EXERCISE

18.0 : Introduction

Q.1. Give the main assumptions of Dalton's atomic theory.

Ans: Assumptions of Dalton's atomic theory:

i. All matter is composed of extremely hard and small indivisible particles called atoms.
ii. Atoms of given element are identical and have a characteristic mass.
iii. Atoms of different elements differ in their properties and have different masses and sizes.
iv. Compounds are formed when atoms of different elements combine with each other in simple numerical ratios such as 1 : 1, 1 : 2, 2 : 3, and so on.
v. Atoms cannot be created, destroyed or transformed into atoms of other elements.
vi. The relative number and kind of atoms are always the same in a given compound.

Q.2. Explain J. J. Thomson's model of atom.

Ans: J.J. Thomson's model of atom:

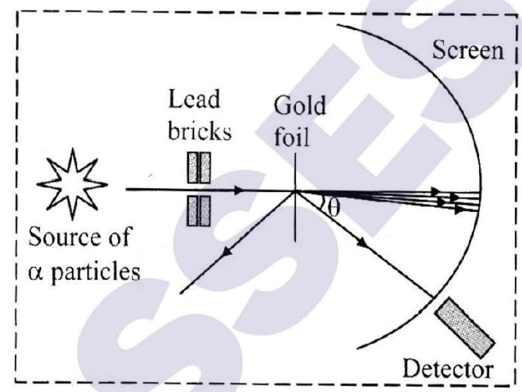
i. J.J. Thomson proposed his atomic model in 1903. According to him, an atom consists of a positively charged sphere with negatively charged electrons embedded in it.
ii. The number of electrons are such that the atom, as a whole, has zero charge.
iii. Thomson could not explain the stable structure of atom and atomic spectra.

18.1 : Geiger-Marsden experiment

Q.3. With the help of neat diagram, describe the Geiger-Marsden experiment.

Ans: Geiger-Marsden experiment:

i. The experimental arrangement is as shown in the figure.
ii. In this experiment, a narrow beam of a-particles from radioactive source was incident on a gold foil.
iii. The scattered a-particles were detected by the detector fixed on rotating stand. Detector used had zinc sulphide screen and microscope.
iv. α -particles produced scintillations on screen which could be observed through microscope.



v. The whole setup is enclosed in an evacuated chamber.

Observation:

i. Most of the a-particles passed undeviated.
ii. Only few a-particles (about 0.14%) were scattered by more than 1^0.
iii. Some α -particles were deflected slightly and very few (1 in 8000) deflected by more than 90°.
iv. Some α -particles were bounced back with $\theta = 180°$.

18.2 : Rutherford's model of atom

Q.4. State the main assumptions of Rutherford's model of atom.

Ans: Assumptions of Rutherford's model of atom:

i. An atom consists of a very small central core called the nucleus.
ii. The nucleus carries all the positive charge and most of the mass of the atom (99.9%).
iii. Ths size of the nucleus is of the order of 10^{-15} m which is very small in comparison to the size of the atom which is of the order of 10^{-10} m. Thus major portion of atom is empty space.
iv. The negatively charged electrons revolve around the nucleus in circular orbits with nucleus at the centre of the orbit. The necessary centripetal force for circular

HENRY CLASSES

motion is provided by the force of attraction between positively charged nucleus and negatively charged electron.

v. Positive charge on nucleus is exactly equal to the total negative charge of all electrons so that atom is electrically neutral.

Q.5. Discuss the demerits of Rutherford's model of atom.

Ans: Demerits of Rutherford's model of atom:

i. The stability of atomic structure and hydrogen line spectrum could not be explained.

ii. His model imitates sun-planets system, however in atom, it is nucleus-electron interactive system where electrons revolve around nucleus in circular orbits.

iii. The circular motion is accelerated motion and according to electromagnetic theory, accelerated charge radiates energy. The energy of accelerated electrons should therefore continuously decrease and follow inward spiral path and finally fall into the nucleus. However, electrons revolve round the nucleus without falling into it. Thus, Rutherford's model of atom could not explain the stability of the atom.

iv. When electrons follow inward spiral path, their angular velocity and hence frequency would increase continuously. Thus they would emit energy with continuously increasing frequency (or continuously decreasing wavelength). That is, atom should emit continuous spectra. But, experimentally observed facts are in contradiction with these expectations. All atoms emit line spectra i.e. spectra of well-defined frequencies or wavelengths.

Q.6. Explain Rutherford model of the structure of atom. Mention any two failures of this model. [Oct 05]

Ans: Assumptions of Rutherford's model of atom:

i. An atom consists of a very small central core called the nucleus.

ii. The nucleus carries all the positive charge and most of the mass of the atom (99.9%).

iii. Ths size of the nucleus is of the order of 10^{-15} m which is very small in comparison to the size of the atom which is of the order of 10^{-10} m. Thus major portion of atom is empty space.

iv. The negatively charged electrons revolve around the nucleus in circular orbits with nucleus at the centre of the orbit. The necessary centripetal force for circular motion is provided by the force of attraction between positively charged nucleus and negatively charged electron.

v. Positive charge on nucleus is exactly equal to the total negative charge of all electrons so that atom is electrically neutral.

Demerits of Rutherford's model of atom:

i. The stability of atomic structure and hydrogen line spectrum could not be explained.

ii. His model imitates sun-planets system, however in atom, it is nucleus-electron interactive system where electrons revolve around nucleus in circular orbits.

iii. The circular motion is accelerated motion and according to electromagnetic theory, accelerated charge radiates energy. The energy of accelerated electrons should therefore continuously decrease and follow inward spiral path and finally fall into the nucleus. However, electrons revolve round the nucleus without falling into it. Thus, Rutherford's model of atom could not explain the stability of the atom.

iv. When electrons follow inward spiral path, their angular velocity and hence frequency would increase continuously. Thus they would emit energy with continuously increasing frequency (or continuously decreasing wavelength). That is, atom should emit continuous spectra. But, experimentally observed facts are in contradiction with these expectations. All atoms emit line spectra i.e. spectra of well-defined frequencies or wavelengths.

18.3 : Bohr model

Q.7. State the postulates of Bohr's theory of hydrogen atom. Write down necessary equations. [Oct 09]

OR

State and explain three postulates of Bohr's theory of hydrogen atom.

[Feb 13 old course]

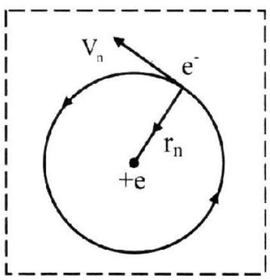
Ans: Bohr's postulate of hydrogen atom:

i. First postulate (Stability condition):

HENRY CLASSES

In a hydrogen atom, the electron revolves round the nucleus in a fixed circular orbit with constant speed.
The centripetal force required for the circular motion is provided by the electrostatic force of attraction between the positively charged nucleus- and negatively charged electron.

Explanation:



Let, r_n = radius of n^{th} orbit of atom
v_n = linear speed of electron in the n^{th} orbit of atom
Force of attraction between electron and nucleus is given by Coulomb's law.

i.e., $F_e = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r^2}$

Centripetal force between nucleus and electrpn is given by, $F_{cp} = \frac{mv^2}{r}$.

From the first postulate,
$F_{cp} = F_e$

$$\therefore \quad \frac{mv_n^2}{r_n} = \frac{e^2}{4\pi\varepsilon_0 r_n^2}$$

where, m = mass of the electron
e = electronic charge
ε_0 = permittivity of free space.

ii. Second postulate (Quantum condition):
The electron can revolve' around the nucleus only in those orbits for which its angular momentum is equal to integral multiple of $\frac{h}{2\pi}$. Those orbits are called as stable or permissible or stationary or quantized or Bohr orbits.
For nth stable orbit, the angular momentum is given by, $L = \frac{nh}{2\pi}$.
where, h = Planck's constant
n = 1, 2, 3, ... principal quantum number
But, $L = I_n \omega_n$

$$\therefore \quad I_n \omega_n = \frac{nh}{2\pi}$$

where, I_n = moment of inertia of electron about the axis of revolution.
ω_n = angular speed of electron

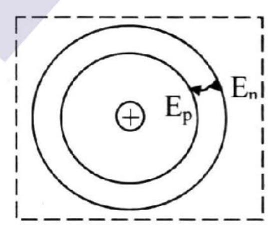
Also, $I_n = mr_n^2$ and $\omega_n = \frac{v_n}{r_n}$

$$\therefore \quad L = mr_n^2 \times \frac{v_n}{r_n} = \frac{nh}{2\pi}$$

$$\therefore \quad mr_n v_n = \frac{nh}{2\pi}$$

iii. Third postulate (Transition and frequency condition):
As long as electron remains in one' of - the stationary orbits, it does not radiate energy. Whenever an electron jumps from higher stationary orbit to lower stationary orbit, it radiates energy equal to the difference in energies of the electron in the two orbits.

Explanation:



Let 'E_n' be the energy of the electron in the higher energy orbit and 'E_p' be the energy of the lower orbit, then radiated energy is given by,
$E_n - E_p = h\nu$
where, ν = frequency of radiation
h = Planck's constant

Note:
Electron does not radiate energy while revolving in circular orbit i.e. energy of electron in stationary orbit remains constant. Hence atomic structure is stable.

Q.8. What is minimum angular momentum of electron in hydrogen atom?

Ans: Minimum angular momentum of electron in hydrogen atom is $\left(\frac{h}{2\pi}\right)$.

HENRY CLASSES

Q.9. If Bohr's quantisation postulate (angular momentum = nh/2 π) is a basic law of nature, it should be equally valid for the case of planetary motion also. Why then do we never speak of quantlsation of orbits of planets around the sun? (NCERT)

Ans: Bohr's quantisation postulate is expressed in terms of Planck's constant (h). But angular momenta associated with planetary motion are approximately of the order of 10^{70} h (for earth). In terms of Bohr's quantisation postulate, this will correspond to n ~ 10^{70}. For such large values of n, the differences in successive energies and angular momenta of the quanti sed levels are so small that the levels may be considered as continuous and not discrete.

Q.10. Which quantity of the orbiting electron has same dimension as that of h?

Ans: Angular momentum of an orbiting electron has the same dimensions as that of h.

Q.11. Obtain an expression for the radius of nth Bohr orbit in hydrogen atom. Hence -show that radius of nth orbit is directly proportional to square of the principal quantum number.

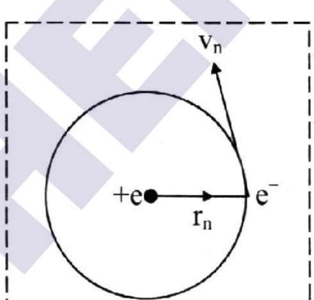
OR

Show that the radius of Bohr orbit is directly proportional to the square of the principle quantum number.

[Mar 11, Oct 11]

Ans: Expression for radius of Bohr orbit in hydrogen atom:

i. Let,
m = mass of electron.
–e = charge on electron
r_n = radius of nthBohr's orbit
+e = charge on nucleus
v_n = linear velocity of electron in nth orbit
n = principal quantum' number



ii. From Bohr's first postulate,
Coulomb's force F_e = Centripetal force F_{cp}

$$\therefore \quad \frac{e^2}{4\pi\varepsilon_0 r_n^2} = \frac{mv_n^2}{r_n}$$

$$\therefore \quad v_n^2 = \frac{e^2}{4\pi\varepsilon_0 r_n m} \qquad \text{....(i)}$$

iii. According to Bohr's second postulate,

$$mr_n v_n = \frac{nh}{2\pi}$$

$$\therefore \quad m^2 v_n^2 r_n^2 = \frac{n^2h^2}{4\pi^2}$$

$$\therefore \quad v_n^2 = \frac{n^2h^2}{4\pi^2 m^2 r_n^2} \qquad \text{....(ii)}$$

iv. From equations (i) and (ii), we have,

$$\frac{n^2h^2}{4\pi^2 m^2 r_n^2} = \frac{e^2}{4\pi\varepsilon_0 r_n m}$$

$$\therefore \quad r_n = \frac{n^2h^2 e_0}{\pi m e^2}$$

$$\therefore \quad r_n = \left(\frac{\varepsilon_0 h^2}{\pi m e^2}\right) n^2 \qquad \text{...(iii)}$$

This is the required expression for radius of nth orbit.

v. In equation (iii), ε_0, π, m, h and e are constants.

$\therefore$ $r_n \propto n^2$

Hence, radius of Bohr orbit is directly' proportional to. the square of the principal quantum number.

