}$$

iii. It is a vector quantity.

iv. Direction: The direction of angular velocity is given by right hand rule and is in the direction of angular displacement.

v. Unit: rad $s^{-1}$

vi. Dimensions: $[M^{\circ}L^{\circ}T^{-1}]$

**Note:** Magnitude of angular velocity is called angular speed.

**Q.9. What is angular acceleration? State its unit and dimension.**

**OR**

***Define angular acceleration.**

**Ans:**

i. *The rate of change of angular velocity with respect to time is called angular acceleration.*

It is denoted by $\vec{a}$ .

ii. If $\vec{w}_0$ and $\vec{w}$ are the angular velocities of a particle performing circular motion at instant $t_0$ and t, then angular acceleration is given by,

$$\vec{a} = \frac{\vec{w} - \vec{w}_0}{t - t_0} = \frac{\overrightarrow{dw}}{dt}$$

iii. **Direction:** The direction of $\vec{a}$ is given by right hand thumb rule or right handed screw rule.

iv. Unit: rad /s in SI system.

v. Dimensions: $[M^{\circ}L^{\circ}T^{-2}]$.

**Q.10. Define the following terms.**

**I. Average angular acceleration**

**II. Instantaneous angular acceleration**

**Ans: Average angular acceleration:**

*Average angular acceleration is defined as the time rate of change of angular velocity*

It is given by $\vec{a}_{avg} = \frac{\vec{w}_2 - \vec{w}_1}{t_2 - t_1} = \frac{d\vec{w}}{dt}$

***ii. Instantaneous angular acceleration:***

Instantaneous angular acceleration is defined as the limiting rate of change of angular velocity

It is given by $\vec{a} = \lim_{dt \to 0} \frac{d\vec{w}}{dt} = \frac{d\vec{w}}{dt}$

**Q.11. Give an example of**

i. **Positive angular acceleration**

ii. **Negative angular acceleration.**

**Ans: i. Positive angular acceleration:**

When an electric fan is switched on, the angular velocity of the blades of the fan increases with time. In this case, angular acceleration will have the same direction as angular velocity. This is an example of positive angular acceleration,

**ii. Negative angular acceleration:**

When an electric fan is switched off, theangular velocity of the blades of fan decreases with time. In this case, angular acceleration will have a direction opposite to that of angular velocity. This is an example of negative angular acceleration.

**Q.12. What happens to the direction of angular acceleration**

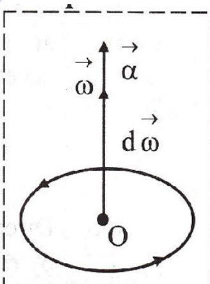
i. **if a particle is speeding up?**

ii. **if a particle is slowing down?**

**Ans:**

**i. Direction of $\vec{a}$ a when the particle is speeding up:**

Consider a particle is moving along a circular path inanticlockwise direction and is speeding up.




ω increases

Fig. (a)

Magnitude of $\vec{w}$ keeps on increasing which results in $d\vec{w}$ to be directed up the plane.

Hence, direction of $\vec{a}$ is upward. As $\vec{w}$ and $\vec{a}$ are ∧ ar to the plane, they are parallel to each other. [See fig. (a)].

**ii. Direction of $\vec{a}$ when the particle is slowing down;**

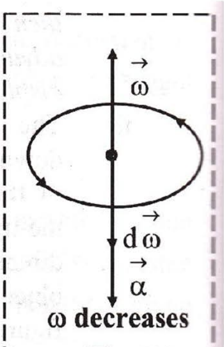



Fig. (b)

Consider a particle is moving along a circular path in anticlockwise direction and is slowing down.

Magnitude of $\vec{w}$ keeps on decreasing which results in $d\vec{w}$ to be directed down the plane.

Hence, direction of a [See fig. (b)].

**Q.13. Write down the main characteristics of angular acceleration.**

**Ans: Characteristics of angular acceleration:**

i. Angular acceleration is positive if angular velocity increases with time.

ii. Angular acceleration is negative if angular velocity decreases with time.

iii. Angular acceleration is an axial vector.

iv. In uniform circular motion, angular velocity is constant, so angular acceleration is zero.

**Note:**

1. When a body rotates with constant angular velocity its instantaneous angular velocity is equal to its average angular velocity, whatever may be the duration of the time interval. If the angular velocity is constant, we write

$$w = |\vec{w}| = \frac{dq}{dt}$$

2. If a body completes one revolution in time interval T, then angular speed,

$$w = \frac{2p}{T} = 2pn,$$

where n = frequency of revolution.

3. $d\vec{q}$, $\vec{w}$ and $\vec{a}$ are called axial vectors, as they are always taken to be along axis of rotation.

4. The direction of $d\vec{q}$ and $\vec{w}$ is always given by right hand thumb rule.

## 1.3 <u>Relation between linear velocity angular velocity:</u>

***Q.14.Show that linear speed of a particle performing circular motion is the product of radius of circle and angular speed of particle.**

**OR**

**Define linear velocity. Derive the relation between linear velocity and angular velocity. [Feb 02, Mar 96,08,12, Oct 09]**

**Ans: Linear velocity:**

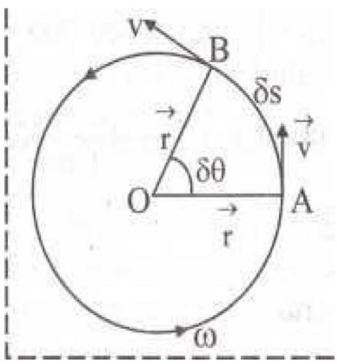
*Distance travelled by a body per unit time in a given direction is called linear velocity.*

It is a vector quantity and is given by,

$$\vec{v} = \frac{d\vec{s}}{dt}$$

**Relation between linear velocity and angular velocity:**

i. Consider a particle moving with uniform circular motion along the circumference of a circle in anticlockwise direction with centre O and radius r as shown in the figure.




ii. Let the particle cover small distance 8s from A to B in small interval dt.

In such case, small angular displacement is

ÐAOB = dq.

iii. Magnitude of instantaneous linear velocity of particle is given by,

$$v = \lim_{dt® 0} \frac{ds}{dt}$$

But ds = rdq

$$\therefore \quad v = \left(\lim_{\delta t \to 0} \frac{\delta\theta}{\delta t}\right) \quad [\because r = \text{constant}]$$

Also $\lim_{\delta t \to 0} \frac{\delta\theta}{\delta t} = \omega$

In vector form, $\vec{v} = \vec{\omega} \times \vec{r}$.

**Q.15. Prove the relation $\vec{v} = \vec{\omega} \times \vec{r}$, where symbols have their usual meaning.**

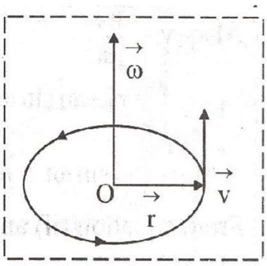
**Ans: Analytical method:**

i. Consider a particle performing circular motion in anticlockwise sense with centre O and radius r as shown in the figure.

ii. Let, $\vec{\omega} =$ angular velocity of the particle

$\vec{v} =$ linear velocity of the particle

r = radius vector of the particle




iii. Linear displacement in vector form is given by,

$$d\vec{s} = d\vec{\theta} \times \vec{r}$$

Dividing both side by dt, we get

$$\frac{d\vec{s}}{dt} = \frac{d\vec{\theta}}{dt} \times \vec{r} \qquad \text{...... (i)}$$

iv. Taking limiting value in equation (i) we get,

$$\lim_{\delta t \to 0} \frac{d\vec{s}}{dt} = \lim_{\delta t \to 0} \frac{d\vec{\theta}}{dt} \times \vec{r}$$

$$\therefore \frac{d\vec{s}}{dt} = \frac{d\vec{\theta}}{dt} \times \vec{r}$$

But, $\frac{d\vec{s}}{dt} = \vec{v}$ = linear velocity,

$\frac{d\vec{\theta}}{dt} = \vec{\omega}$ = angular velocity

$$\therefore \quad \vec{v} = \vec{\omega} \times \vec{r}$$

**Calculus method:**

i. A particle is moving in XY plane with position vector,

$$\vec{r} = r\,\hat{i}\cos\omega t + r\,\hat{j}\sin\omega t \text{ ....(i)}$$

ii. Angular velocity is directed perpendicular to plane, i.e., along Z-axis. It is given by $\vec{\omega} = \omega\hat{k}$,

Where k = unit vector along Z-axis.

iii. $\vec{\omega} \times \vec{r} = \omega\hat{k} \times (r\hat{i}\cos\omega t + r\hat{j}\sin\omega t)$

[From e quation (i)]

$= \omega r\cos\omega t.(\hat{k} \times \hat{i}) + \omega r\sin\omega t.(\hat{k} \times \hat{j})$

$= \omega r\cos\omega t + \omega r(-\hat{i})\sin\omega t$

$[\because \hat{k} \times \hat{i} = \hat{j} \text{ and } \hat{k} \times \hat{j} = -\hat{i}]$

$\therefore \quad \vec{\omega} \times \vec{r} = -r\omega\hat{i}\sin\omega t + \omega r\hat{j}\cos\omega t$

$\therefore \quad \vec{\omega} \times \vec{r} = r\omega(-\hat{i}\sin\omega t + \hat{j}\cos\omega t) \quad \text{.... (ii)}$

Also $\vec{v} = \frac{d\vec{r}}{dt}$

$= r(-\omega\hat{i}\sin\omega t + \omega\hat{j}\cos\omega t)$

$\vec{v} = r\omega(-\hat{i}\sin\omega t + \hat{j}\cos\omega t) \qquad \text{....... (iii)}$

From equation (ii) and (iii), we have,

$$\therefore \quad \vec{v} = \vec{\omega} \times \vec{r}$$

## <u>1.4 Uniform Circular Motion :</u>

**Q.16. What is uniform circular motion? OR**

***Define uniform circular motion?**

**Ans:**

i. The motion of a body along the circumference of a circle with constant speed is called uniform circular motion.

ii. In U.C.M, direction of velocity is along the tangent drawn to the position of particle on circumference of the circle.

iii. Hence, direction of velocity goes on changing continuously, however the magnitude of velocity is constant. Therefore, magnitude of angular velocity is constant.

iv. Examples of U.C.M:

a. Motion of the earth around the sun.

b. Motion of the moon around the earth.

c. Revolution of electron around the nucleus of atom.

**Q.17. State the characteristics of uniform circular motion.**

**Ans: Characteristics of U.C.M:**

i. It is a periodic motion with definite period and frequency.

Speed of particle remains constant but velocity changes continuously.

It is an accelerated motion.

Work done in one period of U.C.M is zero.

**Q.18. Define periodic motion. Why U.C.M is called periodic motion?**

**Ans:**

**i. Definition:**

A type of motion which is repeated after equal interval of time is called periodic motion.

ii. The particle performing U.C.M repeats its motion after equal intervals of time on the same path. Hence, U.C.M is called periodic motion.

**Q.19. Define period of revolution of U.C.M. State its unit and dimension. Derive an expression for the period of revolution of a particle performing uniform circular motion.**

**Ans: Definition:**

The time taken by a particle performing uniform circular motion to complete one revolution is called as period of revolution.

It is denoted by T and is given by, $T = \frac{2p}{w}$

Unit: second in SI system.

Dimensions: $[M^{\circ}L^{\circ}T^{1}]$

**Expression for time period:**

During period T, particle covers a distance equal to circumference *2izr* of circle with linear velocity v.

$$\backslash \quad \text{Time period} = \frac{\text{Distance covered in one revolution}}{\text{Linear velocity}}$$

$$\backslash \quad T = \frac{2pr}{v}$$

But v = rw

$$\backslash \quad T = \frac{2pr}{rw}$$

$$\backslash \quad T = \frac{2p}{w}$$

**Q.20. What is frequency of revolution? Express angular velocity in terms of frequency of revolution.**

**Ans:**

i. *The number of revolutions performed by a particle performing uniform circular motion in unit time is. called as frequency of revolution.*

ii. Frequency of revolution (n) is the reciprocal of period of revolution.

$$n = \frac{1}{T} = \frac{1}{\left(\frac{2p}{w}\right)} = \frac{w}{2p} = \frac{v}{2pr}$$

iii. Unit : herts (Hz), c.p.s., r.p.s. etc.

iv. Dimensions : $[M^{o}L^{o}T^{-1}]$

**Angular velocity in terms of frequency of revolution:**

$$w = \frac{2p}{T} = 2p \times \frac{1}{T}$$

But $\frac{1}{T} = n$

***Q.21.Define period and frequency of a particle performing uniform circular motion. State their SI units.**

***Ans:*** *Refer Q. 19 and Q.20*

## 1.5Acceleration In U.C.M (Radial acceleration)

**Q.22. Define linear acceleration. Write down its unit and dimensions.**

**Ans: i. Definition:**

*The rate of change of linear velocity with respect to time is called linear acceleration.*

It is denoted by $\vec{a}$ and is given by

$$\vec{a} = \frac{d\vec{v}}{dt}$$

ii. Unit: $m/s^2$ in SI system and $cm/s^2$ in CGS system.

iii. Dimensions: $[M^{\circ}L^{1}T^{-2}]$

**Q.23. U.C.M is an accelerated motion. Justify this statement.**

**Ans:**

i. In U.C.M, the magnitude of linear velocity (speed) remains constant but the direction of linear velocity goes on changing i.e. linear velocity changes,

ii. The change in linear velocity is possible only if the motion is accelerated. Hence, U.C.M is an accelerated motion.

***Q.24.0btain an expression for acceleration of a particle performing uniform circular motion.**

**OR**

**Define centripetal acceleration. Obtain an expression for acceleration of a particle performing U.C.M by analytical method.**
**Ans: Definition:**

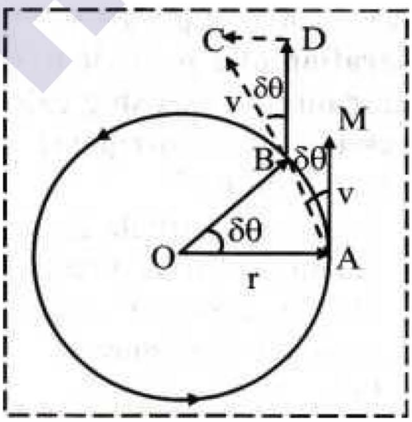
*The acceleration of a particle performing U.C.M which is directed towards the centre and along the radius of circular path is called as centripetal acceleration.*
The centripetal acceleration is directed along the radius and is also called radial acceleration.

**Expression for acceleration in U.C.M by analytical method (Geometrical method):**

i. Consider a particle performing uniform circular motion in a circle of centre O and radius r with a uniform linear velocity of magnitude v.

ii. Let a particle travel a very short distance from A to B in a very short time interval dt.

iii. Let dq be the angle described by the radius vector OA in the time interval dt as shown in the figure.




v. Velocity at B is represented by $\overline{BC}$ while the velocity at A is represented by $\overline{AM}$. [Assuming AM = BD]

vi. Angle between BC and BD is equal to 80 as they are perpendicular to $\overline{OB}$ and $\overline{OA}$ respectively.

vii. Since DBDC ~ DOAB

$$\backslash \quad \frac{DC}{BD} = \frac{AB}{AO} \qquad \backslash \quad \frac{Dv}{v} = \frac{AB}{r}$$

viii. For very small dt, arc length ds of circular path between A and B can be taken as AB

$$\backslash \quad \frac{dv}{v} = \frac{ds}{r} \quad \text{or} \ \backslash \quad dv = \frac{v}{vr} ds$$

where, dv = change in velocity

$$\text{Now,} \quad a = \lim_{dt® 0} \frac{dv}{dt} = \lim_{dt® 0} \frac{v}{r}\frac{ds}{dt}$$

$$a = \frac{v}{r} \lim_{dt® 0} \frac{ds}{dt} = \frac{v}{r} ´ \ v = \frac{v^2}{r}$$

As dt ® 0, B approaches A and dv becomes perpendicular to the tangent i.e. along the radius towards the centre.

ix. Also v = rw

$$a = \frac{r^2 w^2}{r} = w^2 r$$

x. In vector form, $\vec{a} = - \vec{w} \vec{r}$

Negative sign shows that direction of $\vec{a}$ is opposite to the direction of $\vec{r}$.

Also $a = \frac{v^2}{r} \hat{r}_0$, where $\hat{r}_0$ is the unit vector along the radius vector.

**Q.25. Derive an expression for linear acceleration of a particle performing U.C.M. [Mar 98, 08]**

**Ans:** ***Refer Q.24***

**Q.26. Derive an expression for centripetal acceleration of a particle performing uniform circular motion by using calculus method.**

**Ans: Expression for centripetal acceleration by calculus method:**

i. Suppose a particle is performing U.C.M in anticlockwise direction.

The co-ordinate axes are chosen as shown in the figure.