Q.12. Classically, an electron can be in any orbit around the nucleus of an atom. Then what determines the typical atomic size? Why is an atom not, say, thousand times bigger than its typical size? The question had greatly puzzled Bohr before he arrived at his famous model of the atom. To stimulate what he might well have done before his discovery, let us play with the basic constants of nature and see if we can get a quantity with the dimensions of length that is roughly equal to known size of an atom (~10^{-10} m).

i. Construct a quantity with the dimensions of length from the fundamental constants e, me and c. Determine its numerical value.

ii. You will find that the length obtained in

HENRY CLASSES

(i) is many orders of magnitude smaller than the atomic dimensions. Further, it involves c. But energies of atoms are mostly in non-relativistic domain where c is not expected to play any role. This is what may have suggested Bohr to discard c and look for something else to get the right atomic size. Now, the Planck's constant h had already made its appearance elsewhere. Bohr's great insight lay in recognising that h, me and e will yield the right atomic size. Construct a quantity with the dimensions of length from h, me and e and confirm that its numerical value has, indeed the correct order of magnitude. (NCERT)

Ans: i. Using fundamental constants, e, fie and c, we construct a quantity which has the dimensions of length.

This quantity is $\left(\dfrac{e^2}{4\pi\varepsilon_0 m_e c^2}\right)$

$$\text{Now, } \frac{e^2}{4\pi\varepsilon_0 m_e c^2} = \frac{\left(1.6\times10^{-19}\right)^2 \times 9\times10^9}{9.1\times10^{-31}\left(\times10^8\right)^2}$$

$$= 2.82 \times 10^{-15}\ \text{m}$$

This is much smaller than the typical atomic size.

ii. However, when we drop c and use h, me and e to construct a quantity, which has dimensions of length, the quantity we obtain is $\dfrac{4\pi\varepsilon_0 (h/2\pi)^2}{m_e e^2}$

$$\text{Now, } \frac{4\pi\varepsilon_0 (h/2\pi)^2}{m_e e^2}$$

$$= \frac{\left(6.6\times10^{-34}/2\pi\right)^2}{9\times10^9 \times (9.1\times10^{-31})(1.6\times10^{-19})}$$

$$= 0.53 \times 10^{-10}\ \text{m}$$

Q.13. Show that linear velocity of electron in Bohr orbit is inversely proportional to principal quantum number. [Oct 02]

Ans: i. From Bohr's first postulate,

$$\frac{mv_n^{\,2}}{r_n} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n^{\,2}}$$

$$\therefore\quad m\,v_n^{\,2}\,r_n = \frac{1}{4\pi\varepsilon_0} \times e^2 \qquad ....(i)$$

ii. From Bohr's second postulate,

$$mv_n r_n = \frac{nh}{2\pi} \qquad ....(ii)$$

iii. Dividing equation (i) by (ii), we get,

$$\frac{mv_n^{\,2}r_n}{mv_n r_n} = \frac{e^2}{4\pi\varepsilon_0}\left(\frac{2\pi}{nh}\right)$$

$$\therefore\quad v_n = \left(\frac{e^2}{2\varepsilon_0 h}\right)\frac{1}{n} \qquad ....(iii)$$

iv. Since $\dfrac{e^2}{2\varepsilon_0 h}$ = constant

$$\therefore\quad v_n \propto \frac{1}{n}$$

Thus, linear velocity of electron in Bohr's orbit is inversely proportional to principal quantum number.

Q.14. Show that angular velocity of electron in Bohr orbit is inversely proportional to cube of principal quantum number.

Ans: i. According to Bohr's first postulate,

$$\frac{mv_n^{\,2}}{r_n} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n^{\,2}}$$

$$\therefore\quad mv_n^2 r_n = \frac{e^2}{4\pi\varepsilon_0} \qquad ...(i)$$

ii. From Bohr's second postulate,

$$mv_n r_n = \frac{hn}{2\pi} \qquad ...(ii)$$

iii. Dividing equation (i) by (ii), we get,

$$v_n = \frac{e^2}{4\pi\varepsilon_0}\cdot\frac{2\pi}{hn} = \left(\frac{e^2}{2\varepsilon_0 h}\right) \times \frac{1}{n}$$

iv. But $v_n = r_n\,\omega$ and $r_n = \dfrac{\varepsilon_0 h^2 n^2}{\pi m e^2}$

$$\therefore\quad \omega = \frac{v_n}{r_n} = \left(\frac{e^2}{2\varepsilon_0 h}\right)\frac{1}{n}\Big/\frac{\varepsilon_0 h^2 n^2}{\pi m e^2}$$

$$\therefore\quad \omega = \frac{e^2}{2\varepsilon_0 hn} \times \frac{\pi m e^2}{\varepsilon_0 h^2 n^2} = \left(\frac{\pi m e^2}{2\varepsilon_0^{\,2} n^3}\right)\cdot\frac{1}{n^3}$$

HENRY CLASSES

$\therefore \quad \dfrac{\pi m e^4}{2\varepsilon_0{}^2 n^3} = \text{constant}$

$\therefore \quad \omega \propto \dfrac{1}{n^3}$

Hence, angular velocity of electron is inversely proportional to cube of principal quantum number.

Q.15. Derive an expression for frequency of revolution of electron in Bohr orbit. Hence show that time period of revolution of electron is directly proportional to cube of principal quantum number.

Ans: Expression for frequency of revolution of electron in Bohr orbit:

i. Let,
v = linear velocity of electron
r = radius of Bohr orbit
ω = angular velocity of electron
T = time period of revolution
v = frequency of revolution

ii. Angular velocity of electron in nth Bohr orbit is given by,

$$\omega_n = \frac{\pi m e^4}{2\varepsilon_0{}^2 h^3 n^3}$$

But, $\omega_n = 2\pi v$

$$\therefore \quad 2\pi v = \frac{\pi m e^4}{2\varepsilon_0{}^2 h^3 n^3}$$

$$\therefore \quad v = \frac{m e^4}{4\varepsilon_0{}^2 h^3 n^3}$$

This is required expression for frequency.
Time period of revolution,

$$T = \frac{1}{v} = \frac{1}{\left(\dfrac{me^4}{4\varepsilon_0{}^2 h^3 n^3}\right)} = \frac{4\varepsilon_0{}^2 h^3 n^3}{me^4}$$

$$\therefore \quad T = \left(\frac{4\varepsilon_0{}^2 h^3}{me^4}\right) n^3$$

$$\because \quad \frac{4\varepsilon_0{}^2 h^3}{me^4} = \text{constant}$$

$\therefore \quad T \propto n^3$

Hence, time period of revolution is directly proportional to cube of principal quantum number.

Q.16. Show that centripetal acceleration of electron revolving in nth Bohr's hydrogen atom is inversely proportional to fourth power of principal quantum number.

Ans: i. From the Bohr's first postulate,

$$\frac{mv_n{}^2}{r_n} = \frac{1}{4\pi\varepsilon_0} \times \frac{e^2}{r_n{}^2}$$

$$\therefore \quad mv^2_n = \frac{1}{4\pi\varepsilon_0} \times \frac{e^2}{r_n}$$

$$\therefore \quad v_n{}^2 = \frac{1}{4\pi\varepsilon_0} \times \frac{e^2}{r_n m} \quad \text{....(i)}$$

$$\text{and } r_n = \frac{\varepsilon_0 h^2 n^2}{\pi m e^2} \quad \text{....(ii)}$$

ii. Now, centripetal acceleration of the electron

$$a_n = \frac{V_n{}^2}{r_n} \quad \text{....(iii)}$$

Substituting $V_n{}^2$ from equation (i) in equation (iii), we have,

$$\therefore \quad a_n = \frac{\dfrac{1}{4\pi\varepsilon_0} \times \dfrac{e^2}{r_n m}}{r_n} = \frac{1}{4\pi\varepsilon_0} \times \frac{e^2}{m} \times \frac{1}{r_n{}^2}$$

$$\therefore \quad a_n = \frac{1}{4\pi\varepsilon_0} \times \frac{e^2}{m} \times \frac{1}{\left(\dfrac{\varepsilon_0 h^2 n^2}{\pi m e^2}\right)^2}$$

[From equation (ii)]

$$\therefore \quad a_n = \frac{1}{4\pi\varepsilon_0} \times \frac{e^2}{m} \times \frac{1}{\left(\dfrac{\varepsilon_0{}^2 h^4 n^4}{\pi^2 m^2 e^4}\right)}$$

$$\therefore \quad a_n = \frac{1}{4\pi\varepsilon_0} \times \frac{e^2}{m} \frac{\varepsilon_0{}^2 h^4 n^4}{\pi^2 m^2 e^4}$$

$$\therefore \quad a_n = \frac{1}{4\pi\varepsilon_0} \times \frac{e^2}{m} \times \frac{\pi m e^4}{\varepsilon_0{}^2 h^4 n^4}$$

$$\therefore \quad a_n = \left[\frac{\pi m e^6}{4\varepsilon_0{}^3 h^4}\right] \times \frac{1}{n^4} \quad \text{...(iv)}$$

iii. In equation (iv), n, m, ε_0, e and h are constant.

$$\therefore \quad \frac{\pi m e^6}{4\varepsilon_0{}^3 h^4} = \text{constant}$$

HENRY CLASSES

$\therefore \quad a_n \propto \frac{1}{n^4}$

Hence centripetal acceleration is inversely proportional to 4th power of the principal quantum number.

Q.17. Obtain an expression for linear momentum of electron in Bohr orbit. Hence show that linear momentum is inversely proportional to principal quantum number.

Ans: Expression for linear momentum of electron in Bohr orbit:

i. Let, m = mass of electron
V_n = linear velocity of electron in nth Bohr orbit
r_n = radius of n^{th} Bohr orbit
e = charge of electron
p = linear momentum of electron

ii. According to Bohr's first postulate,

$$\frac{mv_n^2}{r_n} = \frac{e^2}{4\pi\varepsilon_0 r_n^2}$$

$$\therefore \quad mv_n^2 r_n = \frac{e^2}{4\pi\varepsilon_0} \quad ...(i)$$

iii. According to Bohr's second postulate,

$$mv_n r_n = \frac{nh}{2\pi} = \quad .... (ii)$$

iv. Dividing equation (i) by equation (ii), we have, $v_n = \frac{e^2}{4\pi\varepsilon_0} \times \frac{2\pi}{nh}$

$$\therefore \quad v_n = \left[\frac{e^2}{2\varepsilon_0 h}\right] \times \frac{1}{n} \quad .... (iii)$$

v. Linear momemtum p of the electron in any orbit is given by, p = mv..

But, $v_n = \frac{e^2}{2\varepsilon_0 h} \times \frac{1}{n}$

$$\therefore \quad p = m \times \frac{e^2}{2\varepsilon_0 h} \times \frac{1}{n}$$

$$\therefore \quad p = \left(\frac{me^2}{2\varepsilon_0 h}\right) \times \frac{1}{n} \quad ...(iv)$$

This is required expression for linear momentum of the electron in Bohr orbit.

vi. In equation (iv),

$$\frac{me^2}{2\varepsilon_0 h} = \text{constant}$$

$$\therefore \quad p = (\text{constant}) \times \frac{1}{n}$$

$$\therefore \quad p \propto \frac{1}{n}$$

Thus, linear momentum of the electron in a Bohr's orbit is inversely proportional to the principal quantum number.

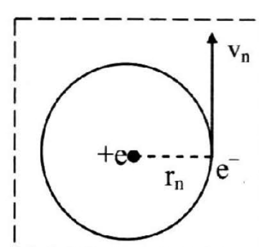
Q.18. Derive an expression for total energy of electron in nth Bohr orbit and show that $E_n \propto \frac{1}{n^2}$. OR

Derive an expression for the total energy of electron in 'n' th Bohr orbit. Hence show that energy of the electron is inversely proportional to the square of principal quantum number. [Oct 14]

Ans: Expression for energy of electron:

i. Kinetic energy:

Let, m = mass of electron
r_n = radius of nth orbit of Bohr's hydrogen atom
v_n = velocity of electron
–e = charge of electron
+e = charge on the nucleus



According to Bohr's first postulate,

$$\frac{mv_n^2}{r_n} = \frac{1}{4\pi\varepsilon_0} \times \frac{e^2}{r_n^2}$$

where, ε_0 is permittivity of free space.

$$\therefore \quad mv_n^2 = \frac{1}{4\pi\varepsilon_0} \times \frac{e^2}{r_n^2} \quad ....(i)$$

The revolving electron in the circular orbit has linear speed and hence it possesses kinetic energy.

It is given by,

HENRY CLASSES

$$K.E = \frac{1}{2}\ mv_n^{\ 2}$$

$$\therefore \quad K.E = \frac{1}{2} \times \left(\frac{1}{4\pi\varepsilon_0} \times \frac{e^2}{r_n^{\ 2}}\right)$$

[From equation (i)]

$$\therefore \quad K.E = \frac{e^2}{8\pi\varepsilon_0 r_n} \qquad .... (ii)$$

ii. Potential energy:

Potential energy of electron is given by, P.E = V(–e)

where, V = electric potential at any point due to charge on nucleus

– e = charge on electron.

$$\therefore \quad P.E = \frac{1}{4\pi\varepsilon_0} \times \frac{e^2}{r_n^{\ 2}} \times (-e)$$

$$\therefore \quad P.E = \frac{1}{4\pi\varepsilon_0} \times \frac{-e^2}{r_n^{\ 2}}$$

$$\therefore \quad P.E = \frac{e^2}{4\pi\varepsilon_0 r_n} \qquad .... (iii)$$

iii. Total energy:

The total energy of the electron in any orbit is its sum of P.E and K.E.

$$\therefore \quad T.E = K.E + P.E$$

$$= \left(\frac{e^2}{8\pi\varepsilon_0 r_n}\right) + \left(-\frac{e^2}{4\pi\varepsilon_0 r_n}\right)$$

[From equations (ii) and (iii)]

$$\therefore \quad T.E. = -\frac{1}{8\pi\varepsilon_0} \times \frac{e^2}{r_n}$$

$$\therefore \quad T.E. = -\frac{e^2}{8\pi\varepsilon_0 r_n} \qquad ...(iv)$$

$$\therefore \quad T.E. \propto \frac{1}{r_n}$$

Thus, the total energy of the electron in Bohr orbit is inversely proportional to the radius of the orbit.

iv. But, $r_n = \left(\frac{\varepsilon_0 h^2}{\pi m e^2}\right) \times n^2$

Substitute for r_n in equation (iv),

$$\therefore \quad T.E = \frac{1}{8\pi\varepsilon_0} \times \frac{e^2}{\left(\frac{\varepsilon_0 h^2}{\pi m e^2}\right) n^2}$$

$$= \frac{1}{8\pi\varepsilon_0} \times \frac{e^2 \pi m e^2}{\varepsilon_0 h^2 n^2}$$

$$\therefore \quad T.E = \left(\frac{me^4}{8\varepsilon_0^{\ 2} h^2}\right) \times \frac{1}{n^2} \qquad .... (v)$$

This is required expression for energy of electron in nth orbit of Bohr's hydrogen atom.

v. The negative sign in equation (v) shows that the electron is bound to the nucleus by an attractive force and hence energy must be supplied to the electron in order to make it free from the influence of the nucleus.

To show: T.E $\propto \frac{1}{n^2}$

As m, e, ε_0, and h in equation (v) are constant,

$$\frac{me^4}{8\varepsilon_0^{\ 2} h^2} = \text{constant}$$

Using equation (v), we get,

$$T.E = \text{constant} \times \frac{1}{n^2}$$

$$T.E. \propto \frac{1}{n^2}$$

Hence, the total energy of electron in a Bohr's orbit is inversely proportional to the square of the principal quantum number.

Q.19. Derive the formula for total energy of an electron in the Bohr orbit. [Feb 06]

Ans: Refer Q.18

Q.20. Show that the energy of an electron in Bohr orbit is inversely proportional to the radius of the orbit. [Feb 01]

Ans: Refer Q. 18

Q.21. Define binding energy. [Oct 14]

OR

What is binding energy of electron?

Ans: i. Binding energy of an electron is the minimum energy required to make it free from the nucleus.

ii. It is also called ionisation energy and is

HENRY CLASSES

always positive as it is supplied from outside agency.

iii. When binding energy is supplied to an electron, the total energy of the system containing nucleus and electron becomes zero.

$\therefore$ B.E + $T.E_n = 0$

$\therefore$ B.E = $-T.E_n$

iv. But $T.E_n = \frac{-me^4}{8\varepsilon_0^2h^2n^2}$

$\therefore$ $B.E. = -\left(-\frac{-me^4}{8\varepsilon_0^2h^2n^2}\right)$

$\therefore$ $B.E = \frac{me^4}{8\varepsilon_0^2h^2n^2}$

Note:

For n = 1, 2, 3, ...

$$T.E = \left(\frac{me^4}{8\varepsilon_0^2h^2}\right) \times 1 = -13.6eV$$

$$T.E_2 = \frac{T.E_1}{(2)^2} = \frac{-13.6}{4} = -3.4eV$$

$$T.E_3 = \frac{T.E_1}{(3)^2} = \frac{-13.6}{9} = -1.51 \text{ eV and so on.}$$

Q.22. Define

i. Stationary orbits (Bohr orbit)

ii. Ionisation energy (ionisation potential) of atom

Ans: i. Stationary orbits (Bohr orbit):

Stationary orbits are any discrete orbit in which electron do not radiate energy while it is revolving in such orbits.

ii. Ionisation energy:

The minimum energy required to remove the least strongly bound electron from a neutral atom to such a distance that there is no electrostatic interaction between the ion and the electron is known as ionisation energy.

Note:

For the innermost orbit, B.E is maximum. An electron in the innermost orbit is bound to the nucleus with maximum attractive force.

Q.23. How do P.E and K.E change when hydrogen atom is raised from ground state to excited state?

Ans: i. In hydrogen atom, K.E is maximum in ground state and P.E is mmimum in ground state.

ii. When hydrogen atom is raised from ground state to excited state, K.E decreases and P.E increases, but total energy remains constant.

Q.24. The gravitational attraction between electron and proton in a hydrogen atom is weaker than the coulomb attraction by a factor of about 10^{-4}. An alternative way of looking at this fact is to estimate the radius of the first Bohr orbit of a hydrogen atom if the electron and proton were bound by gravitational attraction. You will find the answer interesting. [NCERT]

Ans: The radius of the first Bohr orbit of a hydrogen atom is, $r_0 = \frac{4\pi\varepsilon_0(h/2\pi)^2}{m_e e^2}$

If we consider the atom to be bound by the gravitational force $\left(= \frac{Gm_pm_e}{r^2}\right)$, the quantity, $\frac{e^2}{4\pi\varepsilon_0}$ will be replaced by (Gm_pm_e), In that case, radius of first Bohr orbit of hydrogen atom,

$$r_0 = \frac{(h/2\pi)^2}{Gm_pm_e^2}$$

Substituting the standard values, we get

$$r_0 = \frac{(6.63\times10^{-34}/2\pi)^2}{6.67\times10^{-11}\times1.67\times10^{-27}\times(9.1\times10^{-31})^2}$$

$= 1.2 \times 10^{29}$ metre

This value is much greater than the estimated size of the entire universe.

18.4 : Hydrogen spectrum

Q.25. Explain Balmer's theory of hydrogen spectrum. State its impirical relation.

Ans: Balmer's theory of hydrogen spectrum:

i. In 1885, Balmer studied the characteristics of hydrogen spectrum.

ii. When a gas is subjected to strong electric field or suitable radiation, it begins to emit electromagnetic waves of specific wavelengths called characteristic spectrum of the gas.

HENRY CLASSES

iii. Balmer measured the wavelengths of emitted radiation. These radiations lie in the visible region of spectrum and are known as $H_\alpha, H_\beta, H_\gamma$ and H_δ lines, with the wavelengths of 6563 Å, 4868 Å, 4341 Å and 4202 Å respectively.

iv. Balmer stated the relation for these wavelengths called as Balmer's impirical relation.
According to this relation,

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \qquad \text{.... (i)}$$

where, $R = 1.097 \times 10^7\ m^{-1}$ = Rydberg's constant.
Equation (i) is satisfied for the values n = 3, 4, 5 and 6.

Note:
Balmer could not explain theoretically the real cause of hydrogen spectrum.

Q.26. Explain origin of spectral line and obtain Bohr's formula. [Feb 03]

OR

Uslng expression for energy of electron, obtain the Bohr's formula for hydrogen spectral line.

Ans: Origin of spectral line:

i. According to Bohr's third postulate, when an electron in a hydrogen atom jumps from higher energy level to the lower energy level, the difference of energies of the two energy levels is emitted as' a radiation of particular wavelength called spectral line.

ii. The wavelength of the spectral line depends upon the energy associated with the two energy levels, between which the transition of the electron takes place.

iii. If the energy absorbed is equal to difference between the energies of the two levels then it jumps to a higher permitted orbit and revolves in it. In this case, electron is said to be in the excited state.

iv. In the excited state, the electron is not stable and tries to attain stability by going back to the ground state by emitting the extra amount of energy it had gained in one or more jumps.

v. The energy is emitted as electromagnetic waves and produces a spectral line of the corresponding frequency or wavelength.

Bohr's formula for spectral lines in hydrogen spectrum:

i. Let, E_n = Energy of electron in n^{th} higher orbit
E_p = Energy of electron in pth lower orbit

ii. According to Bohr's third postulate,
$E_n - E_p = hv$

$$\therefore \quad v = \frac{E_n - E_p}{h} \qquad \text{....(i)}$$

iii. But $E_n = -\dfrac{me^4}{8\varepsilon_0^2 h^2 n^2}$ ...(ii)

$$E_p = -\frac{me^4}{8\varepsilon_0^2 h^2 p^2} \qquad \text{...(iii)}$$

iv. From equations (i), (ii) and (iii), we have,

$$v = \frac{\dfrac{-me^4}{8\varepsilon_0^2 h^2 n^2} - \left(-\dfrac{-me^4}{8\varepsilon_0^2 h^2 p^2}\right)}{h}$$

$$\therefore \quad v = \frac{me^4}{8\varepsilon_0^2 h^3}\left[-\frac{1}{n^2} + \frac{1}{p^2}\right]$$

$$\therefore \quad \frac{c}{\lambda} = \frac{me^4}{8\varepsilon_0^2 h^3}\left[\frac{1}{p^2} - \frac{1}{n^2}\right] \qquad \left[\because v = \frac{c}{\lambda}\right]$$

where, c = speed of electromagnetic radiation

$$\therefore \quad \frac{1}{\lambda} = \frac{me^4}{8\varepsilon_0^2 h^3 c}\left[\frac{1}{p^2} - \frac{1}{n^2}\right]$$

v. But, $\dfrac{me^4}{8\varepsilon_0^2 h^3 c}$ = R = Rydberg's constant

$$\frac{1}{\lambda} = R\left[\frac{1}{p^2} - \frac{1}{n^2}\right] \qquad \text{...(iv)}$$

Equation (iv) represents Bohr's formula for hydrogen spectrum.

vi. $\dfrac{1}{\lambda}$ is called wave number (v) of the line.

$$\therefore \quad \bar{v} = \frac{1}{\lambda} = R\left(\frac{1}{p^2} - \frac{1}{n^2}\right)$$

Q.27. Explain the origm of spectral line and hence obtain an expression for wave number. [Mar 97]

Ans: Origin of spectral line:

i. According to Bohr's third postulate, when an electron in a hydrogen atom jumps from higher energy level to the lower energy level, the difference of energies of the two energy levels is emitted as' a radiation of particular wavelength called spectral line.

ii. The wavelength of the spectral line depends upon the energy associated with the two energy levels, between which the transition of the electron takes place.

iii. If the energy absorbed is equal to difference between the energies of the two levels then it jumps to a higher permitted orbit and revolves in it. In this case, electron is said to be in the excited state.

iv. In the excited state, the electron is not stable and tries to attain stability by going back to the ground state by emitting the extra amount of energy it had gained in one or more jumps.

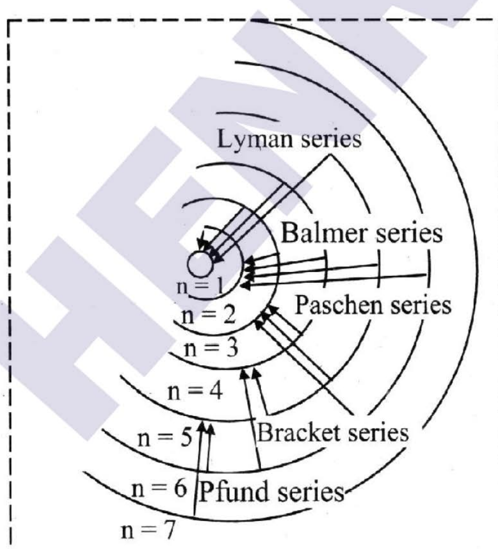
v. The energy is emitted as electromagnetic waves and produces a spectral line of the corresponding frequency or wavelength.

Q.28. Explain the different series of spectral lines in hydrogen spectrum.

Ans: Series of spectral lines:

Different wavelength in hydrogen spectrum can be calculated by formula, $\frac{1}{\lambda} = R\left[\frac{1}{p^2} - \frac{1}{n^2}\right]$.

The transition of electrons from different outer orbits to a certain inner orbit is explained with following series as shown in the figure.



Lyman series:

i. This series corresponds to the transition of an electrori from some higher energy state to the innermost orbit (first Bohr orbit).

ii. For Lyman series, p = 1 and n = 2, 3, 4, .. The wave numbers and the wavelengths of the spectral lines constituting the Lyman series are. given by,

$$\bar{v} = \frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

iii. There are infinite number lines in this senes.

iv. This series lies in the ultraviolet region of spectrum. It is not visible with naked eye, but it can be photographed. Wavelengths for n = 2 and 3 are 1216 Å and, 1025 Å respectively.

Balmer series:

i. The spectral lines of this series correspond to the transition of an electron from some higher energy state to 2nd orbit.

ii. For Balmer series, p = 2 and n = 3, 4, 5, The wave numbers and the wavelengths of spectral lines constituting the Balmer series are given by,

$$\bar{v} = \frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

iii. There are infinite number lines in this series out of which four lines are seen called $H_\alpha, H_\beta, H_\gamma, H_\delta$

iv. This series lies in the visible region. Wavelengths for n = 3 and 4 are 6563 Å and 4868 Å respectively.