Let,

A = initial position of the particle which lies on positive X-axis

P = instantaneous position after time t

$\theta$ = corresponding angular displacement

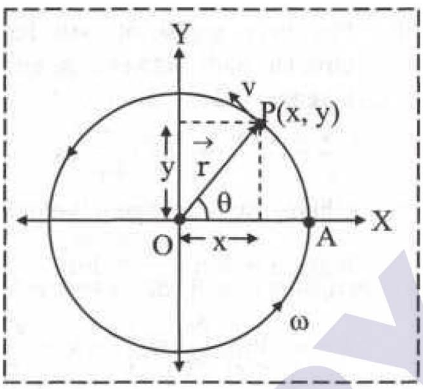
$\omega$ = constant angular velocity

$\vec{r}$ = instantaneous position vector at time t

ii. From the figure,

$\vec{r} = \hat{i}x + \hat{j}y$

where, $\hat{i}$ and $\hat{j}$ are unit vectors along X-axis and Y-axis respectively.




iii. Also, $x = r\cos\theta$ and $y = r\sin\theta$

$\therefore \quad \vec{r} = [r\hat{i}\cos\theta + r\hat{j}\sin\theta]$

But $\theta = \omega t$

$\therefore \quad \vec{r} = [r\hat{i}\cos\omega t + r\hat{j}\sin\omega t]$ ...... (i)

**iv. Velocity of the particle is given as rate of change of position vector.**

$$\therefore \quad \vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}[r\hat{i}\cos\omega t + r\hat{j}\sin\omega t]$$

$$= r\left[\frac{d}{dt}\cos\omega t\right]\hat{i} + r\left[\frac{d}{dt}\sin\omega t\right]\hat{j}$$

$$\therefore \quad \vec{v} = -r\omega\hat{i}\sin\omega t + r\omega\hat{j}\cos\omega t$$

$$\therefore \quad \vec{v} = r\omega(-\hat{i}\sin\omega t + \hat{j}\cos\omega t) \quad \text{..... (ii)}$$

v. Further, instantaneous linear acceleration of the particle at instant t is given by,

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}[r\omega(-\sin\omega t + \hat{j}\cos\omega t)]$$

$$= r\omega\left[\frac{d}{dt}(-\hat{i}\sin\omega t + \hat{j}\cos\omega t)\right]$$

$$= r\omega\left[\frac{d}{dt}(-\sin\omega t)\hat{i} + \frac{d}{dt}(\cos\omega t)\hat{j}\right]$$

$$= r\omega(-\omega\hat{i}\cos\omega t - \omega\hat{j}\sin\omega t)$$

$$= -r\omega^2(\hat{i}\cos\omega t + \hat{j}\sin\omega t)$$

$$\therefore \quad \vec{a} = -\omega^2(r\hat{i}\cos\omega t + r\hat{j}\sin\omega t) \quad \text{.....(iii)}$$

vi. From equation (i) and (iii), we have,

$$\vec{a} = -\omega^2\vec{r}$$

Negative sign shows that direction of acceleration is opposite to the direction of position vector.

**vii.** Magnitude of centripetal acceleration is given by,

$$a = \omega^2 r$$

$$\text{As } \omega = \frac{v}{r}$$

$$\therefore \quad a = \frac{v^2}{r}$$

**Note :**

To show $\vec{a} = \vec{\omega} \times \vec{v}$,

$$\vec{\omega} \times \vec{v} = \omega\hat{k} \times (-r\omega\hat{i}\sin\omega t + r\omega\hat{j}\cos\omega t)$$

$$= -r\omega^2\sin\omega t(\hat{k} \times \hat{i}) + r\omega^2\cos\omega t(\hat{k} \times \hat{j})$$

$$= -r\omega^2\sin\omega t\,\hat{j} + r\omega^2\cos\omega t(-\hat{i})$$

$$[\because \hat{k} \times \hat{i} = \hat{j} \text{ and } \hat{k} \times \hat{j} = -\hat{i}]$$

$$= -r\omega^2\hat{i}\cos\omega t - r\omega^2\hat{j}\sin\omega t$$

$= \vec{a}$ [From equation (iii)]

**Q.27. Derive an expression for centripetal acceleration of a particle performing uniform circular motion. [Feb 02, Feb 06]**

**Ans:** *Refer Q. 26*

***Q.28.Derive the relation between linear acceleration and angular acceleration if a particle performs U.C.M.**

**Ans: Relation between linear acceleration and angular acceleration in U.C.M:**

i. Consider a particle performing U.C.M. with constant angular velocity $\omega$ with path radius r.

ii. Magnitude of linear acceleration is given by,

$$a = \lim_{dt \to 0} \frac{dv}{dt}$$

$$\therefore \quad a = \frac{dv}{dt}$$

iii. But, $v = r\omega$

$$\therefore \quad a = \frac{d}{dt}(r\omega) = r\left(\frac{d\omega}{dt}\right) + \omega\left(\frac{dr}{dt}\right)$$

iv. Since, r = contant

$$\therefore \quad \frac{dr}{dt} = 0$$

$$\therefore \quad a = r\left(\frac{d\omega}{dt}\right)$$

But, $\frac{d\omega}{dt} = \alpha$

$$\therefore \quad a = r\alpha$$

In vector form,

$$\vec{a} = \vec{\alpha} \times \vec{r}$$

This is required relation.

Q.29. Define non-uniform circular motion. Derive an expression for resultant acceleration in non-uniform circular motion.

**Ans: Non-uniform circular motion:**

*Circular motion with variable angular speed is called as non-uniform circular motion. Example: Motion of a body on vertical circle.*

**Expression for resultant acceleration in non-U.C.M:**

**i.** Since, $\vec{v} = \vec{\omega} \times \vec{r}$ **.....(i)**

Differentiating equation (i) with respect to t, we get $\frac{d\vec{v}}{dt} = \frac{d}{dt}(\vec{\omega} \times \vec{r})$

$$\frac{d\vec{v}}{dt} = \vec{\omega} \times \frac{d\vec{r}}{dt} + \frac{d\vec{\omega}}{dt} \times \vec{r} \quad \text{...... (ii)}$$

ii. But $\frac{d\vec{v}}{dt} = \vec{a}, \frac{d\vec{r}}{dt} = \vec{v}, \frac{d\vec{\omega}}{dt} = \vec{\alpha}$

$\therefore$ Equation (ii) becomes,

$$\vec{a} = \vec{\omega} \times \vec{v} + \vec{\alpha} \times \vec{r} \quad \text{....... (iii)}$$

iii. $\vec{\omega} \times \vec{v}$ v is along the radius of the circle, pointing towards the centre, hence it is called radial acceleration $\vec{a}_R$.

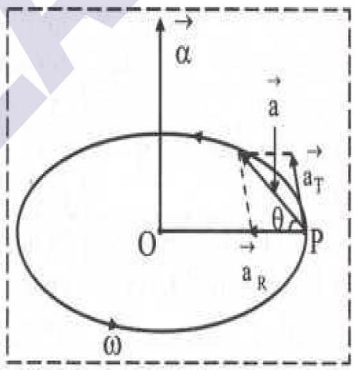
$$\therefore \quad \vec{a}_R = \vec{\omega} \times \vec{v} \quad \text{...... (iv)}$$

iv. $\vec{\alpha} \times \vec{r}$ is along the tangent of the circumference of the circular path, hence it is called tangential acceleration $\vec{a}_T$.

$$\vec{a}_T = \vec{\alpha} \times \vec{r} \quad \text{..... (v)}$$

v. From equation (iii), (iv) and (v), we have,

$$\vec{a} = \vec{a}_R + \vec{a}_T$$




Magnitude of resultant linear acceleration is given by $|a| = \sqrt{a_R^2 + a_T^2}$

Note:

**1. Resultant linear acceleration in different cases**

| Situation | Resultant motion | Resultant linear acceleration |
|---|---|---|
| $a_R = 0, a_T = 0$ | Uniform linear motion | $a = 0$ |
| $a_R = 0, a_T \neq 0$ | Accelerated linear motion | $a = a_T$ |
| $a_R \neq 0, a_T = 0$ | Uniform circular motion | $a = a_R$ |
| $a_R \neq 0, a_T \neq 0$ | Non-uniform circular motion | $a = \sqrt{a_R^2 + a_T^2}$ |

2. In non-uniform circular motion, aR is due to change in direction of linear velocity, whereas $a_T$ is due to change in magnitude of linear velocity.
3. In uniform circular motion, particle has only radial component aR due to change in the direction of linear velocity. It is so because w = constant

$$a = \frac{dw}{dt} = 0, \text{ so } a_T = a \times r = 0$$

4. Since the magnitude of tangential velocity does not change, there is no component of acceleration along the tangent. This means the acceleration must be perpendicular to the tangent, i.e. along the radius of the circle.

***Q.30. What is the difference between uniform circular motion and non uniform circular motion? Give examples.**

**Ans:**

| **Sr. No.** | **U.C.M.** | **Non-U.C.M.** |
|---|---|---|
| 1. | Circular motion with constant angular speed is known as uniform circular motion. | Circular motion with variable angular speed is called as non-uniform circular motion. |
| 2. | For U.C.M, $a = 0$ | For non-U.C.M $a\ ^{1}\ 0$. |
| 3. | In U.C.M, work done by tangential force is zero. | In non-U.C.M, work done by tangential force is not zero. |
| 4. | Example: Motion of the earth around the sun. | Example: Motion of a body on vertical circle. |

## 1.6 Centripetal and centrifugal forces

**Q.31. What is centripetal force? Write down its unit and dimensions.**

**Ans:**

i. *Force acting on a particle performing circular motion along the radius of circle and directed towards the centre of the circle is called centripetal force.*

It is given by $F_{cp} = \frac{mv^2}{r}$

where r = radius of circular path.

ii. **Example:** Electron revolves around the nucleus of an atom. The necessary centripetal force is provided by electrostatic force of attraction between positively charged nucleus and negatively charged electron.

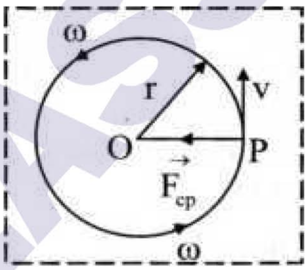
iii. Unit: N in SI system and dyne in CGS system.

iv. Dimensions: $[M^1L^1T^{-2}]$

**Q.32. Derive the formula for centripetal force experienced by a body in case of uniform circular motion. Express the formula in vector form.**

**Ans: Expression for centripetal force:**

i. Suppose a particle performs uniform circular motion. It has an acceleration of magnitude $v^2/r$ or $w^2r$ directed towards the centre of the circle.




ii. According to Newton's second law of motion, acceleration must be produced by a force acting in the same direction.

iii. If m is the mass of particle performing U.C.M then the magnitude of centripetal force is given by,

$F_{CP}$ = Mass of particle × centripetal acceleration

$F_{CP} = maCP$

iv. But, $a_{CP} = \frac{v^2}{r} = vw = rw^2$

$$F_{CP} = \frac{mv^2}{r} = mvw = mrw^2$$

v. Also $w = 2pn$

\ $F_{CP} = mr(2pn)^2 = 4p^2n^2\, mr$

**Centripetal force in vector form:**

$$\vec{F}_{CP} = -\frac{mv^2}{r}\hat{r}_0 = -mrw^2.\hat{r}_0$$

where $\hat{r}_0$ is unit vector in the direction of radius vector $\vec{r}$.

**Q.33. State four examples where centripetal force is experienced by the body.**

**Ans:** i. A stone tied at the end of a string is revolved in a horizontal circle, the tension in the string provides the necessary centripetal force. It is given by equation $T = mr\omega^2$ .

ii. The planets move around the sun in elliptical orbits. The necessary centripetal force is provided to the planet by the gravitational force of attraction exerted by the sun on the planet.

iii. A vehicle is moving along a horizontal circular road with uniform speed. The necessary centripetal force is provided by frictional force between the ground and the tyres of wheel.

iv. Satellite revolves round the earth in circular orbit, necessary centripetal force is provided by gravitational force of attraction between the satellite and the earth.

***Q.34.Define centripetal force. Give its any four examples.**

**OR**

**Define centripetal force and give its any two examples.** [Mar 11]

**Ans:** *Refer Q.31 and Q.33* Q.35.

**Define and explain centrifugal force.**

**Ans: Definition:**

The force acting on a particle performing U.C.M which is along the radius and directed away from centre of circle is called centrifugal force.

Magnitude of centrifugal force is same as that of centripetal force but acts in opposite direction.

**Explanation:**

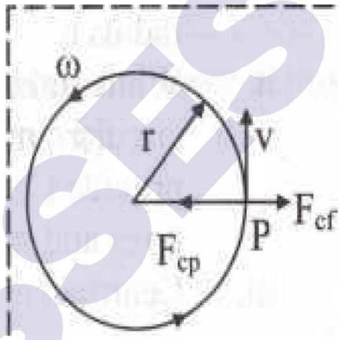
i. U.C.M is an accelerated motion. Thus, a particle performing U.C.M is in an accelerated (non-inertial) frame of reference.

ii. In non-inertial frame of reference, an imaginaiy force or a fictitious or a pseudo force is to be considered in order to apply Newton's laws of motion.

iii. The magnitude of this pseudo force is same as that of centripetal force but its direction is opposite to that of centripetal force. Therefore, this pseudo force is called centrifugal force.

iv. If m is the mass of a particle performing U.C.M then centrifugal force experienced by the body is given by,

$$F_{cf} = \frac{mv^2}{r}$$




But $v = r\omega$

$\therefore$ $F_{cf} = mr\omega^2$

v. In vector form,

$$\vec{F}_{cf} = m\omega^2\vec{r}$$

$$\therefore \quad \vec{F}_{cf} = \frac{mv^2}{r}\hat{r}_0$$

where $\hat{r}_0$ = unit vector in the direction of $\vec{r}$ .

**Q.36. Explain applications of centrifugal force in our daily life.**

**Ans: Applications:**

i. When a car in motion takes a sudden turn towards left, passengers in car experience an outward push to the right. This is due to centrifugal force acting on the passengers.

ii. A bucket full of water is rotated in a vertical circle at a particular speed, so that water docs not fall. This is because, weight of water is balanced by centrifugal force acting on it.

iii. The children sitting in merry-go-round experience an outward pull as merry-go-round rotates about vertical axis. This is due to centrifugal force acting on the Children.

iv. A coin kept slightly away from the axis of rotation of turn table moves away from axis of rotation as the speed of rotation of turn table increases. This is due to centrifugal force acting on coin.

v. The bulging of earth at equator and flattening at the poles is due to centrifugal force acting on it.

vi. Drier in washing machine consists of a cylindrical vessel with perforated walls. As the cylindrical vessel is rotated fast, centrifugal force acts on wet clothes.

This centrifugal force, forces out water through perforations thereby drying wet cloths quickly.

vii. A centrifuge works on principle of centrifugal force. In centrifuge, a test tube containing liquid along with suspended particles is whirled in a horizontal circle. Denser particles are acted upon by centrifugal force, hence they get accumulated at bottom which is on outside while rotating.

***Q.37. Define centrifugal force. Give its any four examples.**

**Ans:** *Refer Q.35 and Q.36*

**Q.38. What is pseudo force? Why centrifugal force is called pseudo force? [Oct99]**

**Ans:**

i. The force whose origin is not defined due to the known natural interactions is called pseudo force.

ii. The known interactions are gravitational force, electromagnetic force, nuclear force, frictional force, etc.

iii. It is directed opposite to the direction of accelerated frame of reference.

iv. The centrifugal force is a fictitious force which arises due to the acceleration of the frame of reference. Therefore it is called a pseudo force;.

**Q.39. Define:**

i. **Inertial frame of reference**

ii. **Non-inertial frame of reference**

**Ans:**

**i. Inertial frame of reference:**

*A frame of reference which is fixed or moving with uniform velocity relative to a fixed frame, is called as inertial frame of reference.*

Newton's laws of motion can be directly applied when an inertial frame of reference is used, without inclusion of pseudo force,

**ii. Non-inertial frame of reference:**

*A frame of reference which is moving with an acceleration relative to a fixed frame of reference is called non-inertial frame of reference,*

In non-inertial frame of reference, Newton's laws of motion can be applied only by inclusion of some fictitious force (pseudo force) acting on the bodies.

***Q.40.Distinguish between centripetal force and centrifugal force.**

**[Mar 05,09,10, Feb 2013 old course] Ans: Difference between centripetal force and centrifugal force**

| Sr. No. | Centripetal force | Centrifugal force |
|---|---|---|
| i. | Centripetal force is directed along the radius towards the centre of a circle | Centrifugal force is directed along the radius away from the centre of a circle. |
| ii. | It is a real force. | It is a pseudo force. |
| iii. | It is considered in inertial frame of reference. | It is considered in non-inertial frame of reference |
| iv. | In vector form, it is given by, $\vec{F} = \frac{mv^2}{r}.\hat{r}_0$ | In vector form, it is given by, $\vec{F} = +\frac{mv^2}{r}.\hat{r}_0$ |

**Note:**

1. If centripetal force, somehow vanishes at any point on its path, the body will fly off tangentially to its path at that point, due to inertia.

2. The work done on the revolving particle by a centripetal force is always zero, because the directions of the displacement and force are perpendicular to each other.

   Thus, W = $\vec{F}.\vec{s}$ = Fs cosq

   But q = 90°

   \ W = F × cos 90° = 0

3. Any one of the real forces or their resultant provide centripetal force.

4. Accelerated frame is used to attach the frame of reference to the particle performing U.C.M.

**#Q.41.Do centripetal and centrifugal force constitute action reaction pair? Explain.**

**Ans:**

i. Centripetal and centrifugal force do not form action reaction pair,

ii. The centripetal force is necessary for the body to perform uniform circular motion. It is real force in inertial frame of reference. The centrifugal force is not a real force. It is the force acting on the same body in non-inertial frame of reference to make Newton's laws of motion true. As both centripetal and centrifugal forces are acting on the same body in different frame of reference, action reaction pair is not possible.

1.7 Banking of roads.

***Q.42. Derive the expression for maximum safety speed with which vehicle should move along a curve horizontal road. State the significance of it.**

**Ans: Expression for maximum safety speed on horizontal curve road:**

i. Consider a vehicle of mass m moving with speed v along a horizontal curve of radius r.

ii. While taking a turn, vehicle performs circular motion. Centripetal force is provided by the frictional force between tyres and road.

iii. Centripetal force is given by,

$$F_{cp} = \frac{mv^2}{r}$$

iv. Frictional force between road and tyres of wheel is given by, $F_s = mN$

where, m = coefficient of friction between tyres of wheels and road.

N = normal reaction acting on vehicle in upward direction.