Paschen series:

i. The spectral lines of this senes correspond to the transition of an electron from some higher energy state to 3rd orbit.

ii. For paschen series, p = 3 and n = 4, 5,... The wave numbers and the wavelengths of the spectral lines constituting the Paschen series are given by,

$$\bar{v} = \frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$$

iii. Paschen series lies in the infrared region of the spectrum which is invisible and contains infinite number of lines,

iv. Wavelengths for n = 4 and 5 are 18750 Å

HENRY CLASSES

and 12820 Å respectively.

Bracket series:

i. The spectral lines of this senes corresponds to the transition of an electron from a higher energy state to the 4th orbit.

ii. For this series, p = 4 and n = 5,6, 7, ... The wave numbers and the wavelengths of the spectral lines constituting the Bracket series are given by,

$$\bar{v} = \frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right)$$

iii. This series lie in the near infrared region of the spectrum and contains infinite number of lines. Wavelengths for n = 5 and 6, are 40518 Å and 26253 Å respectively.

Pfund series:

i. The spectral lines of this series correspond to the transition of electron from a higher energy state to the 5th orbit.

ii. For this series, p = 5 and n = 6, 7, 8 .... The wave numbers and the wavelengths of the spectral lines constituting the Pfund series are given by,

$$\bar{v} = \frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right)$$

iii. This series lie in the far infrared region of the spectrum and contains infinite number of lines. Wavelengths for n = 6 and 7 are 75587 Å and 46533 Å respectively.

Q.29. What is series limit? State series limit for Balmer and Paschen series

Ans: Serles limit:

i. The smallest wavelength emitted in a series is called series limit.

ii. In Bohr's relation, series limit for particular series is found by taking n = ∞ .

iii. Series limit for Balmer series is given by,

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right)$$

$$= \frac{R}{4} \quad [\because \frac{1}{\infty} = 0]$$

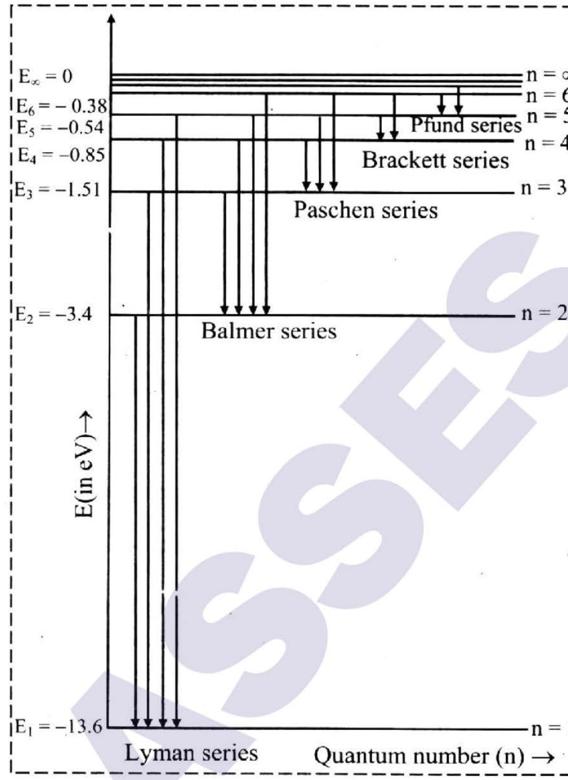
$$\therefore \quad \lambda = \frac{4}{R}$$

iv. Series limit of Paschen series is given by,

$$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{\infty^2}\right)$$

of the hydrogen atom. [Oct 10, Mar 12]

Ans: Energy level diagram



Note:

Energy of electron in. the infinite energy level is maximum i.e. 0 eV because below this value every energy level has negative value.

Q.31. Draw a neat labelled energy diagram for Hydrogen atom, showing Balmer and Paschen series. [Feb 04]

Ans: Refer Q.30 (energy level diagram)

Q.32. How may spectral lines are possible in hydrogen spectrum?

Ans: Infinite spectral lines are possible in hydrogen spectrum.

The series resulting from transition to p = 6 from n = 7, 8, 9, ... etc. is called Humphery series. The further series do exist but the lines corresponding to them appear fainter and fainter as we go further in the increasing order of n. Hence these series have not been named.

Q.33. Which hydrogen spectral series is found first?

Ans: Balmer spectral series is found first.

Q.34. Explain the origin of characteristic X-ray spectrum.

Ans: Origin of characteristic X-ray spectrum:

i. According to Bohr's atomic model, an atom consists of central positively charged nucleus with electrons revolving around it in orbits of definite radii, designated as K, L, M, N, ... shells.

ii. When an electron strikes the target, it penetrates well inside the atom due to its high kinetic energy and knocks out one of the electrons from it say the innermost K-shell. The vacancy created in the K-shell is immediately filled up by the jump of electron from one of the outer shells.

iii. If the vacancy in the K-shell is filled up by the jump of an electron from the L-shell, then X-ray photon is emitted, . whose frequency is given by,
$h\nu = E_L - E_K$
where, E_K and E_L are the energies associated with the K and L-shells respectively.

iv. If an electron from the M-shell jumps to the K-shell, X-ray photon of greater frequency will be emitted. The X-rays photons emitted as a result of jumps of the electrons from outer shells to the K-shell form a small group, called the Kvseries.

v. If the incident electron ejects an electron from the L-shell and the vacancy caused is filled by the jumps of electrons from the outer shells to the L-shell, the X-ray photons emitted again form a small group, called the L-series. However, the frequencies of the spectral lines of the L-series will be smaller than those of the K-series for the same target element.

vi. Similar phenomenon takes place in the M-series, N-series. Thus the characteristic X-rays are produced.

vii. The wavelength of characteristic X-rays may be used to identify the element from which they originate.

Q.35. Explain origin of continuous X-ray spectrum.

Ans: Origin of continuous X-ray spectrum:

i. When electrons are accelerated through a potential difference V, the kinetic energy acquired by the electron is given by, eV = $\frac{1}{2}mv^2$.

ii. When high speed electrons hit a target, X-rays are produced. Some high speed electrons may pass very close to the nuclei of some of the atoms within the target.

iii. In doing so, they experience a strong Coulomb's force of attraction due to the positive nuclei. The motion of the electrons is then suddenly subjected to an intense. retardation and even the direction of its motion may be changed.

iv. During retardation, the X-rays are emitted by the electrons due to inverse photoelectric effect. As the electron undergo all sorts of retardation, the X-rays of all sorts of wavelength from a minimum value onwards are emitted.

v. The maximum frequency V_{max} of the X-rays produced is given by, $eV = h\nu_{max}$ where, h is Planck's constant.

vi. If λ_{min} is the corresponding muumum wavelength then,

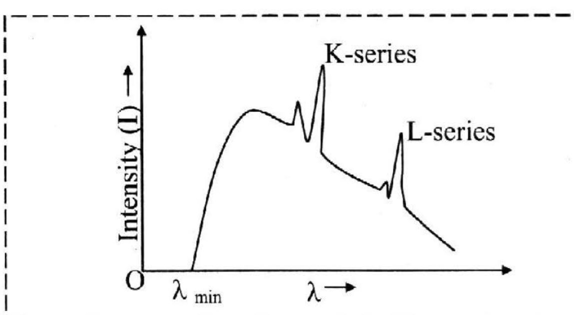
$$eV = \frac{hc}{\lambda_{min}}$$

$$\therefore \quad \lambda_{min} = \frac{hc}{eV}$$

As h, c and e are constants, the nummum wavelength of X-rays produced is inversely proportional to accelerating voltage.

Q.36. Draw intensity vis wavelength graph for X-ray spectrum.

Ans:



Intensity - wavelength graph in X-rays spectrum

Note:
K- series of X-ray spectrum consists of two spectral lines denoted as K K_α and K_β.

18.5 : Composition and size of nucleus

Q.37. What is nucleus? Name the constituent particles of nucleus.

HENRY CLASSES

Ans: i. Nucleus is the small positively charged mass located at the central region of an atom.

ii. The constituent particles of the nucleus are protons and neutrons.

iii. Protons and neutrons when described collectively are called nucleons.

Q.38. Define the following terms.

i. Atomic number

ii. Mass number

Ans: i. Atomic number:

Number of protons present in the nucleus of an atom is called atomic number.

It is denoted by Z.

For neutral atom, Z = P = e.

ii. Mass number:

Sum of number of protons and neutrons present in the nucleus of atom is called mass number.

It is denoted by A.

A = p + n

For an element X, atomic number and mass number is represented as ${}_{Z}^{A}X$.

Example : ${}_{79}^{197}Au$

Q.39. Define the following terms.

i. Isotopes

ii. Isobars

iii. Isotones

Ans: i. Isotopes:

The atoms of element having same atomic number but different mass number are called isotopes.

Examples:

a. ${}_{1}^{1}H, {}_{1}^{2}H, {}_{1}^{3}H$

b. ${}_{8}^{16}O, {}_{8}^{17}O, {}_{8}^{18}O$

All isotopes have same chemical properties.

ii. Isobars:

Atoms having same mass number but different atomic number are called isobars.

Examples:

a. ${}_{1}^{3}H, {}_{2}^{3}He$,

b. ${}_{6}^{14}C, {}_{7}^{14}N$

iii. Isotones:

Atoms having same number of neutrons but different atomic numbers are called isotones.

Examples:

a. ${}_{4}^{9}Be, {}_{5}^{10}B$

b. ${}_{1}^{3}H, {}_{2}^{4}He$

Q.40. Explain how nuclear size of an atom is estimated.

Ans: i. The nucleus diameters are measured by scattering of high energy electrons by nucleus.

ii. It is found that a nucleus of mass number A has radius of $R = R_0 A^{\frac{1}{3}}$ where R_0 is linear constant and has the value of 1.2×10^{-15} m.

iii. This implies that density of nucleus is constant and is independent of mass number A for all nuclei. .

iv. For example,

a. Radius of carbon nucleus:

$$R_C = 1.2 \times 10^{-15} \text{ m} \times (12)^{\frac{1}{3}}$$
$$= 2.7473 \times 10^{-15} \text{ m}$$

b. Radius of uranium nucleus:

$$R_U = 1.2 \times 10^{-15} \text{ m} \times (238)^{\frac{1}{3}}$$
$$= 7.4366 \times 10^{-15} \text{ m}$$

Q.41. Prove that nuclear density for all the nucleus is same.

Ans: i. Let 'm' be the average mass of a nucleon in a nuclide of mass number' A' and radius 'R'.

$\therefore$ $R = R_0 A^{\frac{1}{3}}$

$\therefore$ The volume 'V' of the nuclide,

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi\left(R_0 A^{\frac{1}{3}}\right)^3 = \frac{4}{3}\pi R_0^{\,3} A$$

ii. The approximate mass 'M' of' the nuclide is mA.

$\therefore$ The density 'p' is given by,

$$\rho = \frac{M}{V}$$

$$= \frac{3mA}{4\pi R_0^3 A} = \frac{3m}{4\pi R_0^3} = 4\pi R_0^3 A$$

iii. Since m and R_0 are constants, Nuclear density will be constant for all nuclei.

Q.42. Explain Einstein's mass-energy relation with suitable examples.

Ans: Einstein's mass - energy relation:

i. Einstein proved that mass is a form of energy. Mass and energy are interconvertible.

HENRY CLASSES

ii. The mass 'm' has equivalent energy E given by, $E = mc^2$.
where, c = speed of light.

iii. In a reaction, the law of conservation of energy states that, the initial energy and the final energy are equal, if the energy associated with mass is included.

iv. As mass of particle is not constant but depends upon its velocity, therefore it is more convenient to express mass in the form of energy.

v. For example, energy of electron

$E_e = m_e c^2$

$= (9.1 \times 10^{-31}) \times (3 \times 10^8)^2$ joule

$= \dfrac{9.1 \times 10^{-31} \times 9 \times 10^{16}}{1.6 \times 10^{-19}}$ eV

$= 0.511 \times 10^6$ eV

$\therefore$ $E_e = 0.511$ MeV

Similarly, the energies of proton and neutron are $E_p = 941.1$ MeV and $E_n = 942.2$ MeV.

Q.43. What is unified atomic mass unit?

Ans: i. The unified atomic mass unit is $\left(\dfrac{1}{12}\right)^{th}$ of mass of neutral carbon atom in its lowest energy state.

ii. It is denoted by u and is given by,

$1\ u = 1.66054 \times 10^{-27}$

$= 931\ MeV/c^2$

$\therefore$ Energy equivalent of mass

1 u = 931 MeV

Q.44. What do you mean by mass defect?

Ans: i. The difference between the actual mass of the nucleus and the sum of masses of constituent nucleons is called mass defect.

ii. Let,

M = measured mass of nucleus

A = mass number

Z = atomic number

m_p = mass of hydrogen atom

m_n = mass of free neutron

(A – Z) = number of neutrons

$\therefore$ Mass defect,

$\Delta m = [Zm_p + (A - Z)m_n] - M$

Q.45. What is nuclear binding energy?