But, N = mg

$F_s = mmg$

v. For safe turning of vehicle,

$F_{cp} = F_s$

\ $$\frac{mv^2}{r} = mmg$$

\ $v^2 = mrg$ \ $v = \sqrt{mrg}$

vi. Maximum safe speed of the vehicle without skiding is provided by maximum centripetal force.

$$v_{max} = \sqrt{mrg}$$

This is maximum speed of vehicle.

**Significance:**

i. The maximum safe speed of a vehicle on a curve road depends upon friction between tyres and road.

ii. Friction depends on the nature of the surface and presence of oil or water on the road.

iii. If friction is not sufficient to provide centripetal force, the vehicle is likely to skid off the road.

**Q.43. What force is exerted by a vehicle on the road, when it is at the top of a convex bridge of radius R?**

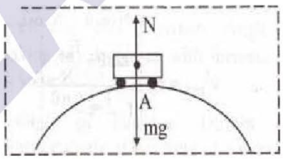
**Ans: Force exerted by the vehicle on the convex bridge:**

i. Let,

m = mass of vehicle

R = radius of convex bridge

g = acceleration due to gravity




In the figure, centripetal force acting on the vehicle is given by,

$$mg - N = \frac{mv^2}{R}$$

$$N = mg - \frac{mv^2}{R}$$

iii. Normal reaction is balanced by the net force exerted on the vehicle.

It is given by, N = f

\ $$F = N = mg - \frac{mv^2}{R}$$

This is the required force on the vehicle.

**Note:**

If bridge is concave then,

$$F = mg + \frac{mv^2}{R}$$

***Q.44.What is banking of road?**

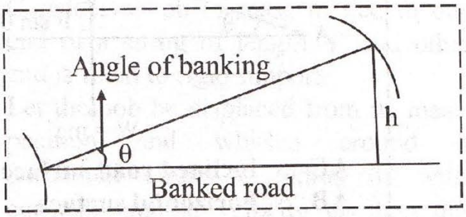
**[Mar 99,12; Oct 01, 06] Explain the necessity of banking of the road.**

**[Mar 99,Oct 01, Oct 06]**

**Ans: Banking of road:**

The process of raising outer edge of a road over its inner edge through certain angle is known as banking of road.

The angle made by the surface of road with horizontal surface of road is called angle of banking.




**Necessity of banking of the road:**

i. When a vehicle moves along horizontal curved road, necessary centripetal force is supplied by the force of friction between the wheels of vehicle and surface of road.

ii. Frictional force is not enough and reliable every time as it changes when road becomes oily or wet in rainy season.

iii. To increase the centripetal force the road should be made rough. But it will cause wear and tear of the tyres of the wheel.

iv. Thus, due to lack of centripetal force vehicle tends to skid.

v. When the road is banked, the horizontal component of the normal reaction provides the necessary centripetal force required for circular motion of vehicle.

vi. Thus, to provide the necessary centripetal force at the curved road, banking of road is necessary.

***Q.45.Show that the angle of banking is independent of mass of vehicle.**

**[Mar 10, Oct 10]**

**OR**

**Obtain an expression for maximum safety speed with which a vehicle can be safely driven along curved banked road.**

**[Mar 10,12; Oct 10]**

**Ans: Expression for angle of banking:**

i. *The angle made by the surface of road with horizontal surface of road is called . angle of banking. It i§ given by angle 0.*

**AC : inclined road surface**

**AB : horizontal surface**

**BC : height of road surface**

**G : centre of gravity of vehicle**

**W : (mg) weight of vehicle**

**N : normal reaction exerted on vehicle.**

**q : angle of banking**

ii. Consider a vehicle of mass m moving with speed v on a banked road banked at an angle q as shown in the figure.

iii. Let F be the frictional force between tyres of the vehicle and road surface. The forces acting on the vehicle are

a. Weightmg acting vertically downward.

b. Normal reaction N in upward direction through C.G.

The frictional force between tyres of vehicle and road surface can be resolved into,

Fcosq - along horizontal direction

Fsinq - along vertically downward direction.

iv. The normal reaction N can be resolved into two components:

a. Ncosq along vertical direction

b. Nsinq along horizontal direction.

v. The component Ncosq balances the weight mg of vehicle and component Fsinq of frictional force.

$N\cos q = mg + F\sin q$

$N\cos q - F\sin q = mg$ ....(i)

vi. The horizontal component Nsinq along with the component Fcosq of frictional force provides necessary centripetal forced $\frac{mv^2}{r}$.

\ $N\sin q + F\cos q = \frac{mv^2}{r}$ ..... (ii)

vii. Dividing equation (ii) by (i) we get,

$$\frac{N\sin q + F\cos q}{N\cos q - F\sin q} = \frac{v^2}{rg}$$

The magnitude of, frictional force depends on speed of vehicle for given road surface and tyres of vehicle.

Let $V_{max}$ be the maximum speed of vehicle, the frictional force produced at this speed is given by,

$F_{max} = m_s N$ ..... (iv)

$$\therefore \frac{v_{max}^2}{rg} = \left[\frac{N\sin\theta + F_{max}\cos\theta}{N\cos\theta - F_{max}\sin\theta}\right] \quad ..... (v)$$

Dividing the numerator and denominator of equation (v) by Ncos G, we have,

$$v_{max}^2 = rg\left[\frac{\frac{N\sin\theta}{N\cos\theta} + \frac{F_{max}\cos\theta}{N\cos\theta}}{\frac{N\cos\theta}{N\cos\theta} - \frac{F_{max}\sin\theta}{N\cos\theta}}\right]$$

$$\therefore v_{max}^2 = rg\left[\frac{\tan\theta + \frac{F_{max}}{N}}{1 - \frac{F_{max}\tan\theta}{N}}\right]$$

$$\therefore v_{max}^2 = rg\left[\frac{\mu_s + \tan\theta}{1 - \mu_s\tan\theta}\right]$$

$$[\because F_{max} = \mu_s N]$$

$$\therefore v_{max} = \sqrt{rg\left[\frac{\mu_s + \tan\theta}{1 - \mu_s\tan\theta}\right]} \quad ..... (vi)$$

ix. For a curved horizontal road, $\theta = 0^0$, hence equation (vi) becomes,

$$v_{max} = \sqrt{\mu_s rg} \quad ..... (vii)$$

Comparing equation (vi) and (vii) it is concluded that maximum safe speed of vehicle on a banked road is greater than that of curved horizontal road/level road. If $|a_s$ = 0, then equation (vi) becomes,

$$V_{max} = V_0 = \sqrt{rg\left[\frac{0 + \tan\theta}{1 - 0\tan\theta}\right]}$$

$$\therefore V_0 = \sqrt{rg\tan\theta}$$

At this speed, the frictional force is not needed to provide necessary centripetal force. There will be a little wear and tear of tyres, if vehicle is driven at this speed on banked road. $v_0$ is called as optimum speed.

xii. From equation (viii) we can write,

$$\tan\theta = \frac{v_0^2}{rg}$$

$$\theta = \tan^{-1}\left(\frac{v_0^2}{rg}\right) \quad ...... (ix)$$

banking of a banked road. Formula for angle of banking does not involve mass . of vehicle m. Thus angle of banking is independent of mass of the vehicle.

Q.46. State the factors which affect the angle of banking.

Ans: Factors affecting angle of banking;

*i.* *Speed of vehicle:* Angle of banking ($\theta$) increases with maximum speed of vehicle.

*ii.* *Radius of path:* Angle of banking ($\theta$) decreases with increase in radius of the path.

iii;; *Acceleration due to gravity:* Angle of banking ($\theta$) decreases with increase in the value of 'g'.

Q.47. Define angle of banking. Obtain an expression for angle of banking of a curved road and show that angle of banking is independent of the mass of the vehicle.
**[Mar 97, Feb 03, Oct 03]**

***Ans:*** Refer Q. 44, 45.

#Q.48. The curved horizontal road is banked at angle '$\theta$'. What will happen for vehicle moving along this road if,

i. **$\theta$ < $\theta$' ii. $\theta$ > $\theta$' ? where $\theta$ is angle of banking for given road.**

**Ans:** -

i. If **$\theta$ < $\theta$'**, the necessary centripetal force will not be provided and the vehicle will tend to skid outward, up the inclined road surface,

ii. If **$\theta$ < $\theta$'**, the centripetal force provided will be more than needed and the vehicle will tend to skid down the banked road.

*Q.49.Define conical pendulum. Obtain an expression for the angle made by the string of conical pendulum with vertical. Hence deduce the expression for linear speed of bob of the conical pendulum.!

Ans : Definition:

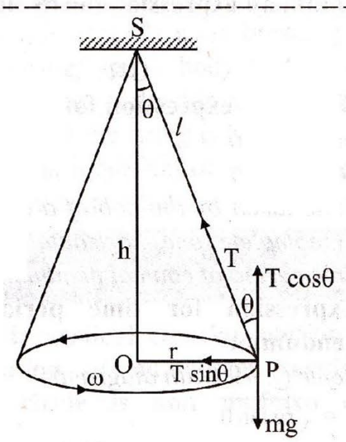
*A simple pendulum, whidh W given such a motion that bob describes a horizontal circle and the string describes a cone is called a conical pendulum.*

**Expression for angle made by string with vertical:**

i. Consider a bob of mass m tied to one end of a string of length T and other end is fixed to rigid support.

ii. Let the bob be displaced from its mean position and whirled around a horizontal circle | of radius 'r' with constant angular velocity co, then the, bob performs y.C.M.

iii. During the motion, string is incljned to the vertical at an angle o as. shown in the




**S : rigid support**
**T : tension in the string**
***l* : length of string**
**h : height of support from bob**
**v : velocity of bob**
**r : radius of horizontal circle**
**θ : semi vertical angle**
**mg : weight of bob**

iv. In the displaced position P, there are two forces acting on the bob.

a. The weight mg acting vertically downwards.

b. The tension T acting upward along the string.

v. The tension (T) acting in the string can be resolved into two components:

a. T cosq acting vertically upwards.

b. T sinq acting horizontally towards centre of the circle.

vi. Vertical component T cosq balances the weight and horizontal component T sinq provides the necessary centripetal force.

$T\cos q = mg$ ..... (i)

$$T\sin q = \frac{mv^2}{r} = mrw^2 \qquad ....(ii)$$

vii. Dividing equation (ii) by (i), we get

$$\tan q = \frac{v^2}{rg} \qquad ....... (iii)$$

Therefore, the angle made by the string with the vertical is $q = \tan^{-1}\left(\frac{v^2}{rg}\right)$

Also, from eq. (iii),

$v^2 = rg \tan q$

$v = \sqrt{rg \tan q}$

This is the expression for the linear speed of the bob of a conical pendulum.

*Q.50.Define period of conical pendulum and obtain an expression for its time period. [Oct 08,09]

OR

Derive an expression for period of a conical pendulum. [Mar 08]

Ans: Definition:

*Time taken by the bob of a conical pendulum to complete one horizontal circle is called time period of conical pendulum.*

**Expression for time period of conical pendulum:**

*Refer Q. 49 (with diagram)*

$\because \quad v = \sqrt{rg \tan q}$

$\because \quad w = \sqrt{\frac{g \tan q}{r}} \qquad [\because v = rw] \qquad ......$ (i)

i. In D SOP, $\tan q = \frac{r}{h}$

From equation (i),

$$w = \sqrt{\frac{gr}{rh}}$$

$$\backslash \qquad w = \sqrt{\frac{g}{h}}$$

ii. If the period of conical pendulum is T then,

$$\omega = \frac{2\pi}{T}$$

$$\therefore \quad \frac{2\pi}{T} = \sqrt{\frac{g}{h}} \qquad ....(ii)$$

iii. Also, In $\Delta$ SOP,

$$\cos\theta = \frac{h}{l}$$

$\therefore$ $h = l \cos\theta$

Substituting h in equation (ii), we get,

$$T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$$

This is required expression for time period of conical pendulum.

Q.51. Discuss the factors on which time period of conical pendulum depends.

**Ans:** Time period of conical pendulum is given by,

where, $l$ = length of the string

g = acceleration due to gravity

$\theta$ = angle of inclination

From equation (i), it is observed that period of conical pendulum depends on following factors.

*i. Length of pendulum (l):* Time period of conical pendulum increases with increase in length of pendulum.

*ii. Acceleration due to gravity (g):* Time period of conical pendulum decreases with increase in g.

*iii. Angle of inclination ($\theta$):* As $\theta$ increases, cos $\theta$ decreases, hence, time period of conical pendulum decreases with increase in $\theta$. (For $0 < \theta < \pi$)

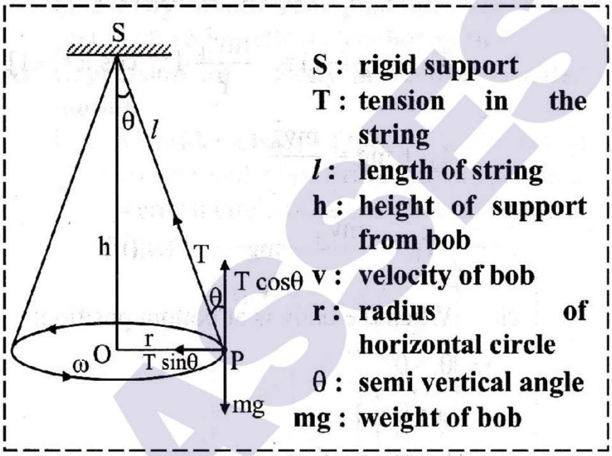
Q.52. Find an expression for tension in the string of a conical pendulum.

Ans: Expression for tension in the string of a conical pendulum:

i. Consider a bob of mass 'm' is tied to one end of a string of length '$l$' and other end fixed to rigid support (S).

ii. Let the bob be displaced from its mean position and whirled around a horizontal circle of radius 'r' With constant angular velocity '$\omega$'.

iii. During the motion, string is inclined to the vertical at an angle $\theta$ as shown in the figure.




iv. In the displaced position P, there are two forces acting oil the bob:

a. The weight mg acting vertically downwards and

b. The tension T acting upwards along the string.

v. The tension (T) acting in the string can be resolved into two components:

a. T cos $\theta$ acting vertically upwards

b. T sin $\theta$ acting horizontally towards centre of the circle

vi. Vertical component T cos $\theta$ balances the weight of the bob and horizontal component T sin $\theta$ provides the necessary centripetal force.

$$T\cos\theta = mg \qquad ...(i)$$

$$T = \sin\theta = \frac{mv^2}{r} \qquad ....(ii)$$

Vii. Squaring and adding equations (i) and (ii), we get,

$$T^2\cos^2\theta + T^2\sin^2\theta = (mg)^2 + \left(\frac{mv^2}{r}\right)$$

$$T^2(\cos^2\theta + \sin^2\theta) = (mg)^2 + \left(\frac{mv^2}{r}\right)^2$$

$$T^2 = (mg)^2 + \left(\frac{mv^2}{r}\right)^2 \qquad \text{...... (iii)}$$

$$[\because \sin^2\theta + \cos^2\theta = 1]$$

viii. Dividing equation (ii) by (i),

$$\tan\theta = \frac{v^2}{rg} \qquad \text{...... (iv)}$$

From figure, $\tan\theta = \frac{r}{h}$

$$\therefore \frac{r}{h} = \frac{v^2}{rg} \qquad \therefore v^2 = \frac{r^2 g}{h}$$

ix. From equation (iii) and (iv), we have

$$T^2 = (mg)^2 + \left[\frac{m}{r}\left(\frac{r^2 g}{h}\right)\right]^2$$

$$\therefore T^2 = (mg)^2 + \left[1 + \left(\frac{r}{h}\right)^2\right]$$

$$\therefore T = mg + \sqrt{1 + \left(\frac{r}{h}\right)^2}$$

This is required expression for tension in the string.

#Q.53. Is there any limitation on semivertical angle in conical pendulum?

**Ans:**

i. For a conical pendulum,

Period $T \propto \sqrt{\cos\theta}$

Tension $F \propto \frac{1}{\cos\theta}$

With increase in angle $\theta$, cos $\theta$ decreases and tan $\theta$ increases. For $\theta = 90°$, T = 0, F = $\infty$ and v = $\infty$ which cannot be possible,

ii. Also, $\theta$ depends upon breaking tension of string, and a body tied to a string cannot be resolved in a horizontal circle such that the string is horizontal. Hence, there is limitation of semivertical angle in conical pendulum.

## 3.8 Vertical circular motion due to earth's gravitation

*Q.54. What is vertical circular motion? Show that motion of an object revolving in vertical circle is non uniform circular motion.

**Ans:**

i. **A** body revolves in a vertical circle such that its motion at different points is different then the motion of the body is said to be vertical circular motion, ii. Consider an object of mass m tied at one end of an inextensible string and whirled in a vertical circle of radius r.

ii. Due to influence of earth's gravitational field, velocity and tension of the body vary in magnitude from maximum at bottom (lowest) point to minimum at the top (highest) point.

iii. Hence motion of body in vertical circle is non uniform circular motion.

## 1.9 Equation for velocity and energy at different positions in vertical circular motion.

*Q.55.Obtain expressions for tension at highest position, midway position and bottom position for an object revolving in a vertical circle.