Ans: i. The amount of energy required to separate all the nucleons from the nucleus is called binding energy of the nucleus.

ii. The B.E of nucleus is very high. It is 2.22 Me V for deutron nucleus, whereas B.E. for an atom, say hydrogen atom in its ground state is 13.6 eV.

iii. It means that B.E of nucleus is about 10,00,000 times larger than B.E. of atom.

Q.46. What is binding energy per nucleon? Express binding energy per nucleon in term of mass defect.

Ans: Binding energy per nucleon:

The average energy required to release a nucleon from the nucleus is called binding energy per nucleon.

Binding energy per nucleon is given by,

$$E = \frac{\text{B.E of nucleus}}{A}$$

Binding energy per nucleon in terms of mass defect:

Since, B.E = $\Delta m \times c^2$ joule

where, Δm = mass defect

c = speed of light.

But, $\Delta m = [Zm_p + (A - Z)m_n] - M$

$\therefore$ B.E of nucleus

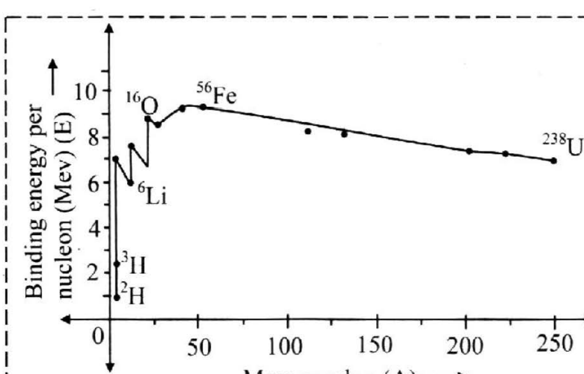
$= [Zm_p + (A - Z)m_n - M]\ c^2$ joule

$\therefore$ The B.E per nucleon,

$$E = \left[\frac{Zm_p + (A - Z)m_n - M}{A}\right] c^2$$

Q.47. Draw a binding energy curve to show the variation of binding energy per nucleon with mass number. What inferences can be drawn from B.E curve?

Ans: The B.E curve is an indicator of nuclear stability. The higher the B.E per nucleon, the greater is the stability of the nucleus.



B.E curve

HENRY CLASSES

Inferences of B.E curve:

i. The B.E per nucleon is practically constant and is independent of mass number for nuclei $30 < A < 170$
ii. It is maximum 8.75 MeV for A = 56 and is 7.6 MeV, for A = 238.
iii. It is low for both light nuclei (A < 30) and heavy nuclei (A > 170). This means that the nucleons of atoms are loosely bound with nucleus.
iv. When heavy nucleus (A = 240) breaks into lighter nuclei (A = 120), B.E. increases i.e. nucleons get more tightly, bound.
v. When very light nuclei A < 10, join to form a heavier nucleus, B.E increases, i.e. nucleons get more tightly bound

18.6 : Radioactivity

Q.48. What is radioactivity?

Ans: i. The phenomenon of spontaneous emission of radiations from radioactive substance is called as radioactivity.
ii. Radioactivity is property of atom and nuclei, hence is unaffected by chemical or physical changes.
iii. Radioactivity of an unstable nucleus undergoes a decay.

Q.49. What are radioactive substances?

Ans: Substances which emit spontaneous emission of radiation from their nuclei are called radioactive substances.
Example: Uranium, Radium, Thorium etc.

Q.50. What is radioactive decay?

Ans: i. The nuclear phenomenon in which an unstable nucleus undergoes a decay is called radioactive decay.
ii. It is classified into three types.
a. α **– decay:** It is a type of decay in which helium nucleus ~He is emitted.
b. β **– decay:** In this decay, electrons or positrons are emitted.
c. γ **– decay:** In this decay, high energy photons are emitted.

Q.51. State the main properties of α – particles.

Ans: Properties of α – particles:

i. They are positively charged particles having charge of magnitude $+3.2 \times 10^{-19}$ C and mass is 6.64×10^{-27} kg.
ii. Being charged particle, it is deflected by electric and magnetic fields.
iii. The speed of emission of α–particles depend upon the nature of radioactive element. It varies from $\left(\frac{1}{10}\right)^{th}$ to $\left(\frac{1}{100}\right)^{th}$ of the speed of light.
iv. They affect photographic plate, produce fluorescence.
v. They ionise gas when passed through it.
vi. The range of a–particles through air varies from 2.7 cm to 8.62 cm for thorium.
vii. They are scattered when incident on mica, aluminium and gold foil.
viii. When an α–particle is emitted by an atom, its atomic number decreases by 2 and mass number decreases by 4.

Example: ${}^{238}_{92}U \xrightarrow{-\alpha} {}^{234}_{90}Th + {}^{4}_{2}He$

Q.52. State properties of β–rays.

Ans: Properties of β–rays:

i. β–rays are fast moving electrons from nucleus.
ii. Their speed ranges from 1% to 99% of the speed of light.
iii. Being charged particles, they are deflected by electric and magnetic fields.
iv. They can ionise gas but its ionisation power is $\left(\frac{1}{100}\right)^{th}$ of that of α – particles.
v. They are more penetrating than α – particles.
vi. Their range in air depends on their speed. A β–particle of 0.5 MeV has a range of 1 m in air.
vii. When β–particle is radiated, the atomic number increases by 1 and mass number does not change.

Example: ${}^{32}_{15}P \xrightarrow{-\beta} {}^{32}_{16}S + {}^{0}_{1}e$

Q.53. State properties of γ–rays.

Ans: Properties of γ–rays:

i. γ–rays are not particles but they are electromagnetic waves (photons) of very short wavelength. Photons originating from the nucleus are called γ–rays.
ii. They are neutral in charge and not affected

HENRY CLASSES

by electric and magnetic fields.

iii. They affect photographic plate and produce fluorescence.

iv. They have very low ionization power about $\left(\frac{1}{1000}\right)^{th}$ of that of α – particles.

v. They have high penetration power and can pass through 25 cm thick iron plates.

vi. They are diffracted by crystals.

Q.54. What do you mean by radioactive disintegration?

Ans: i. The spontaneous breaking of nucleus is called radioactive disintegration.

ii. A radioactive element constantly breaks up into fresh radioactive atoms due to radioactive disintegration.

iii. The new atoms after radioactive disintegration are generally radioactive.

Q.55. State the law of radioactive decay. Hence derive the expression $N = N_0\ e^{-\lambda t}$ where symbols have their usual meanings.

[Feb 13]

Ans: Law of radioactive decay:

The number of nuclei undergoing the decay per unit time is proportional to the number of unchanged nuclei present at that instant. If 'N' is the number of nuclei present at any instant 't', 'dN' is the number of nuclei that disintegrated in short interval of time 'dt', then according to decay law,

$$-\frac{dN}{dt} \propto N$$

$$\therefore \quad \frac{dN}{dt} = -\lambda\ N$$

where, λ is known as decay constant or disintegration constant. The negative sign indicates disintegration of atoms.

Derivation of relation $N = N_0\ e^{-\lambda t}$:

i. Let, N = number of nuclei present at any instant t.
dN = number of nuclei disintegrated in short interval dt.

ii. According to decay law,

$$\frac{dN}{dt} = -\lambda\ N \qquad \text{.... (i)}$$

iii. Integrating both sides of equation (i), we get, $\int \frac{dN}{N} = \int -\lambda\ dt$

$\therefore \quad \log_e N = -\lambda t + c \qquad$ ...(ii)

where, c is constant of integration whose value depends on initial conditions.

iv. At, t = 0, $N = N_0$

$\therefore \quad \log e\ N_0 = 0 + c \qquad$ [From equation (i)]

v. Substituting the value of c in equation (i), we get,

$$\log_e N = -\lambda t + \log_e N_0$$

$$\log_e N - \log_e N_0 = -\lambda t$$

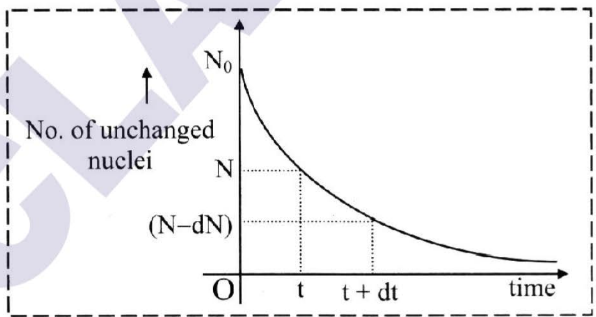
$$\log_e \left(\frac{N}{N_0}\right) = -\lambda t$$

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$N = N_0\ e^{-\lambda t}$$

Q.56. Draw a decay curve showing number of unchanged nuclei per unit time.

Ans: Decay curve:



Q.57. What is decay constant?

Ans: According to decay law,

$$\frac{dN}{dt} = -\lambda N$$

where, λ = proportionality constant called decay constant

$$\lambda = \frac{dN}{dt}/N \qquad \text{...(magmtudewtse)}$$

Hence decay constant is defined as the ratio of the amount of substance disintegrated per unit time to amount of substance present at that time.

Q.58. Define half life period. Derive expression for it.

Ans: Definition:

Half life period of a radioactive substance is defined as the time in which the half substance is

HENRY CLASSES

disintegrated.

Expression for half life period:

From law of radioactive decay,

$N = N_0 e^{-\lambda t}$

at $t = T_{1/2}$, $N = \frac{N_0}{2}$

$\therefore \quad \frac{N_0}{2} = N_0 e^{-\lambda t\ 1/2}$

$\therefore \quad \frac{1}{2} = e^{-\lambda t\ 1/2}$

This is required expression for half life period of radioactive substance.

Q.59. What is nuclear fission?

Ans: i. The process of splitting a heavy nucleus into two lighter nuclei after bombardment with neutrons is called nuclear fission.

ii. The process of nuclear fission was first discovered by German scientists Otto Hahn and Strassman in 1939.

iii. Example of nuclear fission:

$${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{236}_{92}U \rightarrow {}^{144}_{56}Ba + {}^{89}_{36}Kr + 3{}^{1}_{0}n$$

In the above reaction, when ${}^{235}_{92}U$ is bombarded by neutron, it breaks up into two intermediate fragments which emit $\beta-$particles to achieve stable end products.

iv. The energy released in fission first appears as K.E which gets converted into heat in surrounding.

v. Fission energy is being used in nuclear power projects for generation of electricity.

vi. The uncontrolled fission process is used in atom bomb.

Q.60. What is nuclear fusion?

Ans: i. The nuclear reaction in which two lighter nuclei are fused to form a heavier nucleus is called nuclear fusion.

ii. The newly formed nucleus has smaller mass than the sum of masses of fused nuclei.

iii. The mass defect is converted into energy.

iv. For example,

$${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{4}_{2}He + 24MeV$$

When two deuterons get fused, u– particle is emitted and 24 MeV energy is released.

v. Fusion process requires very high temperature of about $10^{5o}C$. These reactions are called thermonuclear reactions. The energy generated in stars is because of nuclear fusion.

Q.61. Write notes on –

a. Nuclear fission

b. Nuclear fusion

Ans: a. Nuclear fission :

i. The process of splitting a heavy nucleus into two lighter nuclei after bombardment with neutrons is called nuclear fission.

ii. The process of nuclear fission was first discovered by German scientists Otto Hahn and Strassman in 1939.

iii. Example of nuclear fission:

$${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{236}_{92}U \rightarrow {}^{144}_{56}Ba + {}^{89}_{36}Kr + 3{}^{1}_{0}n$$

In the above reaction, when ${}^{235}_{92}U$ is bombarded by neutron, it breaks up into two intermediate fragments which emit $\beta-$particles to achieve stable end products.

iv. The energy released in fission first appears as K.E which gets converted into heat in surrounding.

v. Fission energy is being used in nuclear power projects for generation of electricity.

vi. The uncontrolled fission process is used in atom bomb.

b. Nuclear fusion

i. The nuclear reaction in which two lighter nuclei are fused to form a heavier nucleus is called nuclear fusion.

ii. The newly formed nucleus has smaller mass than the sum of masses of fused nuclei.

iii. The mass defect is converted into energy.

iv. For example,

$${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{4}_{2}He + 24MeV$$

When two deuterons get fused, u– particle is emitted and 24 MeV energy is released.

v. Fusion process requires very high temperature of about $10^{5o}C$. These reactions are called thermonuclear reactions. The energy generated in stars is because of nuclear fusion.

Q.62. Distinguish between nuclear fission and

HENRY CLASSES

nuclear fusion.

Ans:

No.	Nuclear fission	Nuclear fusion
i.	It is the process in which a heavy nucleus splits up into two lighter nuclei of nearly equal masses.	It is the process in which two lighter nuclei combine together to form a heavy nucleus.
ii.	About 200 MeV energy available per fission of $^{235}_{92}U$.	Energy available per fusion is much less but the energy per unit mass of material is much greater than that for fission of heavy nuclei.
iii.	Nuclear fission may take place at ordinary temperature.	A very high temperature of the order of million of degree is required.
iv.	The sources of fissionable materials is limited.	The sources or fusion reaction i.e., hydrogen is more plentiful (air and water).
v.	The products of nuclear fission are in general radioactive and hence pose a radiation hazard.	The products of fusion are non-radioactive and pose no radiation hazard.