Ans: Expression for tension in V.C motion:

i. Let a body of mass m be tied at the end of a massless inextensible string and whirled in a vertical circle of radius r in anticlockwise direction.

ii. At any point P the forces acting on it are:

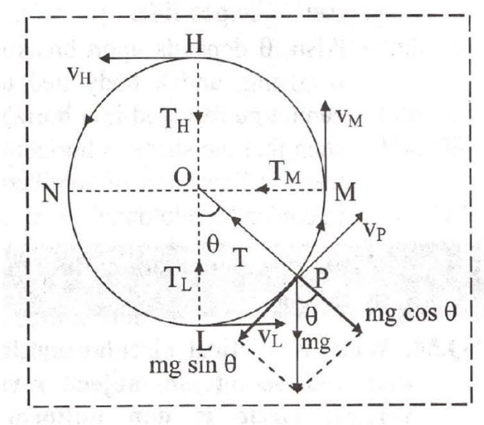
a. Tension T along PO

b. Weight mg along Vertically downward direction.

iii. The weight mg can be resolved into two rectangular components:

a. mg cos $\theta$ acting along OP.

b. mg sin $\theta$ acting tangentially in a direction opposite to velocity at that point.




i. To complete vertical circular path, the necessary centripetal force is provided by the difference in the tension T and mg cos θ.

$$\therefore \quad T - mg\cos\theta = \frac{mv_P^2}{r} \quad \ldots\ldots (i)$$

where, $v_P$ = velocity at point P.

When body is at highest position, tension in the string = $T_H$ and θ = π.

Using equation (i), we have

$$T_H - mg\cos\theta = \frac{mv_H^2}{r}$$

where $v_H$ = velocity at highest point.

$$\therefore \quad T_H - mg(-1) = \frac{mv_H^2}{r} \quad [\because \cos\pi = -1]$$

$$\therefore \quad T_H + mg = \frac{mv_H^2}{r}$$

$$\therefore \quad T_H = \frac{mv_H^2}{r} - mg \quad \ldots\ldots (ii)$$

vi. When the body is at bottom position :

θ = $0^0$

∴ cos θ = 1

From equation (i),

$$T_L - mg\cos 0^0 = \frac{mv_L^2}{r}$$

where $T_L$ = tension at lowest point

$V_L$ = velocity at lowest point

$$\therefore \quad T_L - mg = \frac{mv_L^2}{r}$$

$$\therefore \quad T_L = \frac{mv_L^2}{r} + mg \quad \ldots\ldots (iii)$$

vii. When the body is at midway position,

(M or N)

θ = $90^0$

∴ cos $90^0$ = 0

If tension at horizontal position is $T_M$ then

$$T_M - mg\cos 90^0 \frac{mv_M^2}{r} \quad \text{[From (i)]}$$

$$\therefore \quad T_M - 0 = \frac{mv_M^2}{r}$$

$$\therefore \quad T_M = \frac{mv_M^2}{r} \quad \ldots\ldots (iv)$$

From equation (ii), (iii) and (iv) it is observed that tension is maximum at lowest position and minimum at highest position.

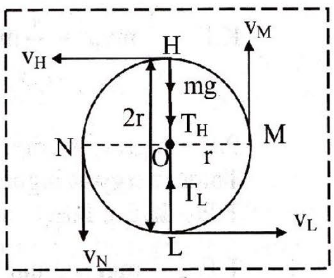
Q.56. Derive expressions for linear velocity at lowest point, midway and top position for a particle revolving in a vertical circle if it has to just complete circular motion without string slackening at top.

**OR**

Obtain an expression for minimum velocity of a body at different positions, so that it just performs vertical circular motion.

Ans: Expression for velocity in vertical circular motion:

i. Consider a body of mass m which is tied to one end of a string and moves in a vertical circle of radius r as shown in the figure.




Let,

$V_H$ = velocity at highest position

$V_L$ = velocity at lowest position

$V_M$ = velocity at midway position

The velocity at any point on the circle is tangential to the circular path.

iii. **Velocity at highest position:**

Tension in the string at highest position

$$T_H - \frac{mv_H^2}{r} - mg \qquad .......(i)$$

In order to continue the circular motion,

$T_H \geq 0$

\ $T_H = 0$

\ Equation (i) becomes

$$- \frac{mv_H^2}{r} - mg = 0 \qquad \backslash \quad \frac{mv_H^2}{r} = mg$$

\ $v_H^2 = rg$

$$v_H = \sqrt{rg} \qquad ..... (ii)$$

Equation (ii) represents minimum velocity at highest point so that string is not slackened.

To continue vertical circular motion,

$v_H$ ³ $\sqrt{rg}$ (at top position).

**iv.** Velocity at lowest position:

**According to law of conservation of energy,**

Total energy at L = Total energy at H

$(K.E)_L + (P.E)_L = (K.E)_H + (P.E)_H$

..... (iii)

At lowest point,

P.E = 0

$$K.E. = \frac{1}{2}mv_L^2$$

At highest point,

P.E. = mg (2r) and $K.E. = \frac{1}{2}mv_H^2$

From equation (iii)

$$\frac{1}{2}mv_L^2 + 0 = \frac{1}{2}mv_H^2 + mg(2r)$$

\ $\frac{1}{2}mv_L^2 = \frac{1}{2}mv_H^2 + \frac{1}{2}(4mgr)$

\ $\frac{1}{2}mv_L^2 = \frac{1}{2}m(v_H^2 + 4gr)$

\ $v_L^2 = v_H^2 + 4gr$ ..... (iv)

To complete vertial circular motion,

$v_H = \sqrt{rg}$

\ $v_L^2 = \left(\sqrt{rg}\right)^2 + 4rg = rg + 4rg$

$v_L^2 = 5rg$

$$v_L = \sqrt{5rg} \qquad ...... (v)$$

Equation (v) represents minimum velocity at the lowest point, so that body can safely travel along vertical circle.

**v. Velocity at midway position :**

At midway position, $K.E. = \frac{1}{2}mv_M^2$ and

P.E. = mgr

Total energy at L = Total energy at M

$(P.E.)_L + (K.E.)_L = (P.E.)_M + (K.E.)_M$

$$0 + \frac{1}{2}m \,´\, 5rg = mgr + \frac{1}{2}mv_M^2$$

\ $\frac{5mgr}{2} = mgr + \frac{1}{2}mv_M^2$

\ $\frac{5}{2}mgr - mgr = \frac{1}{2}mv_M^2$

\ $\frac{3}{2}mgr = \frac{1}{2}mv_M^2$

$\therefore \quad v_M^2 = 3rg$

$\therefore \quad v_M = \sqrt{3rg}$ .... (vi)

Equation (vi) represents minimum velocity of a body at midway position, so that it can safely travel along vertical circle. To continue vertical circulr motion,

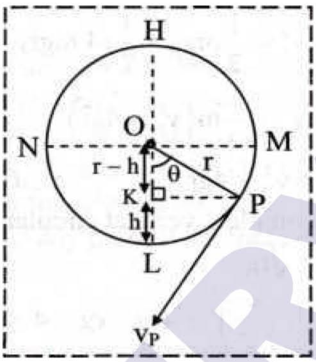
$v_M = \sqrt{3rg}$ .

Q.57. Derive an expression for the minimum velocity of a body at any point in vertical circle so that it can perform vertical circular motion.

Ans: Expression for minimum velocity at any point in V.C. motion:

i. Consider a body of mass 'm' is performing vertical circular motion of path radius r. P is any point on the circle as shown in the figure. We have to find velocity at P.

Let vp = velocity at P




In Δ OKP,

OK = r cos θ

h = r – OK

= r – r cos θ

h = r (1 – cos θ)

iii. From principle of conservation of energy,

Total energy at L = Total energy at P $(P.E)_L + (K.E)_L = (P.E)_P + (K.E)_P$

$$0 + \frac{1}{2}mv_L^2 = mgh + \frac{1}{2}mv_P^2$$

But min. $v_L = \sqrt{5rg}$

$$\therefore \quad \frac{1}{2} \times 5mgr = mgr(1 - \cos\theta) + \frac{1}{2}mv_P^2$$

$$\therefore \quad \frac{1}{2}mv_P^2 = \frac{1}{2} \times 5mgr - mgr(1 - \cos\theta)$$

$$= mrg\left(\frac{5}{2} - 1 + \cos\theta\right)$$

$$\therefore \quad \frac{1}{2}v_P^2 = \frac{rg(5 - 2 + 2\cos\theta)}{2}$$

$$v_P^2 = (3 + 2\cos\theta)rg$$

$$v_P = \sqrt{(3 + 2\cos\theta)rg}$$

*Q.58.0btain expression for energy at different positions in the vertical circular motion. Hence show that total energy in vertical circular motion is constant.

OR

Show that total energy of a body performing vertical circular motion is conserved. [Mar 11]

Ans: Expression for energy at different points in V.C.M:

i. Consider a particle of mass m revolving in a vertical circle of radius r in anticlockwise direction.

**ii.** When the particle is at highest point H:

$$K.E. = \frac{1}{2}mv_H^2 = \frac{1}{2}m \times rg \qquad [\because v_H = \sqrt{rg}]$$

P.E = mg(2r) = 2mgr

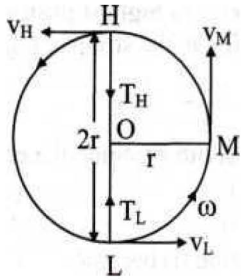
Total energy at highest point

T.E = K.E + P.E

$$\therefore \quad T.E. = \frac{1}{2}mgr + 2mgr = \frac{5}{2}mgr$$

$$\therefore \quad (T.E.)_H = \frac{5}{2}mgr \quad \text{...... (i)}$$

Equation (i) represents energy of particle at the highest point in V.C.M. r




**iii.** When particle is at lowest point L :

P.E. = 0 [$\because$ At lowest point, h = 0]

$$K.E. = \frac{1}{2}mv_L^2 = \frac{1}{2}m \times 5rg = \frac{5}{2}mgr$$

Total energy at lowest point = K.E + P.E

$$= \frac{5}{2}mgr + 0 = \frac{5}{2}mgr$$

$$(T.E.)_L = \frac{5}{2}mgr \quad \text{...... (ii)}$$

Equation (ii) represents energy of particle at lowest point in V.C.M

iv. **When the particle is at midway point in V.C.M:**

P.E = mgh = mgr [$\because$ h = r]

$$K.E. = \frac{1}{2}mv_M^2 = \frac{1}{2}m \times 3rg = \frac{3}{2}mgr$$

Total energy at M = K.E + P.E

$$= \frac{3}{2}mgr + mgr$$

$$\therefore \quad (T.E.)_M = \frac{5}{2}mgr \quad \text{...,(iii)}$$

Equation (iii) represents total energy of particle at midway position in V.C.M

v. From equation (i), (ii) and (iii),

It is observed that total energy at any point in V.C.M is is $\frac{5}{2}mgr$, i.e. constant.

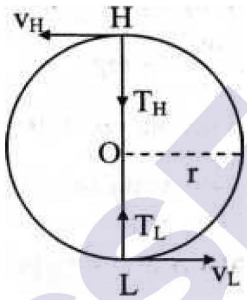
Hence, total energy of a particle performing vertical circular motion remains constant.

Q.59. Show that for a body performing V.C.M., difference in tension at the lowest and highest point on vertical circle is 6mg.

OR

*A particle of mass m, just completes the vertical circular motion. Derive the expression for the difference in tensions at the highest and the lowest points. [Feb 2013]

Ans: i. Suppose a body of mass 'm' performs V.C.M on a circle of radius r as shown in the figure.




Let,

$T_L$ = tension at the lowest point

$T_H$ = tension at the highest point

$V_L$ = velocity at the lowest point

$V_H$ = velocity at the highest point

At lowest point L,

$$T_L = \frac{mv_L^2}{r} + mg$$

At highest point H

$$T_H = \frac{mv_L^2}{r} - mg$$

$$\therefore \quad T_L - T_H = \frac{mv_L^2}{r} + mg - \left(\frac{mv_H^2}{r} - mg\right)$$

$$= \frac{m}{r}(v_L^2 - v_H^2) + 2mg$$

$$\therefore \quad T_L - T_H = \frac{m}{r}(v_L^2 - v_H^2) + 2mg \quad \text{..... (i)}$$

iv. By law of conservation of energy,

(P.E + K.E) at L = (P.E + K.E) at H

$$\therefore \quad 0 + \frac{1}{2}mv_L^2 = mg.2r + \frac{1}{2}mv_H^2$$

$$\therefore \quad v_L^2 = - v_H^2 = 4gr$$

From equation (i) and (ii), we have,

$$T_L - T_H = \frac{m}{r}(4gr) + 2mg = 4mg + 2mg$$

$$T_L - T_H = 6mg$$

### 1.10 Kinematical equation for circular motion in with linear motion

*Q.60. State the kinematical equations for circular motion in analogy with linear motion.

**Ans:** The kinematical equations of circular motion are analogue to the equations of linear motion which is given below:

i. Angular velocity of a particle at any time t is given by, $w = w_0 + at$,

where,

$w_0$ = initial angular velocity of the particle

a = angular acceleration of the particle

It is analogue to the kinematical equation of linear motion,

v = u + at

where, u = initial velocity of particle

v = final velocity of particle

a = constant acceleration of particle

ii. The angular displacement of a particle in rotational motion after time t is given by

$$q = w_0 t + \frac{1}{2} a t^2$$

It is analogous to the kinematic equation of linear motion, $s = ut + \frac{1}{2}at^2$

where,

s = linear displacement

u = initial velocity

a = constant acceleration

t = time interval.

iii. The angular velocity of rotating particle after certain angular displacement 6 is given by,

$$w^2 = w_0^2 + 2aq$$

It is analogous to the kinematic equation of linear motion $v^2 = u^2 + 2as$,

where,

u = initial velocity

v = final velocity

a = constant acceleration

s = linear displacement

## Summary :

1. Motion of a particle along a circumference of a circle is called circular motion.
2. Angle described by a radius vector in a given time at the centre of circle to other position is called as angular displacement.
3. Infinitesimal small angular displacement is a vector quantity. Finite angular displacement is a pseudo vector (scalar). It has magnitude and direction but vector addition is neither commutative nor associative.
4. The rate of change of angular displacement w.r.t time is called angular velocity.

   It is given by $\vec{w} = \frac{d\vec{q}}{dt}$

   Angular velocity relates with linear velocity by the relation, $\vec{v} = \vec{w} ' \vec{r}$ or $v = rw$.
5. The rate of change of angular velocity w.r.t time is called as angular acceleration.

   It is given by relation, $a = \frac{dw}{dt} = \frac{w - w_0}{t}$
6. There are two types of acceleration $a_R$ (radial) and $a_x$ (tangential) in non U.C.M.

   Formula for $a_R = w^2 r$ and $a_T = \frac{dv}{dt} = ra$ ,

   resultant acceleration of a particle in non-U.C.M is given by, $a\,a = \sqrt{a_R^2 + a_T^2}$
7. Centripetal force is directed towards the centre along the radius and makes the particle to move along the circle.
8. Centrifugal force is directed away from the centre along the radius and has the same magnitude as that of centripetal force.
9. The process in which the outer edge of the road is made slightly higher than the inner edge is called as banking of roads.
10. The formula for $v_{max} = \sqrt{mrg}$ and

    $v_{min} = \sqrt{\frac{rg}{m}}$.

    On frictional surface, a body

performing circular motion, the centripetal force is provided by the force of friction given by, $F_s = mmg$.

11. The angle of banking (q) is given by, $\tan q = \frac{v^2}{rg}$.

12. The period of revolution of the conical pendulum is given by,

$$T = 2p\sqrt{\frac{r}{g\tan q}} = 2p\sqrt{\frac{l\cos q}{g}}$$

13. The linear speed of the bob of conical pendulum $v = \sqrt{rg\tan q}$

14. Tension at any point P in vertical circular motion is given by,

$$T = \frac{mv_P^2}{r} + mg\cos q$$

Where, $V_p$ = velocity at any point in V.C.M

**Case 1:** At highest point, q = 180°

$$\text{so, } T_H = \frac{mv_H^2}{r} - mg$$

**Case 2:** At lowest point, 0 = 0°

$$\text{so, } T_L = \frac{mv_L^2}{r} + mg$$

15. Velocity at any arbitrary point is given by,

$$v = \sqrt{rg(3 + 2\cos q)}$$

**Case 1:** At highest point, q=180°

$V_L = \sqrt{5rg}$

**Case 2:** At lowest point, q = 0° $V_L = \sqrt{5rg}$

**Case 3:** At horizontal point, q = 90°

\ $V_M = \sqrt{3rg}$

16. Energy of a particle at any point in vertical circular motion is given by T.E = $\frac{5}{2}$ mgr

## :Formulae :

1. **In U.C.M angular velocity:**

**i.** $w = \frac{v}{r}$ **ii.** $w = \frac{q}{t}$

**iii.** $w = 2pn$ **iv.** $w = \frac{2p}{T}$

2. **Angular displacement:**

i. $q = wt$ ii. $q = \frac{2pt}{T}$

3. Angular acceleration :

i. $a = \frac{w_2 - w_1}{t}$ ii.