18.7 : de Broglie's hypothesis

Q.63. Explain de Broglie hypothesis of matter wave. Find. expression for de Broglie wavelength of matter wave.

Ans: de Broglie matter wave:

i. In 1924, Louis de Broglie suggested that if radiant energy has both the wave nature and particle nature, then particle (matter) must have wave associated with its motion.

ii. He believed that energy and matter must have some symmetrical character.

Expression for de Broglie wavelength of matter wave:

i. Wavelength associated with a particle of mass 'm' moving with velocity 'v' is given by $\lambda = \frac{h}{p} = \frac{h}{mv}$ .... (i)

where, h = Planck's constant
p = mv = momentum of particle

ii. According to Planck's quantum theory,

$$E = h\nu = \frac{hc}{\lambda} \quad .... (ii)$$

where, c = speed of light
ν = frequency of light
λ = wavelength of light

iii. From Einstein's mass energy relation,
$E = mc^2$ .... (iii)

iv. Comparing equation (ii) and (iii),
we have,

$$mc^2 = h\nu = \frac{hc}{\lambda}$$

$$\therefore \quad mc = \frac{h}{\lambda}$$

$$\therefore \quad \lambda = \frac{h}{mc} = \frac{h}{p} \quad .... (iv)$$

where, p = mc = momentum of photon
Equation (iv) represents de Broglie wavelength of matter wave.

Q.64. Explain the de Broglie concept of matter waves.

Ans: de Broglie concept of matter wave:

i. According to de Broglie, every moving particle is associated with a wave of wavelength given by,

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

These waves are called matter waves.

ii. As $v \to 0$, $\lambda \to \infty$ and as $v \to \infty$, $\lambda \to 0$.
This implies that, matter waves are associated with material particles only if they are moving.

iii. Greater the momentum of the particle, the shorter is the wavelength.

iv. Matter waves travel faster than light. The velocity of matter wave is not constant because it depends upon the velocity of particle.

v. The de Broglie wavelength is independent of the charge of particle.

Q.65. State the conclusion drawn from the de Broglie equation of matter wave.

Ans: Conclusion of de Broglie equation:

According to de Broglie equation, $\lambda = \frac{h}{mv}$.

HENRY CLASSES

From the formula following conclusions can be drawn out.

i. Lighter the particle, greater is its de Broglie wavelength.
ii. The faster the particle moves, the smaller is its de Broglie wavelength.
iii. The de Broglie wavelength is independent of charge or nature of the particle.
iv. The matter waves are not electromagnetic in nature. The electromagnetic waves are produced only by accelerated charged particles.
v. The de Broglie wavelength can be measured in the case of subatomic particles like electrons, protons etc. but cannot be measured in the case of heavy bodies.
vi. Two velocities are associated with a material particle in motion, i.e, linear velocity and the velocity of matter wave but they have different values.
vii. The energy is carried by the moving particles like electrons and not by the matter waves associated with it.
viii. Matter waves are different from the mechanical waves as well as the electromagnetic waves.

Q.66. Explain Bohr's second postulate on the basis of de Broglie hypothesis.

Ans: i. The distance travelled by electron in one complete revolution in nthorbit of radius r_n is $2\pi r_n$. It should be integral multiple of wavelength.

$$2\pi r_n = n\lambda \quad ...(i)$$

where, n = 1,2,3,4 ....

ii. By de Broglie hypothesis,

$$\lambda = \frac{h}{p} = \frac{h}{mv_n}$$

iii. Substituting this value of 'λ' in equation (i), we get,

$$2\pi r_n = n\frac{h}{mv_n}$$

$$\therefore \quad mv_n r_n = \frac{nh}{2\pi} = \text{angular momentum of electron.}$$

Hence, Bohr's postulate is verified.

18.8 : Wavelength of an electron

Q.67. On the basis of de Broglie hypothesis, obtain the relation for wavelength of an electron accelerated by a potential difference of 'V' volt.

[Feb 13old course]

OR

Derive an expression for the de Broglie wavelength associated with electron accelerated from rest through a certain potential difference 'V'.

[Oct 08]

Ans: de Broglie wavelength of electrons:

i. Consider an electron of mass 'm' and charge 'e' at rest accelerated through a potential difference of, V' volt.'
ii. The work done by the electron in the electric field increases its K.E. Energy of electron is given by, E = e V

$$\therefore \quad E = \frac{1}{2}mv^2 = eV$$

$$\therefore \quad mv^2 = 2\,eV = 2E$$

$$\therefore \quad m^2v^2 = 2mE$$

$$\therefore \quad mv = \sqrt{2mE} = \sqrt{2meV} \quad .... (i)$$

iii. From de Broglie relation,

$$\lambda = \frac{h}{mv} \quad ...(ii)$$

Substituting equation (i) in equation (ii), we get,

$$\lambda = \frac{h}{\sqrt{2meV}} \quad ...(iii)$$

Equation (iii) gives the wavelength of electron in de Broglie wave.

iv. Substituting the numerical values of h, m and e in equation (iii), we get,

$$\lambda = \frac{6.63\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.6\times10^{-19}\times V}}$$

$$= \sqrt{\frac{6.63\times6.63\times10^{-68}}{2\times9.1\times10^{-31}\times1.6\times10^{-19}\times V}}$$

$$\therefore \quad \lambda = \sqrt{\frac{150}{V}}\times10^{-10}\text{ m}$$

$$\therefore \quad \lambda = \sqrt{\frac{150}{V}}\text{ Å} = \frac{12.27}{\sqrt{V}}\text{ Å}$$

Note:

i. If an electron is accelerated with P.D of 100 volt, then de–Broglie wavelength will be 1.22 Å.

2. Lighter particles have a larger measure of de–Broglie wavelength.

3. For fast moving particles, de Broglie wavelength is small.

4. Mass 'm' in the de Broglie relation is the relativistic mass of the particle.

5. de Broglie formula for wavelength of electron $\lambda = \frac{h}{\sqrt{2meV}}$ is useful to determine wavelength at low voltage only. At high voltage (in kV), the electron velocity becomes very large and electron momentum has to be calculated relativistically.

6. When the velocity of a particle is comparable to the velocity of light, then its mass is called relativistic mass and is given by,

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

where, mo is the rest mass of the particle and c is the velocity of light.

18.9 : Davisson and Germer experiment

Q.68. Describe Davisson and Germer experiment to explain the wave nature of electron.

Ans: Davisson and Germer Experiment:

i. C.J Davisson and L.H. Germer in 1927 performed an experiment to explain indirect proof of matter waves associated with electron. Diffraction effect of waves is well explained and well understood on the basis of wave theory. In a similar way electrons produce diffraction effect.

ii. Experimental arrangement is as shown in fig. (a). It consists of an electron gun, a crystal holder and an electron detector. Whole apparatus is enclosed in an evacuated chamber.

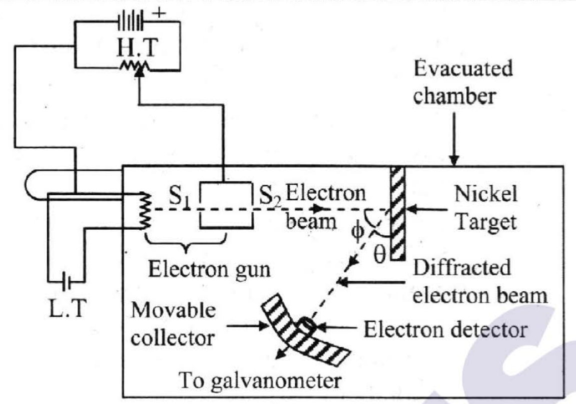


Figure (a) Davisson's experimental setup

iii. The electron beam is produced by thermionic emission from a tungsten filament coated with barium oxide and heated by a low tension battery.

iv. Electrons emitted by the filament are accelerated to a desired velocity by applying suitable potential from a high tension variable battery to narrow slits S_1 and S_2.

v. Electrons are collimated to a fine beam by S_1 and S_2. The collimated fine beam is allowed to incident on nickel target. The electrons are scattered in all directions of nickel atoms.

vi. In detecting system, scattered electrons are collected by electron detector called collector, which is connected to sensitive galvanometer (not shown in figure).

vii. The detector can be rotated on a circular scale. The deflection of the galvanometer is proportional to the intensity of the electron beam entering the collector.

viii. By rotating the detector on the circular scale at different positions, the intensity of the scattered electron beam is measured for different values of scattering angle ϕ which is the angle between the incident and the scattered electron beam.

ix. The variation of the intensity I of the scattered electrons with the angle of scattering ϕ is obtained for different accelerating voltages. A bump or a kink occurs in the curve at $\phi = 50°$ as shown in fig. (b).

HENRY CLASSES

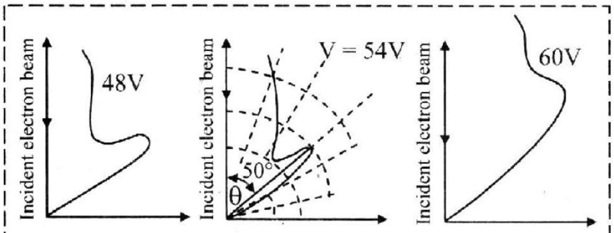


Figure (b) Intensity of scattered electrons

The size of the bump becomes maximum, when accelerating voltage is 54 V.

×. The appearance of the peak in a particular direction is due to the constructive interference of electrons scattered from different layers of the regularly spaced atoms of the crystal.

xi. According to Bragg's diffraction formula for 1st order diffraction maxima,

$\lambda = 2\,d \sin\theta$

where, d = spacing between atomic planes

λ = wavelength of associated wave

θ = glancing angle

xii. For scattering angle, $\phi = 50°$, glancing angle is, e = 65° (glancing angle $90° - \phi/2$) and spacing between atomic planes for nickel is, d = 0.91 Å.

$\lambda = 2 \times 0.91 \times 10^{-10} \times \sin 65°$

$\lambda = 1.65$ Å ....(i)

This is wavelength of electron wave found experimentally.

xiii. By using de Broglie relation, the wavelength of electron at accelerating voltage 54 V is given by,

$$\lambda = \frac{12.27}{\sqrt{V}}\text{ Å} = \frac{12.27}{\sqrt{54}} = 1\ 67\text{ Å} \quad ...(ii)$$

This is in excellent agreement between experimental and theoretical value of wavelength of electron. This explains the wave nature of electron.

Q.69. Draw a neat labelled diagram of Davisson and Germer experiment, for diffraction of electron wave. [Mar 10, 14]

Ans: Refer Q.68. (only fig. (a))

Q.70. State the importance of Davisson and Germer experiment.

Ans: Importance of Davisson and Germer experiment:

i. The Davisson and Germer experiment was the first observed phenomenon of diffraction of electrons.

ii. It directly indicates the wave nature of material particles.

iii. The experiment verified the de Broglie hypothesis for the existence of matter waves.

Summary :

1. Bohr's atomic model is the modified version of Rutherford's model which is based on the principle of quantum theory.
2. Bohr's first postulate states that electron revolves in a fixed circular orbit. Centripetal force is provided by electrostatic force between nucleus and electron.
3. Angular momentum of electron in a fixed circular orbital is given by, $L = mvr = \frac{nh}{2\pi}$ wehre, n = i ,2,3 (principal quantum number).
4. Electron radiates energy when it jumps from higher energy orbit to lower energy orbit.
5. Radius of circular orbit in which electron revolves is given by $r = \left(\frac{\varepsilon_0 h^2}{\pi m e^2}\right) n^2$ i.e. $r \propto n^2$.
6. Total energy of electron in n^{th} Bohr's orbit is given by $E_n = \left[\frac{-me^4}{8\varepsilon_0{}^2 h^2}\right] \times \frac{1}{n^2}$ i.e $E_n \propto \frac{1}{n^2}$
7. Maximum number of spectral lines obtained on account of transition of electron present in n^{th} orbit to various lower orbits is given by $\frac{n(n-1)}{2}$.
8. The basic formula for wave number is given by $\frac{1}{\lambda} = R\left(\frac{1}{p^2} - \frac{1}{n^2}\right)$ where R is Rydberg's constant.
9. Wavelength will be minimum for $n = \infty$ and it will be maximum for the nearest number which is 1st member of series.
10. Relation between energy (E_n) and Rydberg constant (R) is given by $E_n = \frac{-Rhc}{n^2}$
11. If the original number of nuclei present in a radio active sample is N_0 and its halflife is T, then

HENRY CLASSES

a. number of nuclei left after n half lives

$$N = N_0 \left(\frac{1}{2}\right)^n \text{ and}$$

b. number of half lives in t seconds

$$n = \frac{t}{T}$$

12. de Broglie hypothesis is based on two observations:

a. The whole energy in the universe is in the form of electromagnetic radiations and matter.

b. As nature loves symmetry, so the two physical quantities i.e. matter and energy must be symmetrical.