$a = \frac{2p}{t}(n_2 - n_1)$

4. Linear velocity:

**i.** $v = r\ w$ **ii.** $v = 2pnr$

5. **Centripetal acceleration or radial acceleration:**

$a = \frac{v^2}{r} w^2 r$

6. **Tangential acceleration:** $\vec{a}_T = \vec{a} \times \vec{r}$

7. **Centripetal force:**

i. $F_{CF} = \frac{mv^2}{r}$

ii. $F_{CP} = mrw^2$

*iii.* $F_{CP} = 4p^2 mrn^2$

**iv.** $F_{CP} = \frac{4p^2 mr}{T^2}$

**v.** $F_{CP} = mmg = mw^2 r$

8. **Inclination of banked road :**

$q = \tan^{-1}\left(\frac{v^2}{rg}\right)$

9. Maximum velocity of vehicle to avoid skidding on a curve unbanked road:

$v_{max} = \sqrt{mrg}$

10. Maximum safe velocity on banked road :

i. $v_{max} = \sqrt{rg\left[\frac{m_s + \tan q}{1 - m_s \tan q}\right]}$ (Presence of friction)

ii. $v_{max} = \sqrt{rg \tan q}$ (absence of friction)

11. Height of inclined road : **$h = l \sin q$**

12. Conical Pnedulum :

i. Angular velocity of the bob of conical pendulum,

$$w = \sqrt{\frac{g}{l \cos q}} = \sqrt{\frac{g \tan q}{r}} = r\sqrt{\frac{g}{h}}$$

ii. Linear velocity of the bob of conical pendulum

$$v = \sqrt{rg \tan q}$$

iii. Period of conical pendulum

a. $T = 2p\sqrt{\frac{l \cos q}{g}}$

b. $T = 2p\sqrt{\frac{l}{g}}$ (q is small)

c. $T = 2p\sqrt{\frac{r}{g \tan q}}$

13. **Minimum velocity at lowest point to complete V.C.M:** $v_L = \sqrt{5rg}$

14. **Minimum velocity at highest point to complete V.C.M:** $v_H = \sqrt{rg}$

15. **Minimum velocity at midway point to complete in V.C.M:** $v_M = \sqrt{3rg}$

16. Tension at highest point in V.C

$$T_H = \frac{mv_H^2}{r} - mg$$

17. Tension at lowest point in V.C.M :

$$T_L = \frac{mv_L^2}{r} + mg$$

18. **Total energy at any point in V.C.M:**

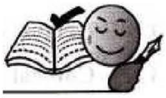
$$\textbf{T.E} = \frac{5}{2}\,\textbf{mgr}$$

19. **Kinematic equations of linear motion:**

i. $w = w_0 + at$

ii. $q = w_0 t + \frac{1}{2} at^2$

iii. $w^2 = w_0^2 + 2aq$



***Example 1***

What is the angular displacement of second hand in 5 seconds ?

***Solution:***

*Given:* T = 60 s, t = 5 s

*To find:* Angular displacement (q)

*Formula:* $q = \frac{2pt}{T}$

*Calculation:* From formula,

**q = 0.5237 rad**

**Ans:** The angular displacement of second hand in 5 seconds is **0.5237** rad.

***Example 2***

Calculate the angular velocity of earth due to its spin motion.

***Solution:***

*Given:* T = 24 hour = 24 × 3600 s

*To find:* Angular velocity (w)

Formula: $q = \frac{2p}{T}$

*Calculation* : From formula,

$$w = \frac{2p}{24 ´ 3600}$$

$$= \frac{2(3.142)}{24 ´ 3600}$$

$w = 7.27 \times 10^{-5}$ rad/s

**Ans:** The angular velocity of earth due to iii spin motion is ***7.27*** **× $10^{-5}$ rad/s.**

Example 3 :

A particle, initially at ret, perform circular motion with uniform angular acceleration 0.2 rad/s$^2$. What speed will it attain in 10 seconds ?

Solution :

**Given :** $w_1 = 0$, $a = 0.2$ rad/s$^2$, t = 10 s

**To find: Speed ($w_2$)**

**Formula :** $w_2 = w_1 + at$

**Calculation : From formula,**

$$w_2 = 0 + (0.2) \times 10$$

$$w_2 = 2 \text{ rad/s}$$

Ans: **Speed attained by the particle in 10 seconds is 2rad/s.**

**Example 4**

The frequency of a particle performing circular motion changes from 60 r.p.m to 180 r.p.m in 20 second. Calculate the angular acceleration.

**Solution:**

Given : $n_1 = 60 \text{ r.p.m.} = \frac{60}{60} = 1 \text{ rev/s}$,

$n_2 = 180 \text{ r.p.m} = \frac{180}{60} = 3 \text{ rev/s}$,

t = 20 s

To find : Angular acceleration (a)

Formula : $a = \frac{w_2 - w_1}{t}$

Calculation : From formula,

$$a = \frac{2pn_2 - 2pn_1}{t} = \frac{2p(3-1)}{20}$$

$$= \frac{2 ´ 3.142 ´ 2}{20}$$

$$a = \frac{3.142}{5}$$

**a = 0.6284 rad/s$^2$**

**Ans:** Angular acceleration[1] of the particle is **0. 6284** rad/s$^2$.

**** Example 5***

**The length of hour hand of a wrist watch is 1.5 cm:'Find magnitude of £ angular velocity % linear velocity**

**i. angular acceleration**

**ii. radial acceleration**

**iii. tangential acceleration**

**iv. linear acceleration of a particle on tip of hour hand.**

**Solution:**

*Givens* T = 12 × 60 × 60 = 43200 s,

r = 1.5cm = 1.5 × $10^{-2}$ m

To find:

i. Angular velocity (w)

ii. Linear velocity (v)

iii. Angular acceleration (a)

iv. Radial acceleration ($a_R$)

v. Tangential acceleration ($a_T$)

vi. Linear acceleration (a)

*Formulae:*

*i.* $w = \frac{2p}{T}$ *ii.* $v = rw$

*iii.* $a = \frac{dw}{dt}$ *iv.* $aR = vw$

*v.* $a_T = ar$ *vi.* $a = a_R + a_T$

Calculation:

i. From formula (i),

$$w = \frac{2 ´ 3.142}{43200}$$

\ $w = 1.454 \times 10^{-4}$ rad/s

ii. From formula (ii),

$v = 1.5 \times 10^{-2} \times 1.46 \times 10^{-4}$

$v = 2.19 \times 10^{-6}$ m/s

iii Since angular velocity of hour hand is constant.

\ a = 0

iv. From formula (iv),

$a_R = 2.182 \times 10^{-6} \times 1.454 \times 10^{-4}$

$a_R = 3.175 \times 10^{-2}$

v. From formla (v),

$a_T = 0 \times 1.5 \times 10^{-2}$

\ $a_T = 0$

vi. From formula (vi),

$a = 3.175 \times 10^{-10} + 0$

$a = 3.175 \times 10^{-10}$ m/s$^2$

**Ans:**

i. The hour hand of the wrist watch has angular velocity **$1.454 \times 10^{-4}$ rad/s.**

ii. The hour hand of the wrist watch has linear velocity **$2.19 \times 10^{-6}$ m/s.**

iii. The hour hand of the wrist watch has angular acceleration 0.

iv. The hour hand of the wrist watch has radial acceleration **$3.175 \times 10^{-10}$ m/s$^2$.**

v. The hour hand of the wrist watch has tangential acceleration 0.

vi. Linear acceleration of the particle on tip of hour hand is $3.175 \times 10^{-10}$ **m/s$^2$.**

**Example 6**

**A one kg mass tied at the end of the string 0.5 m long is whirled in a horizontal circle, the other end of the string being fixed. The breaking tension in the string is 50 N. Find the greatest speed that can be given to the mass.**

**Solution:**

*Given:* Breaking tension, F = 50 N,

m = 1 kg, r = 0.5 m

*To find:* Maximum speed ($v_{max}$)

*Formula :* B.T. max C.F. $= \frac{mv_{max}^2}{r}$

*Calculation:* From formula,

$$v_{max}^2 = \frac{F ´ r}{m}$$

$$v_{max}^2 = \frac{50 ´ 0.5}{1}$$

\ $v_{max} = \sqrt{50 ´ 0.5}$

\ $v_{max} = 5$ m/s

**Ans:** The greatest speed that can be given to the mass is 5 m/s.

**Example 7**

What is the angular speed of the minute hand of a clock? If the minute hand is 5 cm long. What is the linear speed of its tip ? [Oct 04]

**Solution:**

*Given:* Length of minute hand, r = 5 cm,

T = 60 min = 60 x 60 = 3600 s

*To find:* i. Angular speed (w)

ii. Linear speed (v)

*Calculation:* From formula (i),

$$w = \frac{2 ´ 3.142}{3600}$$

**w = $1.74 \times 10^{-3}$ rad/s**

From formula (ii),

$v = rw = 5 \times 10^{-2} \times 1.74 \times 10^{-3}$

$v =$ **$8.7 \times 10^{-5}$ m/s**

**Ans:** The minute hand of the clock has angular speed **$1.74 \times 10^{-3}$ rad/s** and linear speed **$8.7 \times 10^{-5}$ m/s.**

**Example 8**

**A** coin kept on a horizontal rotating disc has its centre at a distance of 0.1 m from the axis of rotation of the disc. If the coefficient of friction between the coin and the disc is 0.25, find the angular speed of the disc at which the coin would be about to slip off.

**Solution:**

*Given:* r = 0.1 m, m = 0.25, g = 9.8 m/s$^2$

*To find:* Angular speed (w)

*Formula:* $mrw_{max}^2 = mmg$

*Calculation:* From formula,

**$w_{max}$ = 4.949 rad/s**

**Ans:** The angular speed of the disc at which the c o i n would be about to slip off is **4.949 rad/s.**

**Example 9**

Calculate the maximum speed with which a car can be safely driven along a curved road of radius 30 m and banked at 30° With the horizontal [g = 9.8 m/s$^2$] [Mar 96]

**Solution:**

*Given:* , r = 30 m, 0 = 30°, g = 9.8 m/s$^2$ ,

*To find:* Maximum speed ($v_{max}$)

*Formula:* $v_{max}$ = ^rg tan 0

*Calculation:* From formula,

Vmax = ^30 x 9.8 x tan (30°)

30x9.8 x~j=

30x9.8

1.732 Vmax = **13.028 m/s Ans:** The maximum speed with which the car can drive safely is **13.028 m/s.**

**Example 10**

A circular race track of radius 300 m is banked at an angle of 15°. If the coefficient of friction between the wheels of a race-car and the road is 0.2. What is the optimum speed of the race car to avoid wear and tear on its tyres? (NCERT)

***Solution:***

*Given:* r = 300 m, q = 15°, g = 9.8 m $s^{-2}$

*To find:* Optimum speed ($v_0$)

*Formula:* $v_0 = \sqrt{rg \tan q}$

*Calculation:* From formula,

$$v_0 = \sqrt{300 ´ \ 9.8 ´ \ \tan 15^0} = \sqrt{787.78}$$

$\mathbf{v_0} = 28.07\ ms^{-1}$

**Ans:** The optimum speed of the race car to avoid wear and tear on its tyres is **28.07** m **s’¹.**

**Example 11**

An aircraft executes a horizontal loop at a speed of 720 km h’¹ with its wings banked at 15°. What is the radius of the loop? (NCERT)

**Solution:**

Given: $v = 720\ km\ h^{-1} = 720 ´ \ \frac{5}{18} = 200\ m/s$,

$q = 15^0$

To find: *Radus (r)*

Formula: $\tan q = \frac{v^2}{rg}$

Calculation : *From formula* ,

$$r = \frac{v^2}{g \tan q}$$

$$\backslash \quad r = \frac{(200)^2}{9.8 ´ \ \tan 15^0} = 15232m$$

\ *r = 15.23 km*

**Ans:** The radius of the loop is 15.23 km.

***Example 12**

Calculate the angular velocity and linear velocity of a tip of minute hand of length 10 cm.

**Solution:**

*Given:* T = 60 min = 60 × 60 s = 3600 s,

*l* = 10 cm = 0.1 m

*To find:* i. Angular velocity (w)

ii. Linear velocity (v)

Formulae : i. $w = \frac{2p}{T}$ ii. v = rw

*Calculation:* From formula (i),

$$w = \frac{2p}{T} = \frac{2 ´ \ 3.142}{3600}$$

w = 1.744 **x** $10^{-3}$ rad/s

From formula (ii),

v = rw = 0.1 × 1.745 x $10^{3}$

v **= 1.745** x **10** $^{4}$ **m/s**

**Ans:** The tip. of the minute hand has angular velocity $1.744 \times 10^{-3}$ rad/s and linear velocity $1.745 \times 10^{-4}$ m/s.

**Example 13**

A coin placed on a revolving disc, with its centre at a distance of 6 cm from the axis of rotation just slips off when the speed of the revolving disc exceeds 45 r.p.m. What should be the maximum angular speed of the disc, so that when the coin is at a distance of 12 cm from the axis of rotation, it does not slip?

**Solution:**

*Given:* $r_1$ = 6 cm,

$r_2$ = 12 cm,

$n_1$ = 45 r.p.m

*To Find:* Maximum angular speed ($n_2$)

*Formula:* $F = mrw^2$

*Calculation:* Since, $mr_1w_1^2 = mr_2w_2^2$

[As mass is constant]

\ $r_1w_1^2 = r_2w_2^2$

$$\frac{w_2}{w_1} = \sqrt{\frac{r_1}{r_2}}$$

$$\therefore \quad \frac{2\pi n_2}{2\pi n_1} = \sqrt{\frac{r_1}{r_2}}$$

$$\therefore \quad \frac{n_2}{n_1} = \sqrt{\frac{r_1}{r_2}}$$

$$\therefore \quad n_2 = n_1\sqrt{\frac{r_1}{r_2}} = 45\sqrt{\frac{6}{12}} \text{ r.p.m.}$$

**Ans:** The maximum angular speed of the disc should be **31.8 r.p.m.**

***Example 14***

A coin kept on a horizontal rotating disc has its centre at a distance of 0.1 m from the axis of the rotating disc. If the coefficient of friction between the coin and the disc is 0.25; find the angular speed of disc at which the coin would be about to slip off. (Given g = 9.8 $m/s^3$)
[Oct11]

***Solution:***

*Given:* r = 0.1 m, $\mu = 0.25$

*To find:* Angular speed ($\omega$)

*Formula:* $v = r\omega$

*Calculation:* Using formula,

$$\omega = \frac{v}{r} = \frac{\sqrt{\mu rg}}{r} = \frac{\sqrt{\mu g}}{r}$$

$$= \left(\frac{0.25 \times 9.8}{0.1}\right)^{1/2}$$

= 4.949 rad/s

**Ans:** The angular speed of the disc at which the coin would be about to slip off is **4.949 rad/s.**

**Example 15**

**A train runs along an unbanked circlar track of radius 30 m at a speed of 54 km $h^{-1}$. The mass of the train $10^6$ kg.**

**i. What provides the centripetal force required for this prupose ? The engine or the rails ?**

**ii. What is the angle of banking required to prevent wearing out of the rail ? (NCERT)**

**Solution :**

i. The centrial force is provided by the lateral force ction due to rails on the wheels of the train.

ii. $v = 54 \text{ km h}{-1} = 54 \times \frac{5}{18} = 15 \text{ m/s}$, r = 30 m

$$\tan\theta = \frac{v^2}{rg}$$

$$\tan\theta = \frac{(15)^2}{30 \times 9.8}$$

$\theta = \tan^{-1}(0.7653)$

$\theta = 37^0 25$

***Example 16***

An aircraft takes a turn along a circular path of radius 1500 m. .If the linear speed of the aircraft is 300 m/s, find its angular speed and time taken by it to complete $\left(\frac{1}{5}\right)^{th}$ of circular path.

***Solution:***

*Given:* r = 1500 m, v = 300 m/s

*To find:* i. Angularspeed ($\omega$)

ii. Time taken for $\left(\frac{1}{5}\right)^{th}$ of circular path (t)

Formulae : i = $r\omega$ ii. $\omega = \frac{\theta}{t}$

*Calculation:* From formula (i),

$$\omega = \frac{v}{r} = \frac{300}{1500}$$

$$\therefore \quad \omega = \frac{300}{1500} = \frac{1}{5} = 0.2 \text{ rad/s}$$

The angular displacement ($\theta$), of the aircraft to complete $\left(\frac{1}{5}\right)^{th}$ of the circular path is

$$\theta = \frac{2\pi}{5} \text{ rad}$$

From formula (ii),

$$t = \frac{\theta}{\omega} = \frac{2\pi/5}{0.2}$$

$$\therefore \quad t = \frac{2 \times 3.142}{5 \times 0.2} = 6.284\,\text{s}$$

**Ans:** The angular speed of the aircraft is **0.2 rad/s** and time taken by it to. complete $\left(\frac{1}{5}\right)^{th}$ of circular path is **6.284 s.**

***Example 17***

A mass of 5 kg is tied' at -the' end of a string 1.2m long revolving in a h o r i z o h t a P t e i r e l e . If the breaking tension in the string is 300 N, find the maximum number of revolutions per minute the mass can make.

***Solution:***

*Given:* Length of the string, r = 1.2 m,
Mass attached, rh = 5 kg,
Breaking pension, T = 300 N

*To find:* Maximum'number of revolution's per minute ($n_{max}$)

*Formula* : $T_{max} = mr\omega^2_{max}$

*Calculation:* From formula,

$\therefore \quad 5 \times 1.2 \times (2\pi n)^2 = 300$

$\therefore \quad 5 \times 1.2 \times 4\pi^2 n^2 = 300$

$$\therefore \quad n^2_{max} = \frac{300}{4 \times (3.142)^2 \times 60} = 1.26618$$

$$\therefore \quad n_{max} = \sqrt{1.26618} = 1.125\,\text{rev/s}$$

$\therefore \quad n_{max} = 67.5$ rev/min

**Ans:** The maximum number of revolutions per minute made by the mass is **67.5 rev/min.**

*Example 18*

**A cyclist speeding at 18 km/h on a level road takes a sharp circular turn of radius 3 m without reducing the speed. The co-efficient of static friction between the tyres and the road is 0.1. Will the cyclist slip while taking the turn ? (NCERT)**

Solution:

The condition for the cyclist not to slip is given by,

$v^2 < \mu_s rg$ ..... (i)

Now, r = 3 m, g = 9.8 $ms^{-2}$, $\mu_s = 0.1$,

$$v = 18\ \text{km/h} = 18 \times \frac{2}{18} = 5\ \text{m/s}$$

From equation (i),

$(5)^2 \leq 0.1 \times 3 \times 9.8$

$\therefore \quad 25 \leq 2.94$

The condition is not obeyed.

$\therefore$ The cyclist will slip while taking the circular turn.

*Example 19*

**A stone of mass 0.25 kg tied to the end of a string is whirled in a circle of radius 1.5 m with a speed of 40 revolutions/min in a horizontal plane. What is the tension in the string? What is the maximum speed with which the stone can be whirled around if the string can withstand a maximum tension of 200 N? (NCERT)**

*Solution:*

*Given:* m = 0.25 kg, r = 1.5 m, $T_{max}$ = 200 N

$$n = 40\ \text{rev. min}^{-1} = \frac{40}{60}\ \text{rev s}^{-1}$$

*To find:* i. Tension (T)
ii. Maximum speed ($v_{max}$)

*Formulae:* i. $T = mr\omega^2$

ii. $$T_{max} = \frac{mv^2_{max}}{r}$$

*Calculation:* Since, $\omega = 2\pi n = \frac{2\pi \times 40}{60}$

$\omega = 1.33\,\pi$ rad $s^{-1}$

From formula (i),

$T = 0.25 \times 1.5 \times (1.33\pi)^2$

**T = 6.55** N

From formula (ii),

$$v_{max}^2 = \frac{T_{max} r}{m}$$

$$v_{max}^2 = \frac{200 \times 1.5}{0.25}$$

$$v_{max} = \sqrt{1200}$$

Vmax =**34.64 ms**$^{-1}$

**Ans:** i. The tension in the string is **6.55** N

ii. The maximum speed with which the stone can be whirled around is **34.64 ms$^{-1}$.**

*Example 20*

**The radius of curvature of meter gauge railway line at a place where the train is moving with a speed of 10 m/s is 50 m. If there is no side thrust on the rails, find the elevation of the outer rail above the inner rail.**

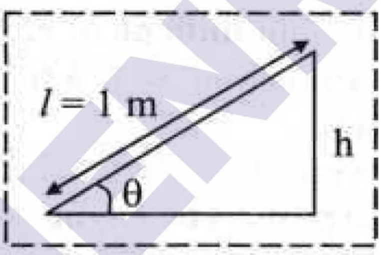
*Solution:*

*Given:* Radius of curve, r = 50 m,

Speed of train, v = 10 m/s

Formulae: i. $\tan\theta = \frac{v^2}{rg}$ ii. $h = l \sin\theta$

Calculation :




From formula (i),

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

$$= \tan^{-1}\left(\frac{100}{50 \times 9.8}\right) = \tan^{-1}(0.2041)$$

$\therefore$ $\theta = 11^0 32'$

From formula (ii),

$h = l \sin\theta$

$h = 1 \times \sin(11^0 32')$

$= 1 \times (0.2000) = 0.2$ m

**h = 20 cm**

**Ans:** The elevation of the outer rail above the inner rail is **20 cm.**

* Example 21

**A circular race course track has a radius of 500 m and is banked to 10°. If the coefficient of friction between tyres of vehicle and the road surface is 0.25. Compute.**

**i. the maximum speed to avoid slipping.**

**ii. the optimum speed to avoid wear and tear of tyres, (g = 9.8 m/s²)**

Solution:

***Given:*** r = 500 m, $\theta = 10°$, $\mu = 0.25$

**To Find :**

**i. Maximum speed to avoid slipping ($v_{max}$)**

**ii. Optimum speed to avoid wear and tear of tyres ($v_0$)**

**Formulae : i.** $v_{max} = \sqrt{rg\left[\frac{\mu_s + \tan\theta}{1 - \mu_s \tan\theta}\right]}$

**ii.** $v_0 = \sqrt{rg\tan\theta}$

**Calculation : i. From fromula (i),**

$$v_{max} = \sqrt{500 \times 9.8\left[\frac{0.25 + \tan 10^0}{1 - 0.25 \times \tan 10^0}\right]}$$

\ $v_{max}$ = **46.72 m/s**

ii. From formula (i),

$$v_0 = \sqrt{500 ´ 9.8 ´ \tan 10^0}$$

$$= \sqrt{500 ´ 9.8 ´ 1.76}$$

\ $v_0$ = 29.37 m/s

**Ans:**

i. The maximum speed to avoid slipping is **46.72 m/s.**

ii. The optimum speed to avoid wear and tear of tyres is **29.37 m/s.**

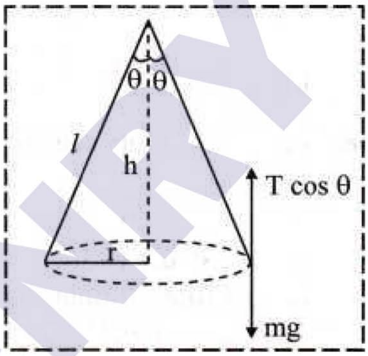
*Example 22*

**A conical pendulum has length 50 cm. Its bob of mass 100 g performs uniform circular motion in horizontal plane, so as to have radius of path 30 cm. Find**

i. the angle made by the string with vertical

ii. the tension in the supporting thread and

iii. the speed of bob.




*Given:* $l$ = 50 cm = 0.5 m,

r = 30 cm = 0.3 m,

m = 100 g = 100 × $10^{-3}$ kg = 0.1 kg

*To find:* i. Angle made by the string with vertical (q)

ii. Tension in the supporting thread (T)

iii. Speed of bob (v)

*Formulae:* i. $\tan q = \frac{r}{h}$

ii. $\tan q = \frac{v^2}{rg}$

*Calculation:* **By Pythagoras theorem,**

$l^2 = r^2 + h^2$

$h^2 = l^2 - r^2$

$h^2$ = 0.25 – 0.09 = 0.16

\ h = 0.4 m

i. From formula (i),

$$\tan q = \frac{0.3}{0.4} = 0.75$$

q = $\tan^{-1}$ (0.75)

**q** = 36°52'

ii. The weight of bob is balanced by vertical component of tension T T cosq = mg

\ T cos q = mg

$$\cos q = \frac{h}{l} = \frac{0.4}{0.5} = 0.8$$

\ $T = \frac{mg}{\cos q} = \frac{0.1 ´ 9.8}{0.8}$

\ T = 1.225 N

iii. From fromula (ii),

$v^2$ = rg tan q

\ $v^2$ = 0.3 × 9.8 × 0.75 = 2.205

v = **1.485 m/s**

**Ans:**

i. Angle made by the string with vertical is **36°52'.**

ii. Tension iii the supporting thread is **1.225 N**.

iii. Speed of the bob is **1.485 m/s.**

*Example 23*

**A motorcyclist rounds a curve of radius 25 m at the speed of 36 km/hr. The combined mass of motorcycle and motorcyclist is 150 kg.**

**(g = 9.8 m/s²)**

**a. What is centripetal force exerted on the motorcyclist ?**

**b. What is upward force exerted on the motorcyclist ?**

**c. What angle the motorcycle makes**

**with vertical ?**
**|Feb 2013 old course]**

*Solution:*

*Given:* $r = 25$ m, $v = 36 \times \frac{5}{18} = 10$ m/s

$m = 150$ kg

*To find:* i. Centripetal force (F)
ii. Upward force (N)
iii. Angle motorcycle makes with .,
vertical ($\theta$) ,

Formulae : i. $F = \frac{mv^2}{r}$

ii. $N \cos \theta = mg$

iii. $\tan \theta = \frac{v^2}{rg}$

*Calculation :* From formula (i),

$$F = \frac{150 \times (10)^2}{25}$$

F = **600 N**

From formula (ii),

N = 9.8 × 150 = **1470 N**

From fromula (iii),

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan^{-1}\left(\frac{10^2}{25 \times 9.8}\right)$$

$\theta = 22^0 12'$

**Ans:**

a. Centripetal force exerted on the motorcyclist is 600 N.

b. Upward force exerted on the motorcyclist is 1470 N.

c. Angle the motorcycle makes with vertical is 22°12'.

***Example 24***

**Propeller blades in aeroplane are 2 m long**

**i. When propeller is rotating at 1800 rev/min, compute the tangential velocity of tip of the blade.**

**ii. What is the tangential velocity at a point on blade midway between tip and axis?**

***Solution :***

***Given :*** $l = 2$m, $n = 1800$ r.p.m.

$= \frac{1800}{60} = 30$r.p.s,

$r_1 = l = 2$m, $r_2 = l/2 = 1$m

To find :

i. Tangential velocity of tip of the blade ($v_{T_1}$)

ii. Tanential velocity at a point on blade midway between tip and axis ($v_{T_2}$).

Formula : $v = 2\pi nr$

Calculation : i From formula, Tangential velocity of the tip of blade,

$v_{T_2} = 2\pi nr_1 = 2 \times 3.14 \times 30 \times 1$

$\therefore$ $v_{T_2} = 188.4$ m/s

**Ans:**

i. The tangential velocity of tip of the blade is **376.8 m/s.**

ii. The tangential velocity at a point on blade midway between tip and axis is **188.4 m/s.**

***Example 25***

**A car of mass 2000 kg moves round a curve of radius 250 m at 90 km/hr. Compute its**

**i. Angular speed.**

**ii. Centripetal acceleration.**

**iii. Centripetal force**

***Solution :***

Given : $m = 2000$ kg, $r = 250$ m,

$v = 90 \text{ km/h} = 90 \times \frac{5}{18} = 25 \text{ m/s}$

*To find :* i. Angular speed ($\omega$)
ii. Centripetal acceleration ($a_{CP}$)
iii. Centripetal force ($F_{CP}$)

*Formulae :* i. $w - \frac{v}{r}$ ii. $a_{CP} - w^2 r$

iii. $F_{CP} = \frac{mv^2}{r}$

Calculation : i. From formula (i),

$$w = \frac{25}{250}$$

\ $w = 0.1$ rad/s

ii. From formula (ii),

$a_{CP} = (0.1)^2 \times 250$

\ $\mathbf{a_{CP} = 2.5\ m/s^2}$

*Example 26*

**A car of mass. 1500 kg rounds a curve of radius 250m at 90 Km/hour. Calculate the centripetal force acting on it. [Feb 2013]**

*Solution:*

*Given:* **m** = 1500 kg, r = 250 m,

$v = 90$ km/h $= 90 \times \frac{5}{18} = 25$ m/s 18

**To find :** **Centripetal force ($F_{cp}$)**

**Formula :** $F_{CP} = \frac{mv^2}{r}$

*Calculation:* From formula,

$$F_{CP} = \frac{1500 ´ (25)^2}{250}$$

\ $F_{CP} = \mathbf{3750}$ N

**Ans:** The centripetal force acting on the car is **3750 N**

* *Example 27*

**A racing car completes 5 rounds of circular track in 2 minutes. Find the radius of the track if the car has uniform centripetal acceleration of $p^2$ m/s².**

Solution:

***Given:*** **5 rounds = 2pr(5), t = 2 minutes = 120 s**

***To find :*** **Radius (R)**

***Formulae :*** $\mathbf{a_{cp} = w^2 r}$

***Calulation:*** **From formula,**

$$\mathbf{a_{cp} = w^2 r}$$

\ $p^2 = \frac{v^2}{r}$

**But** $v = \frac{2pr(5)}{t} = \frac{10pr}{t}$

\ $p^2 = \frac{100p^2 r^2}{rt^2}$

\ $r = \frac{120 ´ 120}{100} = 144\,m$

Ans: **The radius of the track is** 144m**.**

* *Example 28*

**A bucket containing water is whirled in a vertical circle at arms length. Find the minimum speed at top to ensure that no water spills out. Also find corresponding angular speed.**

**[Assume r = 0.75 m]**

Solution:

*Given:* $r = 0.75$ m, $g = 9.8$ m/s²

*To find:* i. Minimum speed ($v_H$)

ii. Angular speed ($w_H$)

***Formulae:*** **i.** $v_H = \sqrt{rg}$

ii. $w_H = \frac{v_H}{r}$

*Calculation:* From formula (i),

$v_H = \sqrt{0.75 ´ 98}$

$v_H = \mathbf{2.711\ m/s}$

From formula (ii),

$$w_H = \frac{2.711}{0.75}$$

\ $w_H = \mathbf{3.615\ rad/s}$

**Ans:** i. For no water to spill out, the minimum speed at top should be **2.711 m/s.**

ii. The angular speed of the bucket is **3.615 rad/s.**

**Example 29*

**A motor cyclist at a speed of 5 m/s is describing a circle of radius 25 m. Find his inclination with vertical. What is the value of coefficient of friction between tyre and ground?**

*Solution:*

*Given:* $v_{,}= 5$ m/s, r = 25 m, g = 9.8 m/s$^2$

*To find:* i. Inclination with vertical ($\theta$)

ii. Coefficient of friction ($\mu$)

*Formulae:* i. $\tan\theta = \frac{v^2}{rg}$ ii. $\frac{v^2}{r} = \mu g$

*Calculation:* From formula (i),

$$\tan\theta = \frac{(5)^2}{25 \times 9.8} = \frac{1}{9.8} = 0.1021$$

$\theta = \tan^{-1}(0.1021) = 5°50'$

From formula (ii),

$$\mu = \frac{v^2}{rg} = 0.1021$$

**Ans:**

i. The inclination of the motor cyclist with vertical is **5°50'.**

ii. The value of coefficient of friction between tyre and ground **0.1021.**

*Example 30

A **motor van weighing** 4400 **kg rounds a level curve of radius** 200 **m on an unbanked road at 60 km/hr. What should be minimum value of coefficient of friction to prevent skidding? At what angle the road should be banked for this velocity?**

Solution:

**Given : m = 4400 kg, r = 200 m,**

$$v = 60 \text{ km/hr} = 60 \times \frac{5}{18} = \frac{50}{3} \text{ m/s},$$

**g = 9.8 m/s$^2$**

**To find : i. Coefficient of friction ($\mu$)**

**ii. Angle of banking ($\theta$)**

**Formulae:** **i. . $v = \sqrt{\mu r g}$**

ii. $\tan\theta = \frac{v^2}{rg}$

*Calculation:* From formula (i),

$$\mu = \frac{v^2}{rg} = \frac{(50/3)^2}{200 \times 9.8} = \frac{25}{18 \times 9.8}$$

$\therefore$ $\mu = 0.1417$

From formula (ii),

$\tan\theta = 0.1417$

$\therefore$ $\theta = \tan^{-1}(0.1417)$

$\therefore$ $\theta = 8°\ 4'$

**Ans:**

i. The minimum value of coefficient of friction to prevent skidding 0.1417.

ii. The angle at which the road should be banked is 8° 4'.

*Example 31

**A stone weighing** 1 **kg is whirled in a vertical circle at the end of a rope of length** 0.5 **m.**

**Find the tension at**

**i. lowest position**

**ii. mid position**

**iii. highest position**

***Solution:***

***Given :*** **m = 1 kg, r = 1 = 0.5 m, g = 9.8 m/s$^2$**

***To find :*** **i. Tension at lowest position ($T_L$)**

ii. Tension at mid position ($T_M$)

iii. Tension at highest position ($T_H$)

**Formulae:** **i.** $T_L = \frac{mv_L^2}{r} + mg$

**ii.** $T_M = \frac{mv_M^2}{r}$

**iii.** $T_H = \frac{mv_H^2}{r} mg$

*Calculation:* Since, $v_L{}^2 = 5rg$

From formula (i),

$$T_L = m\left[\frac{5rg}{r} + g\right] = 6mg$$

$= 6 \times 1 \times 9.8 = 58.8N$

$T_1 =$ **58.8 N**

Since, $v_M{}^2 = 3rg$

From formula (ii),

$$T_M = m\left(\frac{3rg}{r}\right) = 3mg$$

$= 3 \times 1 \times 9.8 = 29.4$ N

$\therefore$ **$T_M$= 29.4 N**

Since, $v_H{}^2 = rg$

From formula (iii),

$$T_H = m\left[\frac{rg}{r} - g\right] = 0$$

$\therefore$ $T_H = 0$

Ans:

i. The tension at lowest position in the vertical circle is **58.8 N.**

ii. The tension at mid position in the vertical circle is 29.4 N.

iii. The tension at highest position in the vertical circle is 0.