13. Like radiation, matter also has dual nature. The wave associated with matter is called matter wave. The wavelength is given by,

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

14. Davisson and Germer experiment was the first experiment which practically shows the wave nature of particles used to find out the wavelength of electron.

$$\lambda = \frac{12.27}{\sqrt{V}},$$

where, V = Applied potential difference.

Formulae :

1. Angular momentum:

$$L = mvr = \frac{nh}{2\pi}$$

2. Energy difference between two successive energy level:

$$\Delta E = E_2 - E_1 = h\nu$$

3. Radius of nth Bohr orbit : $r_n = \frac{\varepsilon_0 n^2 h^2}{\pi m e^2}$

i.e. $r_n \propto n^2$

$$\therefore \frac{r_1}{r_2} = \left(\frac{n_1}{n_2}\right)^2$$

4. Energy of nth Bohr orbit:

i. $E_n = -\frac{me^4}{8\varepsilon_0^2 n^2 h^2}$

i.e. $E_n \propto \frac{1}{n^2}$

$$\therefore \frac{E_1}{E_2} = \left(\frac{n_2}{n_1}\right)^2$$

ii. $E_n = \frac{-Rch}{n^2}$

iii. $E_n = \frac{E_1}{n^2}$

5. Period of revolution in nth Bohr orbit:

$$T_n = \frac{4\varepsilon_0^2 h^3}{me^4} n^3 \text{ i.e., } T \propto n^3$$

6. Frequency of revolution in nth Bohr orbit:

$$f_n = \frac{v}{2\pi r} = \frac{1}{T} = \left(\frac{me^4}{4\varepsilon_0^2 h^3}\right)\frac{1}{n^3} \text{ i.e. } f \propto \frac{1}{n^3}$$

7. Velocity of electron in nth orbit:

$$v_n = \frac{e^2}{2\varepsilon_0 nh} \text{ i.e., } v_n \propto \frac{1}{n^4}$$

8. Centripetal acceleration of electron in n^{th} orbit:

$$a_n = \left(\frac{\pi me^6}{4\varepsilon_0^3 h^4}\right)\frac{1}{n^4} \text{ i.e. } a_n \propto \frac{1}{n^4}$$

9. Angular velocity of electron in n^{th} orbit:

$$\omega_n = \left(\frac{\pi me^4}{2\varepsilon_0^2 h^3}\right)\frac{1}{n^3} \text{ i.e. } \omega \propto \frac{1}{n^3}$$

10. Linear momentum of electron in nth orbit:

$$p_n = mv_n = \left(\frac{me^2}{2\varepsilon_0 h}\right)\frac{1}{h} \text{ i.e. } p \propto \frac{1}{h}$$

11. Wave number:

$$\bar{v} = \frac{1}{\lambda} = R\left(\frac{1}{p^2} - \frac{1}{n^2}\right)$$

12. K.E of n^{th} orbit:

$$K.E = -\frac{me^4}{8\varepsilon_0^2 n^2 h^2} = -\frac{Rch}{n^2} = \frac{-13.6}{n^2}\text{eV}$$

where, $R = \frac{me^4}{8\varepsilon_0^2 ch^3}$ i.e., $K.E \propto \frac{1}{n^2}$

13. Potential energy in n^{th} orbit:

$$P.E = \frac{me^4}{4\varepsilon_0^2 n^2 h^2} \text{ i.e., } P.E \propto \frac{1}{n^2}$$

14. Relation between K.E, P.E and T.E:

HENRY CLASSES

i. P.E = –2 K.E
ii. P.E = 2 T.E
iii. T.E = P.E + K.E

15. Minimum (cut–off) wavelength for ×–rays:

$$\lambda_{min} = \frac{ch}{E}$$

16. Radius of a nucleus:

$$R = R_0 A^{\frac{1}{3}}$$

17. Mass defect:

$$\Delta m = [Zm_p + (A - Z)\, m_n] - M$$

18. Binding energy:

$$B.E = \Delta mc^2 J = \frac{\Delta mc^2}{e} eV$$

19. Binding energy per nucleon:

$$E = \left[\frac{Zm_p + (A - Z)m_n - M}{A}\right] c^2 \text{ J/nucleon}$$

20. Radioactive decay:

i. For α – decay: ${}^A_Z X \xrightarrow{-\alpha} {}^{A-4}_{Z-2} Y$

ii. For β – decay: ${}^A_Z X \xrightarrow{-\beta} {}^{A}_{Z+1} Y + \overline{v}_e$

21. Decay law:

$$\frac{dN}{dt} = -\lambda N$$

22. Number of undecayed nuclei left after time t:

$$N = N_0 e^{-\lambda t}$$

23. Half life period:

$$T_{1/2} = \frac{0.693}{\lambda}$$

24. de Broglie wavelength:

i. $\lambda = \frac{h}{\sqrt{2meV}}$

ii. $\lambda = \frac{h}{mv} = \frac{h}{p}$

Solved Problems :

Example 1

A difference of 2.3 eV separates two energy levels in an atom. What is the frequency of radiation emitted, when the atom makes a transition from the upper level to the lower level? (NCERT)

Solution:

Given : $E = 2.3\ eV = 2.3 \times 1.6 \times 10^{-19}$ J $= 3.68 \times 10^{-19}$ J

To find: Frequency of radiation (v)

Formula : E = hv

Calculation : From formula,

$$v = \frac{E}{h} = \frac{3.68 \times 10^{-19}}{6.63 \times 10^{-34}}$$

$\therefore$ $v = \mathbf{5.55 \times 10^{14}\ Hz}$

Ans: The frequency of radiatioo emitted, when the atom makes a transition from the upper level to the lower level is **5.55×10^{14} Hz**.

Example 2

The radius of the first orbit of electron in H–atom is 0.5 Å. What will be the radius of the third orbit?

Solution:

Given : $r_1 = 0.5$ Å $= 0.5 \times 10^{-10}$ m

To find : Radius of third orbit (r_3)

Formula : $r_n = \left(\frac{\varepsilon_0 h^2}{\pi me^2}\right) n^2$

Calculation : From formula,

$$\therefore \frac{r_3}{r_1} = \left(\frac{n_3}{n_1}\right)^2$$

$$\therefore r_3 = \left(\frac{n_3}{n_1}\right)^2 \times r_1$$

$$= \left(\frac{3}{1}\right)^2 (0.5 \times 10^{-10})$$

$\therefore$ $r_3 = 4.5 \times 10^{-10}$ m $=$ **4.5 Å**

Ans: The radius of the third orbit is **4.5 Å**.

Example 3

Calculate the radius of second Bohr orbit in hydrogen atom from the given data.
Mass of electron = 9.1×10^{-31} kg
Charge on the electron = 1.6×10^{-19} C
Planck's constant = 6.63×10^{-34} J–s
Permittivity of free space = 8.85×10^{-12} C²/Nm²
[Mar 14]

Solution:

Given : $m = 9.1 \times 10^{-31}$ kg,
$e = 1.6 \times 10^{-19}$ C,
$h = 6.63 \times 10^{-34}$ J–s,
$\varepsilon_0 = 8.85 \times 10^{-12}$ C²/Nm², n = 2

To find : Radius of 2nd Bohr orbit (r_2)

HENRY CLASSES

Formula : $r_n = \dfrac{\varepsilon_0 n^2 h^2}{\pi m e^2}$

Calculation : From formula,

$$r_2 = \frac{8.85\times10^{-12}\times22\times(6.63\times10^{-34})^2}{3.14\times9.1\times10^{-31}\times(1.6\times10^{-19})^2}$$

$$= \frac{8.85\times4\times(6.63)2\times10^{-12}\times10^{-68}}{3.14\times9.1\times(1.6)^2\times10^{-31}\times10^{-38}}$$

$$= \frac{8.85\times4\times(6.63)2}{3.14\times9.1\times(1.6)^2}\times10^{-11}$$

$$= 21.27 \times 10^{-11}$$

$$= 2.127 \times 10^{-10}$$

$\therefore\ r_2 = \mathbf{2.127\ Å}$

Ans: Radius of second Bohr orbit is **2.127 Å**.

Example 4

Energy of the electron in the first Bohr orbit is –13.6 eV. Calculate energy of the electron in third orbit.

Solution:

Given : $E_1 = -13.6$ eV

To find : Energy of electron in third orbit (E_3)

Formula : $E_n = -\left(\dfrac{me^4}{8\varepsilon_0{}^2h^2}\right)\dfrac{1}{n^2}$

Calculation : From formula,

$$\frac{E_3}{E_1} = \left(\frac{n_1}{n_3}\right)^2$$

$$\therefore\ E_3 = \left(\frac{n_1}{n_3}\right)^2 \times E_1 = \left(\frac{1}{3}\right)^2(-13.6)$$

$$\therefore\ E_3 = \frac{-13.6}{9} = \mathbf{-1.51\ eV}$$

Ans: Energy of the electron in third orbit is **–1.51 eV**.

Example 5

Calculate the energy of the second Bohr orbit from given data
[m = 9.1 × 10⁻³¹ kg, e = 1.6 × 10⁻¹⁹ C,
h = 6.63 × 10⁻³⁴ Js, ε_0 = 8.85 × 10⁻¹² SI units]

Solution:

Given : $m = 9.1 \times 10^{-31}$ kg, n = 2,
$e = 1.6 \times 10^{-19}$ C,
$\varepsilon_0 = 8.85 \times 10^{-12}$ SI units,
$h = 6.63 \times 10^{-34}$ Js

To find : Energy of electron in second orbit (E_2)

Formula : $E_n = \dfrac{-me^4}{8\varepsilon_0{}^2n^2h^2}$

Calculation : From formula,

$$E_2 = \frac{-9.1\times10^{-31}\times(1.6\times10^{-19})^4}{8\times(8.85\times10^{-12})^2\times(2)^2\times(6.63\times10^{-34})^2}$$

$$= \frac{-9.1\times1.6\times1.6\times1.6\times1.6\times10^{-107}}{32\times8.85\times8.85\times6.63\times6.63\times10^{-92}}$$

$$= \frac{-9.1\times1.6\times1.6\times1.6\times1.6}{32\times8.85\times8.85\times6.63\times6.63}\times10^{-15}$$

$\therefore\ E_2 = -5.41 \times 10^{-19}$ J

$$= \frac{5.41\times10^{-19}}{1.6\times10^{-19}}\text{ eV} = \mathbf{-3.38\ eV}$$

Ans: The energy of the second Bohr orbit is **–3.38 eV.**

Example 6

Find the angular momentum of the electron in the first orbit in a hydrogen atom.
[h = 6.63 × 10⁻³⁴ Js]

Solution:

Given : $h = 6.63 \times 10^{-34}$ Js, n = 1

To find : Angular momentum of electron in 1st orbit (L_1)

Formula : $L_n = n\dfrac{h}{2\pi}$

Calculation : From formula,

$$L_1 = 1\frac{h}{2\pi} = \frac{6.63\times10^{-34}}{2\times3.14}$$

$$= \frac{6.63}{6.28}\times10^{-34}$$

$\therefore\ L_1 = 1.056 \times 10^{-34}$ kg m²/s

Ans: The angular momentum of the electron in the first orbit in a hydrogen atom is **1.056 × 10⁻³⁴ kg m²/s**.

Example 7

Find the value of energy of electron in eV in the third Bohr orbit of hydrogen atom.
[Rydberg's constant (R) = 1.097 × 10⁷ m⁻¹,
Planck's constant (h) = 6.63 × 10⁻³⁴ J – s,
Velocity of light in air (c) = 3 × 10⁸ m/s.]
[Feb 13]

HENRY CLASSES

Solution:

Given : $R = 1.097 \times 10^7\ m^{-1}$, $h = 6.63 \times 10^{-34}$ J – s, $c = 3 \times 10^8$ m/s, $n = 3$, $e = 1.6 \times 10^{-19}$ C

To find : Energy of electron in 3rd orbit (E_3)

Formula : $E = \frac{-hRC}{n^2}$

Calculation : From Formula,

$$E_3 = \frac{-6.63\times10^{-34}\times1.097\times10^7\times3\times10^8}{9\times1.6\times10^{-19}}$$

$\therefore$ E_3 = **1.515 eV**

Ans: The value of energy of electron in the third Bohr orbit of hydrogen atom is **1.515 eV**.

Example 8

Calculate the value of the linear momentum of the electron in the first Bohr orbit of the hydrogen atom, if the radius of that orbit is 5.3×10^{-11} m.

Solution:

Given : $r_1 = 5.3 \times 10^{-11}$ m

To find : Linear momentum of electron in 1st orbit (P_1)

Formula : $P_n = \frac{nh}{2\pi r_n}$

Calculation : From formula, For 1" orbit, n = 1,

$$p_1 = \frac{h}{2\pi r_n} = \frac{6.63\times10^{-34}}{2\times3.14\times5.3\times10^{-11}}$$

$$= \frac{6.63}{6.28\times5.3}\times10^{-23}$$

p_1 = **1.992×10^{-24} kg m/s**

Ans: The value of the linear momentum of the electron in the first Bohr orbit of the hydrogen atom is **1.992×10^{-24} kg m/s**.