* Example 32

**An object (stone) of mass 0.5 kg attached to a rod of length 0.5 m is whirled in a vertical circle at constant angular «peed. If the maximum tension in the string is 5 kg wt. Calculate**

**i. speed of the stone**

**ii. maximum number of revolutions it can complete in a minute.**

Solution:

*Given:* m = 0.5 kg,

r = *l* = 0.5 m,

$T_{max}$ = 5 kg wt. = 5 × 9.8 N

*To find:* i. Speed (v)

ii. Maximum number of revolutions ($n_{max}$)

**Formulae:** **i.** $T_{max} = \frac{mv^2}{r} + mg$ **i i .**

$$n = \frac{v}{2\pi r}$$

**Calculation: i. From formula (i),**

$$v^2 = \frac{r}{m}(T - mg)$$

$$\therefore \quad = 0.5\left(\frac{5 \times 9.8}{0.5} - 9.8\right)$$

**= 49 – 4.9 = 44.1**

**v =** $\sqrt{44.1}$ **= 6.64** m/s

ii. From formula (ii),

$$n_{max} = \frac{v}{2\pi r} \qquad [\because v = r\omega]$$

$$= \frac{6.64}{2 \times 3.14 \times 0.5}$$

= 2.115 r.p.s

$\therefore$ $n_{max}$ = 2.115 × 60

= **126.9 r.p.m**

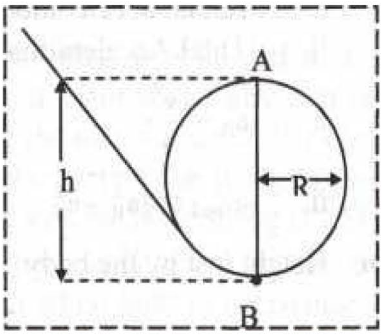
**Ans:**

i. The speed of the stone is **6.64 m/s.**

ii. The maximum number of revolutions the stone can complete in a minute is **126.9 r.p.m.**

* *Example 33*

**A ball is released from height h along the slope and moves along a circular track of radius R without falling vertically downwards as shown in the figure. Show that h = $\frac{5}{2}$ R.**




*Solution:*

The total energy of any body revolving in a vertical circle $= \frac{5}{2}$ mgR

When a ball is released from a height 'h' along the slope and moves along a circular track of radius R without falling vertically downwards, its potential energy (mgh) gets converted into kinetic energy.

$$\left(\frac{1}{2}mv^2\right) = \frac{5}{2}mgR$$

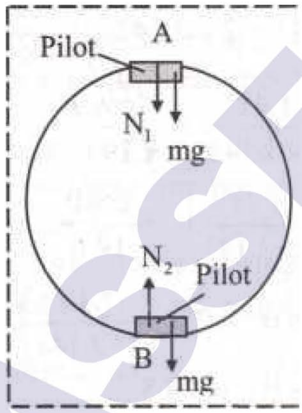
Hence according to law of conservation of energy, $mgh = \frac{5}{2}mgR$

$$\therefore \quad gh = \frac{5}{2}gR$$

$$\therefore \quad h = \frac{5}{2}R$$

* *Example 34*

**A pilot of mass 50 kg in a jet aircraft is executing a loop-the-loop with constant speed of 250 m/s. If the radius of circle is 5 km, compute the force exerted by seat on the pilot at the top of loop. ii. at the bottom of loop**




***Solution :***

*Given :* m = 50 kg, v = 250 m/s,
r = 5 km = $5 \times 10^3$m

i. Force at the top of loop ($F_{top}$)

ii. Force at the bottom of loop ($F_{bottom}$)

*Formulae:* i. $F_{top} = \frac{mv^2}{r} - mg$

ii. $F_{bottom} = \frac{mv^2}{r} + mg$

***Calculation:*** **i. From formula (i),**

$$F_{top} = \frac{50 \times (250)^2}{5 \times 10^3} - 50 \times 9.8$$

$= 625 + 490$ $F_{top} = 135$ N

ii. From formula (ii),

$$F_{bottom} = \frac{50 \times (250)^2}{5 \times 10^3} + 50 \times 9.8$$

$= 625 + 490$

$F_{bottom} = 1115$ N

**Ans:**

i. The force exerted by seat on the pilot at the top of loop is **135 N**.

ii. The force exerted by seat on the pilot at the bottom of loop is **1115 N.**

*Example 35*

**A string of length 0.5 m carries a bob of mass 0.1 kg at its end. It is used as a conical pendulum with a period 1.41 sec. Calculate angle of inclination of string with vertical and tension in the string.**

***Solution:***

*Given:* $l = 0.5$ m, m = 0.1 kg, T = 1.41 sec

*To find:* i. Angle of inclination ($\theta$)

ii. Tension in the string (T)

*Formulae:* i. $T = 2\pi\sqrt{\dfrac{l\cos\theta}{g}}$

ii. Tension, $T' = \dfrac{mg}{\cos\theta}$

*Calculation:* From formula (i),

$$1.41 = 2 \times 3.142\sqrt{\frac{0.5 \times \cos\theta}{9.8}}$$

$$\therefore \quad \frac{1.41}{2 \times 3.142} = \sqrt{\frac{\cos\theta}{19.6}}$$

$$\therefore \quad \left(\frac{1.41}{2 \times 3.142}\right)^2 = \frac{\cos\theta}{19.6}$$

$$\therefore \quad \cos\theta = 19.6\left(\frac{1.41}{2 \times 3.142}\right)^2$$

From formula (ii),

$$T' = \frac{0.1 \times 9.8}{\cos 9^{\circ}19'} = \frac{0.98}{0.9868}$$

$T' = 0.993$ N

**Ans:** i. The angle of inclination of string with vertical is **9°19'.**

ii. The tension in the string is **0.993 N.**

*Example 36*

**A 70 kg man stands in contact against the inner wall of a hollow cylindrical drum of radius 3 m rotating about its vertical axis with 200 rev/min. The coefficient of friction between the wall and his clothing is 0.157 What is the minimum rotational speed of the cylinder to enable the man to remain stuck to the wall (without falling) when the floor is suddenly removed? (NCERT)**

*Solution:*

The horizontal force N of the wall on the man provides the necessary centripetal force.

$$N = \frac{mv^2}{r} = mr\omega^2$$

The frictional force ' f' acting upwards balances the weight 'mg' of the man.

i. e. $f \le \mu N$ or $mg \le \mu mr\omega^2$

$$\frac{g}{\mu r} \le \omega^2 \quad \text{or} \quad \omega^2 \ge \frac{g}{\mu r}$$

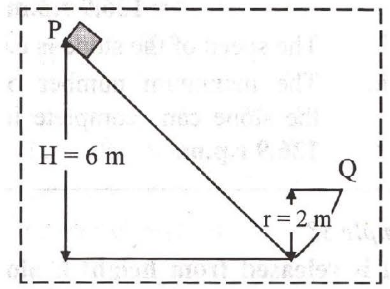
So, the minimum angular velocity of rotation of the drum is given by,

$$\omega_{min} = \sqrt{\frac{g}{\mu r}} = \sqrt{\frac{9.8}{0.15 \times 3}}$$

$\omega_{min} =$ **4.667 rad s$^{-1}$**

* *Example 37*

**A block of mass 1 kg is released from P on a frictionless track which ends in quarter circular track of radius 2 m at the bottom as shown in the figure. What is the magnitude of radial acceleration and total acceleration of the block when it arrives at Q ?**




*Solution:*

H = 6 m, r = 2m,

Given: H = 6 m, r = 2 m,

*To find:* i. Radial acceleration ($a_R$)

ii. Total Acceleration ($a_{Total}$)

***Formulae:* i.** $a_R = \frac{v^2}{r}$

**ii.** $a_{Total} = \sqrt{a_R^2 + a_T^2}$

*Calculation:* Height lost by the body = 6 – 2

= 4m

From equation of motion,

$v^2 = u^2 + 2gh$

$v^2 = 0 + 2 \times 9.8 \times 4 = 78.4$

From formula (i),

$a_R = \frac{78.4}{2}$

$a_R = 39.2$ **m/s**$^2$

$a_T = g = 9.8$ m/s$^2$

From formula (ii),

$a_{Total} = \sqrt{(39.2)^2 + (9.8)^2}$

$= \sqrt{1536.64 + 96.04}$

$= \sqrt{1632.68}$

$a_{Total}$ = 40.4 **m/s**$^2$

**Ans:**

i. The magnitude of radial acceleration of the block is **39.2 m/s**$^2$

ii. The total acceleration of the block is **40.4 m/s**$^2$**.**

## EXERCISE 3

**[Secetion : A - PracticeProblems]**

1. Calculate the angular speed of minute hand of a clock of length 2 cm.
2. Determine the angular acceleration of a rotating body which slows down from 500 r.p.m to rest in 10 seconds.
3. The minute hand of a clock is 5 cm long. Calculate the linear speed of an ant sitting at the tip.
4. Calculate the angular speed and linear speed of tip of a second hand of clock if second hand is 5 cm long.
5. A 0.5 kg mass is rotated in a horizontal circle of radius 20 cm. Calculate the centripetal force acting on it, if its angular speed of rotation is 0.6 rad/s.
6. A body of mass 1 kg is tied to a string and revolved in a horizontal circle of radius 1 m. Calculate the maximum number of revolutions per minute, so that the string does not break. Breaking tension of the string is 9.86 N.
7. A coin just remains on a disc rotating at 120 r.p.m when kept at a distance of 1.5 cm from the axis of rotation. Find the coefficient of friction between the coin and the disc.
8. A coin kept on a horizontal rotating disc has its centre at a distance of 0.25 m from the axis of rotation of the disc. If m = 0.2, find the angular velocity of the disc at which the coin is about to slip off. [g = 9.8 m/s$^2$]
9. With what maximum speed a car be safely driven along a curve of radius 40 m on a horizontal road, if the coefficient of friction between the car tyres and road surface is 0.3? [g = 9.8 m/s$^2$]
10. A vehicle is moving along a circular road which is inclined to the horizontal at 10°. The maximum velocity with which it can move safely is 36 km/hr. Calculate the radius of the circular road.
11. Calculate angle of banking for circular track of radius 50 m as to be suitable for driving a car with maximum speed of 72 km/hr.
12. At what angular speed should the earth rotate

about its axis so that apparent weight of a body on the equator will be zero? What would be the length of the day at that periodic time? [Radius of earth = 6400 km, g = 9.8 $m/s^2$]

13. A body of mass 200 gram performs circular motion of radius 50 cm at a constant speed of 240 r.p.m. Find its linear speed.

14. A body of mass 2 kg is tied to a string 1.5 m long and revolved in a horizontal circle about the other end. If it performs 300 r.p.m, calculate its linear velocity, centripetal acceleration and force acting on it.

15. A string breaks under a tension of 10 kg-wt. If the string is used to revolve a body of mass 12 gm in a horizontal circle of radius 50 cm, what is the frequency of revolution and linear speed with which the body can be revolved ? [g = 9.8 $m/s^2$]

16. The breaking tension of a string is 80 kg-wt. A mass of 1 kg is attached to the string and rotated in a horizontal circle on a horizontal surface of radius 2 m. Find the maximum number of revolutions made without breaking, [g = 9.8 $m/s^2$]

17. Find the angle of banking of a railway track of radius of curvature 250 m, if the optimum velocity of the train is 90 km/hr. Also find the elevation of the outer track over the inner track if the two tracks are 1.6 m apart.

18. The distance between two rails of rail track i s 1.6 m along a curve of radius 800 m. The outer rail is raised about the inner rail by 10 cm. With what maximum speed can a train be safely driven along the curve?

19. A particle, initially at rest, performs circular motion with uniform angular acceleration 0. 18 $rad/s^2$. What speed, in r.p.m will it attain in a time of 10 seconds? What is angular displacement in this time?

20. Determine the force that presses the pilot against his seat at the upper and lower points of a loop, if the weight of the pilot is 75 kgf., the radius of the loop is 200 m and the velocity of the plane looping the loop is constant and equal to 360 km $h^{-1}$.

[g = 10 $m/s^2$]

21. A 50 g mass is attached to a string and. rotated in a vertical circle of radius 1.8 m. What is the minimum speed the mass must have at the top of the circle in order that the string may not slacken? What will be the velocity of mass and the tension in the bottom of the circle under the above conditions?

[g = 9.8 $m/s^2$ ]

22. A body of mass 1 kg tied to string is whirled in a vertical circle.of radius 1 m. Find velocity and tension in the string.

i. at the top of.the circle r

ii. at the bottom of thc circle and

iii. at a point level with the centre.

Assume that the mass just goes around the circle at the top with minimum speed without the string slackening.

**[Section B : Theoretical Board Questions]**

1. Define angle of banking. Draw a neat labelled diagram showing different forces and their components acting on a vehicle fnovirig on a banked road. [Oct 97]

2. Obtain an expression for optimum speed of a car on a banked road. [Oct 99]

3. Explain

   i. Centripetal force

   ii. Centrifugal force. [ M a r 00]

4. What is banking of road ? Obtain an expression for angle of banking. On what factors does it depend?

   [Oct 00, 02,04, Feb 06]

5. Derive the expression for the maximum speed of a vehicle on the banked road. Slate the factors on which the optimum speed depends.
[Feb 01]

6. Derive an expression for radial acceleration of a particle performing uniform circular motion.
Why is it so called ?
[Feb 04 Oct 05]

7. What is centripetal and centrifugal force ?
[Feb 04]

8. A certain body remains stationary on the vertical inner wall of a cylindrical drum of radius 'r', rotating at a constant speed. Show that the minimum angular speed of the drum is

$\sqrt{\frac{g}{mr}}$ where "m" is coefficient of friction between the body and surface of the wall. [Oct 06]

9. For a conical pendulum prove that $\tan q = \frac{v^2}{rg}$
[Oct 09]

10. Obtain an expression for maximum speed with which a vehicle can be driven safely on a banked road. Show that the safety speed limit is independent of the mass of the vehicle. [**Mar 10, Oct 10**]

11. Derive an expression for linear velocity at ; lowest point and at highest point for a particle revolving in vertical circular motion. [**Oct 11**]

**[ Section C: Numerical Board Problems ]**

1. A. train rounds a curve of radius 150 m at a speed of 20 m/s. Calculate the angle of banking so that there is no side thrust on the rails. Also find the elevation of the outer rail over the inner rail, if the distance between the rails is 1 m. **[Oct 96]**

2. An object of mass 400 g is whirled in a x horizontal circle of radius 2 m. If it performs 60 r.p.m, calculate the centripetal force acting on it. **[ Oct 96, Feb 01]**

3. Find the angle which the bicycle and its rider will make with the vertical when going round a curve at 27 km/hr on a horizontal curved road of radius 10 m. [g = **9.8** $m/s^2$]
**[Mar 98]**

4. Find the angle of banking of curved railway track of radius 600 m, if the maximum safety speed limit is 54 km/hr. If the distance between the rails is 1.6 m. find the elevation of the outer track above the inner track, [g = **9.8** $m/s^2$]
**[Oct 98|**

5. The vertical section of a road over a bridge in the direction of its length is in the form of an arc of a circle of radius 4.4 m. Find the greatest velocity at Which a vehicle can cross the bridge without losing contact with the road at the highest point, if the center of the vchicle is 0.5 m from the ground.
[Given: g = **9.8** $m/s^2$]
**[Oct 01]**

6. If the frequency of revolution of an object changes; from 2 Hz to 4 Hz in 2 second, calculate its angular acceleration.

[Oct 03]

7. The minute hand of a clock is 8 cm long.
   Calculate the linear speed of an ant sitting at its tip.
   [Mar 05]

8. The frequency of a spinning top is 10 Hz. If it is brought to rest in 6.28 sec, find the angular acceleration of a particle on its surface.
   [Oct 05]

9. Calculate the angle of banking for a circular track of radius 600 m as to be suitable for driving a car with maximum speed of 180 km/hr. [$g = 9.8$ m/s$^2$]
   **[Feb 06]**

10. A vehicle is moving along a curve of radius 200 m. What should be the maximum speed with which it can be safely driven if the angle of banking is 17°? (Neglect friction)
    **[g = 9.8 m/s$^2$] [Mar 07]**

11. An object of mass 1 kg is tied to one end of a string of length 9 m and Whirled in a verticle circle. What is the minimum speed required at the lowest position to complete a circle ? [Oct 08]

12. An object of mass 2 kg attached to wire of length 5 m is revolved in a horizontal circle. If it makes 60 r.p.m. Find its
    i. angular speed
    ii. linear speed
    iii. centripetal acceleration
    iv. centripetal force **[Mar 09]**

13. A stone of mass one kilogram is tied to the end of a string of length 5 m and whirled in a verticle circle. What will be the minimum speed required at the lowest position to complete the circle?
    [Given: g - **9.8** m/s$^2$] **[Oct 10]**

14. In a conical pendulum, a string of length 120 cm is fixed at rigid support and carries a mass of 150 g at its free end. If the mass is revolved in a horizontal circle of radius 0.2 m around a vertical axis, calculate tension in the string, ($g = 9.8$ m/s$^2$) [Oct 13]

**[Section: D Multiple ChoiceQuestion]**

1. When a particle moves in a circle with a uniform speed
   (A) its velocity and acceleration both are constant.
   (B) its velocity is constant but the acceleration changes.
   (C) its acceleration is constant but the velocity changes.
   (D) its velocity and acceleration both change.

2. A particle is performing a U.C.M along a circle of radius R. In half the period of revolution, its displacement and distance covered are
   (A) R, pR (B) 2R, pR
   (C) 2R, pR (D) $\sqrt{2R}$, 2pR

3. When a body performs a U.C.M it has
   (A) a constant velocity
   (B) a constant acceleration
   (C) an acceleration of constant magnitude but variable direction
   (D) an, acceleration, which changes with time

4. When a body performs a U.C.M
   (A) its velocity remains constant
   (B) Work done oh it is zero
   (C) work done on it is negative
   (D) no force acts on it

5. Angular speed of the second hand of a watch is
   (A) p/60 rad/s (B) p/30 rad/ s
   (C) p rad/s (D) 2p/3 rad/s

6. When a particle moves in a uniform circular motion. It has
   (A) radial velocity & radial acceleration.
   (B) tangential velocity & radial acceleration.
   (C) tangential velocity & tangential acceleration.
   (D) radial velocity & tangential acceleration.

7. For a particle moving along a circular path, the angular velocity vector (w) is directed
(A) along the radius towards the centre.
(B) along the radius but away from the centre.
(C) along the tangent to the circular path.
(D) along the axis of rotation.

8. The ratio of the angular speeds of the hour hand and the minute hand of a clock is
(A) 1 : 12 (B) 1 :6
(C) 1 : 8 (D) 12 : 1

9. A wheel having radius one metre makes 30 revolutions per minute. The linear speed of a palticle on the circumference will be
(A) p/2 m/s (B) p m/s
(C) 30p m/s (D) $6p^2$ m/s

10. A particle starts from rest and moves with an angular acceleration of 3 rad/$s^2$ in a circle of radius 3 m. Its linear speed after 5 seconds will be
(A) 15 m/s (B) 30 m/s
(C) 45 m/s (D) 7.5 m/s

11. Angular speed of a minute hand of a wrist watch in rad/sec is **[Oct 10]**
(A) $\frac{p}{60}$ (B) $\frac{p}{900}$
(C) $\frac{p}{1800}$ (D) $\frac{p}{3600}$

12. To enable a particle to describe a circular path, what should be the angle between its velocity and acceleration?
(A) 0° (B) 90°
(C) 45° (D) 180°

13. A flywheel rotates at a constant speed of 2400 r.p.m The angle in radian described by the shaft in one second is
**(A) 2400 p (B) 80 p**
**(C) 20 p (D) 4800 p**

**14. A body is revolving with a uniform speed v in a circle of radius r. The tangential acceleration is (A) (A) v/r**
**(B) Zero**
**(C) $v^2/r$ (D) $v/r^2$**

15. For keeping a body in uniform circular motion, the force required is
(A) centrifugal (B) radial
(C) tangential (D) centripetal

16. The magnitude of centripetal force cannot be expressed as
(A) $mrw^2$ (B) $\frac{4p^2mr}{T^2}$
(C) mvw (D) mv/w

17. Particle A of mass M is revolving along a circle of radius R. Particle B of mass m is revolving in another circle of radius r. If they take the same time to complete one revolution, then the ratio of their angular velocities is
(A) R/r (B) r/R
(C) 1 (D) $\left(\frac{R}{r}\right)^2$

18. If a cycle wheel of radius 0.4 m completes one revolution in 2 seconds, then acceleration of the cycle is
**[Mar 11]**
(A) 0.4 p m/s (B) 0.4 $p^2$ m/$s^2$
(C) $\frac{p^2}{0.4}$ m/$s^2$ (D) $\frac{0.4}{p^2}$ m/$s^2$

19. A particle is performing circular motion. Its frequency of revolution changes from 120 rpm to 180 rpm in 10 sec. The angular acceleration of the particle is
(A) 1 rad/$s^2$ (B) 0.628 rad/$s^2$
(C) 0.421 rad/$s^2$ (D) 0.129 rad/$s^2$.

20. Which of the following force is a pseudo force?
(A) Force acting on a falling body.
(B) Force acting on a charged particle placed in an electric field.
(C) Force experienced by a person standing on a merry-go- round.
(D) Force which keeps the electrons moving in circular orbits.

**21. In uniform circular motion, the angle**

**between the radius vector and centripetal acceleration is**

**(A) 0°** **(B) 90°**

**(C) 180°** **(D) 45°**

**22. The centripetal force acting on a mass m moving with a uniform velocity v on a circular orbit of radius r will be**

**(A)** $\frac{mv^2}{2r}$ **(B)** $\frac{1}{2}mv^2$

**(C)** $\frac{1}{2}mrv^2$ **(D)** $\frac{mv^2}{r}$

23. A body performing uniform circular motion has [Oct **08**]

(A) constant velocity

(B) constant acceleration

(C) constant kinetic energy

(D) constant displacement

24. Which of the following statements about the centripetal and centrifugal forces is correct?

(A) Centripetal force balances centrifugal force.

(B) Both centripetal force and centrifugal force act in the same frame of reference.

(C) Centripetal force is directed opposite to centrifugal force.

(D) Centripetal force is experienced by the observer at the centre of the circular path described by the body.

25. The linear acceleration of the particle of mass 'm' describing a horizontal circle of radius r, with angular speed 'co' is

(A) w/r (B) rw

(C) $rw^2$ (D) $r^2w$

26. An unbanked curve has a radius of 60 m. The maximum speed at which a car can make a turn, if the coefficient of static friction is 0.75, is

(A) 2.1 m/s (B) 14 m/s

(C) 21 m/s (D) 7 m/s

27. Centrifugal force is

(A) a real force acting along the radius.

(B) a force whose magnitude is less than that of the centripetal force.

(C) a pseudo force acting along the radius and away from the centre.

(D) a force which keeps the body moving along a circular path with uniform speed.

28. A stone is tied to a string and rotated in a horizontal circle with constant angular velocity. If the string is released, the stone flies **[Oct 09, Mar 10]**

(A) radially inward

(B) radially outward

(C) tangentially forward

(D) tangentially backward

29. A particle performs a uniform circular motion in a circle of radius 10 cm. What is its centripetal acceleration if it takes 10 seconds to complete 5 revolutions ?

(A) 2.5 $p^2$ $cm/s^2$ (B) $5p^2$ $cm/s^2$

(C) 10 $p^2$ $cm/s^2$ (D) $20p^2$ $cm/s^2$

30. When a car takes a turn on a horizontal road, the centripetal force is provided by the

(A) weight of the car.

(B) normal reaction of the road.

(C) frictional force between the surface of the road and the tyres of the car.

(D) centrifugal force.

31. On being churned the butter separates out of milk due to

(A) centrifugal force (B) adhesive force

(C) cohesive force (D) frictional force

32. When a particle moves on a circular path then the force that keeps it moving with uniform velocity is

(A) centripetal force (B) atomic force

(C) internal force (D) gravitational force

33. A car is moving along a horizontal curve of radius 20 m and coefficient of friction between the road and wheels of the car is 0.25. If the acceleration due to gravity is 9.8

m/s , then its maximum speed is .
[Mar **08]**

(A) 3 m/s (B) 5 m/s
(C) 7 m/s (D) 9 m/s

34. A particle of mass m is observed from an inertial frame of reference and is found to move in a circle of radius r with a uniform speed v. The centrifugal force on it is

(A) $\frac{mv^2}{r}$ towards the centre

(B) $\frac{mv^2}{r}$ away from the centre

(C) $\frac{mv^2}{r}$ along the tangent through the particle

(D) zero

35. If a cyclist goes round a circular path of circumference 34.3 in $\sqrt{22}$ s, then the angle made by him with the vertical Will be

(A) 42° (B) 43°
(C) 49° (D) 45°

36. A motor cycle is travelling on a curved track of radius 500 m. If the coefficient of friction between the tyres and road is 0.5, then the maximum speed to avoid skidding will be [g = 10 m/s$^2$]

(A) 500 m/s (B) 250 m/s
(C) 50 m/s (D) 10 m/s

37. A coin placed on a rotating turntable just slips if it is placed at a distance of 4 cm from the centre. If the angular velocity of the turntable is doubled, it will just slip at a distance of

(A) 1 cm (B) 2 cm
(C) 4 cm (D) 8 cm

38. Two bodies of mass 10 kg and 5 kg are moving in concentric orbits of radius R and r. If their time periods are same, then the ratio of their centripetal acceleration is

(A) R/r (B) r/R
(C) $R^2/r^2$ (D) $r^2/R^2$

39. A body is moving in a horizontal circle with constant speed. Which one of the following, statements is correct ?

(A) Its P.E is constant.
(B) Its K.E isconstant.
(C) Either P.E or K.E of the body is constant.
(D) Both P.E and K.E of the body are constant.

40. A cyclist bends while taking a turn to

(A) reduce friction
(B) generate required centripetal force
(C) reduce apparent weight
(D) reduce speed

41. A cyclist has to bend inward while taking a turn but a passenger sitting inside a car and taking the same turn is pushed outwards. This is because

(A) the car is heavier than cycle.
(B) centrifugal force acting on both the cyclist and passenger is zero.
(C) the cyclist has to balance the centrifugal force but the passenger cannot balance the centrifugal force hence he is pushed outward.
(D) the speed of the car is more than the speed of the cycle.

42. The minimum velocity (in ms$^{-1}$) with which a car driver must traverse a flat curve of radius 150 m and coefficient of friction 0.6 to avoid skidding is
($g == 10 m/s^2$)

(A) 60 (B) 30
(C) 15 (D) 25

43. Maximum safe speed does not depend on

(A) mass of the vehicle
(B) radius of curvature
(C) angle of inclination (banking)
(D) acceleration due to gravity

44. A motor cyclist moving with a velocity of 72 km per hour on a flat road takes a tom on the road at a point where the radius of curvature of the road is 20 metres. The acceleration due to gravity is

$10\ m/s^2$. In order to avoid skidding, he must not bend with respect to the vertical plane by an angle greater than

(A) $\theta = \tan^{-1}(6)$

(B) $\theta = \tan^{-1}(2)$

(C) $0 = \tan^{-1}(25.92)$

(D) $\theta = \tan^{-1}(4)$

45. A car of mass 1500 kg is moving with a speed of 12.5 m/s on a circular path of radius 20 m on a level road. What should be the coefficient of friction between the car and the road, so that the car does not slip ?

(A) 0.2 (B) 0.4

(C) 0.6 (D) 0.8

46. A particle is moving in a circle of radius r with constant speed v. Its angular acceleration will be (A) vr ( B ) v/r

(C) zero (D) $vr^2$

47. A hollow sphere has radius 6.4 m. Minimum velocity required by a motor cyclist at bottom to complete the circle will be

(A) 17.7 m/s (B) 12.4 m/s

(C) 10.2 m/s (D) 16.0 m/s

48. A curved road having a radius of curvature of 30 m is banked at the correct angle. If the speed of the car is to be doubled, then the radius of curvature of the road should be

**(A) 62 m** **(B) 120m**

**(C) 90 m** **(D) 15m**

**49. The time period of conical pendulum is ...... . [Oct 1]**

**(A)** $\sqrt{\frac{l\cos\theta}{g}}$ **(B)** $2\pi\sqrt{\frac{l\sin\theta}{g}}$

**(C)** $2\pi\sqrt{\frac{l\cos\theta}{g}}$ **(D)** $\sqrt{\frac{l\sin\theta}{g}}$

50. A stone of mass m is tied to a string and is moved in a vertical circle of radius r making n revolutions per minute. The total tension in the string when the stone is at its lowest point is

(A) $m(g + \pi nr^2)$

(B) $m(g + \pi nr^2)$

(C) $m(g + n^2 r^2)$

(D) $m[g + (\pi^2 n^2 r)/900]$

51. A car is moving on a curved path at a speed of 20 km/ hour. If it tries to move on the same path at a speed of 40 km/hr then the chance of toppling will be

(A) half (B) twice

(C) thrice (D) four times

52. Consider a simple pendulum of length 1 m. Its bob performs a circular motion in horizontal plane with its string making an angle 60° with the vertical. The period of rotation of the bob is (Take $g = 10\ m/s^2$)

(A) 2s (B) 1.4 s

(C) 1.98 s (D) none of these

53. The period of a conical pendulum is

(A) equal to that of a simple pendulum of same length $l$

(B) more than that of a simple pendulum of same length $l$.

(C) less than that of a simple pendulum of same length $l$.

(D) independent of length of pendulum

54. When a car crosses a convex bridge, the bridge exerts a force on it. It is given by

(A) $F = mg + \frac{mv^2}{r}$ (B) $F = \frac{mv^2}{r}$

(C) $F = mg - \frac{mv^2}{r}$ (D) $F = mg + \left(\frac{mv^2}{r}\right)^2$

55. Out of the following equations which is WRONG? **[Mar 12]**

(A) $\vec{\tau} = \vec{r} \times \vec{F}$ (B) $\vec{a}_r = \vec{\omega} \times \vec{v}$

(C) $\vec{a}_r = \vec{a} \times \vec{r}$ (D) $\vec{v} = \vec{r} \times \vec{\omega}$

56. A car is moving with a speed of 30 m/s on a

circular path of radius 500 m. Its speed is increasing at the rate Of 2 m/s The acceleration of the car is

(A) 2 m/s (B) 9.8 $m/s^2$

(C) 2.7 $m/s^2$ (D) 1.8 $m/s^2$

57. A ball of mass 250 gram attached to the end of a string of length 1.96 m is moving in a horizontal circle. The string will break if the tension is more than 25 N. What is the maximum speed with which the ball can be moved ?

(A) 5 m/s (B) 7 m/s

(C) 11 m/s (D) 14 m/s

58. A 500 kg car takes a round turn of radius 50 m with a speed of 36 km/hr. The centripetal force acting on the car will be

(A) 1200 N (B) 1000 N

(C) 750 N (D) 250 N

59. Angle of banking does not depend upon

(A) Gravitational acceleration

(B) Mass of the moving vehicle

(C) Radius of curvature of the circular path

(D) Velocity of the vehicle

60. What would be the maximum speed of a car on a road turn of radius 30 m, if the coefficient of friction between the tyres and the road is 0.4 ?

(A) 6.84 m/s (B) 8.84 m/s

(C) 10.84 m/s (D) 4.84 m/s

61. In a conical pendulum, when the bob moves in a horizontal circle of radius r, with uniform speed v, the string of length L describes a cone of semivertical angle q. The tension in the string is given by

(A) $T = \frac{mgL}{(L^2 - r^2)}$ (B) $\frac{(L^2 - r^2)^{1/2}}{mgL}$

(C) $T = \frac{mgL}{\sqrt{L^2 - r^2}}$ (D) $T = \frac{mgL}{(L^2 - r^2)^2}$

62. In a conical pendulum, the centripetal force $\left(\frac{mv^2}{r}\right)$ acting on the bob is given by

(A) $\frac{mgr}{\sqrt{L^2 - r^2}}$ (B) $\frac{mgr}{L^2 - r^2}$

(C) $\frac{(L^2 - r^2)}{mgr}$ (D) $\frac{mgL}{(L^2 - r^2)^{1/2}}$

63. A metal ball tied to a string is rotated in a vertical circle of radius d. For the thread to remain just tightened the minimum velocity at highest point will be

(A) $\sqrt{5gd}$ (B) gd

(C) $\sqrt{3gd}$ (D) $\sqrt{gd}$

64. Which quantity is fixed of an object which moves in a horizontal circle at constant speed?

(A) Velocity (B) Acceleration

(C) Kinetic energy (D) Force

65. A particle of mass 0.1 kg is rotated at the end of a string in a vertical circle of radius 1.0 m at a constant speed of 5 $ms^{-1}$. The tension in the string at the highest point of its path is

(A) 0.5 N (B) 1.0 N

(C) 1.5 N (D) 15 N

66. A stone of mass 1 kg tied to a light inextensible string of length L = (10/3) metre in whirling in a circular path of radius L in a vertical plane. If the ratio of the maximum tension in the string to the minimum tension is 4 and if g is taken to be 10 $m/s^2$, The speed of the stone at the highest point of the circle is

(A) 20 m/s (B) $10\sqrt{3}$ m/s

(C) $5\sqrt{2}$ m/s (D) 10 m/s

67. Water in a bucket is whirled in a vertical circle with a string attached to it. The water does not fall down even when the bucket is inverted at the top of its path. We conclude that in this position.

(A) $mg = mv^2/r$

(B) mg is greater than $mv^2/r$

(C) mg is not greater than $mv^2/r$

(D) mg is not less than $mv^2/r$

68. Let q denote the angular displacement of a simple pendulum oscillating in a vertical plane. If the mass of the bob is m, the tension in the string at extreme position is

(A) mg sin q (B) mg cos q

(C) mg tan q (D) mg

69. Kinetic energy of a body moving in vertical circle is

(A) constant at all points on a circle.

(B) different at different points on a circle.

(C) zero at all the point on a circle.

(D) negative at all the points.

70. A body of mass 1 kg is moving in a vertical circular path of radius 1 m. The difference between the kinetic energies at its highest and lowest position is

(A) 20 J (B) 10 J

(C) $4\sqrt{5}$ J (D) $10(\sqrt{5} - 1)$ J

71. A circular road of radius 1000 m has banking angle 45°. The maximum safe speed of a car having mass 2000 kg will be, if the coefficient of friction between tyre and road is 0.5

(A) 172 m/s (B) 124 m/s

(C) 99 m/s (D) 86 m/s

72. For a particle in circular motion the centripetal acceleration is

(A) less than its tangential acceleration.

(B) equal to its tangential acceleration.

(C) more than its tangential acceleration.

(D) may be more or less than its tangential acceleration.

## ANSWERS

**Section A**

1. $1.74 \times 10^{-3}$ rad/s
2. $-5.237$ rad/s$^2$
3. $8.72 \times 10^{-5}$ m/s
4. $1.07 \times 10^{-1}$ rad/s, $5.235 \times 10^{-3}$ m/s
5. 0.036 N
6. 30
7. 0.2418
8. **2.8 rad/s**
9. **10.84** m/s
10. **57.87 m**
11. $39^0$ 12'
12. **$1.237 \times 10^{-3}$** rad/s, **5080** s
13. **12.57** m/s
14. **47.13** m/s, **1480** m/s$^2$,**2.960**
15. **20.34** rev/s, **63.95** m/s
16. **3.150** rev/s
17. **14°19', 0.3955** m
18. **22.16** m/s
19. **17.18** r.p.m, **1.43** rad
20. **300 kgf, 450 kgf**
21. **42** m/s, **9.39** m/s, **2.94** N
22. i. **3.13** m/s, zero
    ii. **7** m/s, **58.8** Ns
    iii. **5.42** m/s, **29.4** N

**Section C**

1. 15°13', 0.2625 m
2. 31.59N
3. 29°52'
4. 2° 12', 0.061 m
5. 6.429 m/s
6. 6.28 rad/s$^2$
7. $1.396 \times 10^{-2}$ cm/s
8. 10 rad/s$^2$
9. 23 °2'
10. 24.48 m/s
11. 21 m/s
12. i. 6.28 rad/s
    ii. 31.4 m/s
    iii. 197.192 m/s$^2$
    iv. 394.384 N
13. 15.65 m/s
14. 1.47 N

**Section D**

1. **(D)** 2. **(C) 3. (C) 4. (B)**

**5. (B)** 6. **(B) 7. (D)** 8. **(A)**

**9.** **(B)** 10. **(C)** 11. **(C)** 12. **(C)**

**13. (B) 14. (B) 15. (D) 16. (D)**

**17. (C) 18. (B) 19. (B)** 20. **(C)**

21. **(C)** 22. **(D) 23. (C) 24. (C)**

**25. (C) 26. (C) 27. (C) 28. (C)**

**29. (C) 30. (C) 31. (A) 32. (A)**

**33. (C) 34. (D) 35. (D) 36. (C)**

**37. (A) 38. (A) 39. (D) •40. (B)**

**41. (C) 42. (B) 43. (A) 44. (B)**

**45. (D) 46. (C) 47. (A) 48. (B)**

**49. (C) 50. (D) 51. (D) 52. (B)**

**53. (C) 54. (C) 55. (D) 56. (C)**

**57. (D) 58. (B) 59. (B) 60. (C)**

**61. (C) 62. (A) 63. (D) 64. (C)**

**65. (C)** 66. **(D) 67. (C)** 68. **(C)**

**69. (B) 70. (A) 71. (A) 72. (D)**