Example 9

The period of revolution of the electron in the third orbit in a hydrogen atom is 4.132×10^{-15} s. Find the period in the fourth orbit.

Solution:

Given : $T_1 = 4.132 \times 10^{-15}$ s, $n_1 = 3$, $n_2 = 4$

To find : Period of electron in 4th orbit (T_2)

Formula : $\frac{T_1}{T_2} = \left(\frac{n_1}{n_2}\right)^3$

Calculation : From formula,

$$T_2 = T_1\left(\frac{n_2}{n_1}\right)^3$$

$$= 4.132 \times 10^{-15} \times \left(\frac{4}{3}\right)^3$$

$$= 4.132 \times 10^{-15} \times \frac{64}{27}$$

$\therefore$ T_2 = **9.794×10^{-15} s**

Ans: The period of revolution of the electron in the fourth orbit is **9.794×10^{-15} s**

Example 10

Calculate the frequency of the revolution of the electron in the first Bohr orbit of hydrogen atom.

[Given: r_1 = 0.5 Å, $v_1 = 2.245 \times 10^6$ m/s]

[Mar 94, Oct 98, 06]

Solution:

Given : r_1 = 0.5 Å, $v_1 = 2.245 \times 10^6$ m/s

To find : Frequency of revolution in 1st orbit (v_1)

Formula : $v = \frac{v}{2\pi r}$

Calculation : From formula,

$$v_1 = \frac{v_1}{2\pi r_1} = \frac{2.245\times10^6}{2\times3.14\times0.5\times10^{-10}}$$

$\therefore$ v_1 = **7.149×10^{15} Hz**

Ans: The frequency of the revolution of the electron in the first Bohr orbit of hydrogen atom is **7.149×10^{15} Hz**

Example 11

Energy of the electron in the first Bohr orbit is –13.6 eV. Calculate energy of the electron in third orbit.

Solution:

Given : $E_1 = -13.6$ eV

To find : Energy of electron in third orbit (E_3)

Formula : $E_n = -\left(\frac{me^4}{8\varepsilon_0{}^2h^2}\right)\frac{1}{n^2}$

Calculation : From formula,

HENRY CLASSES

$$\frac{E_3}{E_1} = \left(\frac{n_1}{n_3}\right)^2$$

$$\therefore \quad E_3 = \left(\frac{n_1}{n_3}\right)^2 \times E_1 = \left(\frac{1}{3}\right)^2 (-13.6)$$

$$\therefore \quad E_3 = \frac{-13.6}{9} = \mathbf{-1.51\ eV}$$

Ans: Energy of the electron in third orbit is **–1.51 eV**.

Example 12

Calculate the change in angular momentum of electron when it jumps from 3rd orbit to 1st orbit in hydrogen atom.

Solution:

Given : $n_1 = 1$ (first orbit), $n_2 = 3$ (third orbit)

To find : Change in angular momentum $(L_3 - L_1)$

Formula : $L = mvr = \frac{nh}{2\pi}$

Calculation : From formula,

$$L_1 = mv_1r_1 = \frac{n_1h}{2\pi}$$

$$L_3 = mv_3r_3 = \frac{n_3h}{2\pi}$$

$$\therefore \quad L_3 - L_1 = mv_3r_3 - mv_1r_1$$

$$= \frac{n_3h}{2\pi} - \frac{n_1h}{2\pi} = \frac{h}{2\pi}(n_3 - n_1)$$

$$= \frac{h}{2\pi}(3-1) = \frac{h}{\pi} = \frac{6.63\times10^{-34}}{3.142}$$

$$\mathbf{L_3 - L_1 = 2.11 \times 10^{-34}\ kg\ m^2/s}$$

Ans: The change in angular momentum of electron when it jumps from 3rd orbit to is 1st orbit in hydrogen atom is **2.11×10^{-34} kg m²/s**

Example 13

Find the frequency of revolution of electron in 2nd Bohr orbit if the radius and the speed of electron in that orbit are 2.14×10^{-10} m and 1.09×10^6 m/s respectively.

Solution:

Given : $r_2 = 2.14 \times 10^{-10}$ m, n = 2, $v_2 = 1.09 \times 10^6$ m/s

To find : Frequency of revolution (ν_2)

Formula : $\nu = \frac{v}{2\pi r}$

Calculation : From formula,

$$\nu_2 = \frac{v_2}{2\pi r_2} = \frac{1.09\times10^6}{2\times3.142\times2.14\times10^{-10}}$$

$$\nu_2 = \mathbf{8.11 \times 10^{14}\ Hz}$$

Ans: The frequency of revolution of electron in 2nd Bohr orbit is **8.11×10^{14} Hz.**

Example 14

A hydrogen atom initially in the ground level absorbs a photon, which excites it to the n = 4 level. Determine the wavelength and frequency of photon.

Solution:

Given : $E_1 = -13.6$ eV, n = 4

To find : i. Wavelength (λ)

ii. Frequency (ν)

Formula : i. $E = h\nu$ ii. $c = \nu\lambda$

Calculation : Since $E_n = -\frac{13.6}{n^2}$

$$\therefore \quad E_4 = -\frac{13.6}{4^2} = -0.85\ \text{eV}$$

$$\Delta E = E_4 - E_1$$
$$= -0.85 - (-13.6) = 12.75\ \text{eV}$$
$$= 12.75 \times 1.6 \times 10^{-19}$$
$$= 2.04 \times 10^{-18}\ \text{J}$$

From formula (i),

$$\nu = \frac{\Delta E}{h} = \frac{2.04\times10^{-18}}{6.63\times10^{-34}}$$

$$\therefore \quad \nu = \mathbf{3.08 \times 10^{15}\ Hz}$$

From formula (ii),

$$\lambda = \frac{c}{\nu} = \frac{3\times10^8}{3.08\times10^{15}}$$

$$\therefore \quad \lambda = \mathbf{9.74 \times 10^{-8}\ m}$$

Ans: i. Wavelength of the photon is **9.74×10^{-8} m**

ii. Frequency of the photon is **3.08×10^{15} Hz.**

Example 15

Calculate the linear velocity and frequency of revolution of an electron in the first Bohr orbit of hydrogen atom [Radius of first Bohr orbit = 0.53×10^{-10} m and m = 9.1×10^{-31} kg]

Solution:

Given : $r_1 = 0.53 \times 10^{-10}$ m, $m = 9.1 \times 10^{-31}$ kg

HENRY CLASSES

To find : i. Linear velocity (v)
ii. Frequency (ν)

Formulae : i. $v = \dfrac{nh}{2\pi mr}$ ii. $\nu = \dfrac{v}{2\pi r}$

Calculation : From formula (i),

$$v = \frac{1 \times 6.63 \times 10^{-34}}{2 \times 3.14 \times 9.1 \times 10^{-31} \times 0.53 \times 10^{-10}}$$

$$= \frac{6.63}{2 \times 3.14 \times 9.1 \times 0.53} \times 10^{7}$$

v = **2.188×10^{6} m/s**

From formula (ii),

$$\nu = \frac{2.188 \times 10^{6}}{2 \times 3.14 \times 0.53 \times 10^{-10}}$$

$$= \frac{2.188 \times 10^{16}}{2 \times 3.142 \times 0.53}$$

∴ ν = **6.574×10^{15} Hz**

Ans: i. The linear velocity of an electron in the first Bohr orbit of hydrogen atom is **2.188×10^{6} m/s.**

ii. The frequency of revolution of an electron in the first Bohr orbit of hydrogen atom is **6.574×10^{15} Hz.**

Example 16

The velocity of electron in the first Bohr orbit of radius 0.5 A.U. is 2.24×10^{6} m/s. Calculate the period.of revolution of electron in the same orbit. [Mar 09]

Solution :

Given : $r = 0.5$ Å $= 0.5 \times 10^{-10}$
$m = 5 \times 10^{-11}$ m,

To find : Period of revolution (T)

Formula : $T = \dfrac{2\pi r}{v}$

Calculation : Using formula (i) we get,

$$T = \frac{2 \times 3.14 \times 5 \times 10^{-11}}{2.24 \times 10^{6}}$$

$$= \frac{31.4}{2.24} \times 10^{-17} = 14.02 \times 10^{-17}$$

T = **1.402×10^{-16} s**

Ans: Period of revolution of electron in the first Bohr orbit is **1.402×10^{-16} s.**

Example 17

i. Using the Bohr's model, calculate the speed of the electron in a hydrogen atom in the n = 1, 2 and 3 levels.

ii. Calculate the orbital period in each of these levels. (NCERT)

Solution:

Given : $n_1 = 1, n_2 = 2, n_3 = 3$

To find : i. Speeds of electrons in 1st, 2nd and 3rd levels (v_1 v_2,v_3)

ii. Orbital periods in 1st 2nd and 3rd levels (T_1 T_2, T_3)

Formula : i. $v_n = \dfrac{e^2}{2\varepsilon_0 hn}$

ii. $T = \dfrac{4\varepsilon_0^{\,2} h^3 n^3}{me^4}$

Calculation :

i. From formula (i),

$$v_1 = \frac{(1.6 \times 10^{-19})^2}{2 \times 8.85 \times 10^{-12} \times 6.63 \times 10^{-34} \times 1}$$

∴ v_1 = **2.181×10^{6} ms^{-1}**

Since the velocity of electron in an orbit of a hydrogen atom is inversely proportional to its quantum number 'n'.

$$v_2 = \frac{v_1}{2} = \frac{2.181 \times 10^{6}}{2} = \mathbf{1.091 \times 10^{6}\ ms^{-1}}$$

$$v_3 = \frac{v_1}{3} = \frac{2.181 \times 10^{6}}{3} = \mathbf{7.27 \times 10^{5}\ ms^{-1}}$$

From formula (ii),

$$T_1 = \frac{4\varepsilon_0^{\,2} h^3 n^3}{me^4}$$

$$= \frac{4 \times (8.85 \times 10^{-12})^2 \times (6.63 \times 10^{-34})^3 \times (1)^3}{9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^4}$$

∴ T_1 = **1.53×10^{-16} s**

As orbital period of an electron in H–atom is directly proportional to the cube of its quantum number.

∴ $T_2 = 2^3 \times T_1 = 8 \times 1.53 \times 10^{-16}$

∴ $T_2 = 1.22 \times 10^{-15}$ s

$T_3 = 3^3 \times T_1 = 27 \times 1.53 \times 10^{-16}$

∴ $T_3 = 4.13 \times 10^{-15}$s

Ans: i. The speeds of the electron in a hydrogen atom in the n = 1, 2 and 3 levels are **2.181×10^{6} ms^{-1}, 1.091×10^{6} ms^{-1} and 7.27×10^{5} ms^{-1}** respectively.

ii. The orbital periods of the electron in a hydrogen atom in the n = 1, 2 and 3 levels are **1.53×10^{-16} s, 1.22×10^{-15} s and**

HENRY CLASSES

4.13 × 10^{-15} s respectively.

Example 18

Calculate the energy radiated by the electron in the hydrogen atom during its transition from the third Bohr orbit to the first Bohr orbit. Hence determine frequency, wavelength and wave number of the corresponding spectral line.
[Given: E_1 = –13.6 eV, h = 6.63 × 10^{-34} Js, c = 3 × 10^8 m/s]

Solution:

Given : $E_1 = -13.6$ eV,
$h = 6.63 \times 10^{-34}$ Js,
$c = 3 \times 10^8$ m/s

To find : i. Energy radiated (E)
ii. Frequency (v)
iii. Wavelength (λ)
iv. Wave number (v)

Formulae : i. $\Delta E = E_3 - E_1$

ii. $v = \frac{\Delta E}{h}$

iii. $c = v\lambda$

iv. $\bar{v} = \frac{1}{\lambda}$

Calculation : Since $E_n = \frac{E_1}{n^2}$

For 3rd orbit, n = 3

$\therefore E_3 = \frac{E_1}{(3)^2} = \frac{-13.6}{9}$

$\therefore E_3 = -1.511$ eV

Energy radiated is given by,

$\Delta E = E_3 - E_1$
$= -1.511 - (-13.6)$
$= 13.6 - 1.511$
$= 12.08$ eV
$= 12.08 \times 1.6 \times 10^{-19}$

ΔE = **19.328 × 10^{-19} J**

From formula(ii),

$$v = \frac{19.328 \times 10^{-19}}{6.63 \times 10^{-34}}$$

$$= \frac{19.328}{6.63} \times 10^{15}$$

$\therefore$ v = **2.92 × 10^{15} Hz**

From formula (iii),

$$\lambda = \frac{c}{v}$$

$$= \frac{3 \times 10^8}{2.92 \times 10^{15}} = \frac{3}{2.92} \times 10^{-7}$$

$= 1.027 \times 10^{-7}$ m

λ = **1027 Å**

From formula (iv),

$$\bar{v} = \frac{1}{1.027 \times 10^{-7}}$$

$$= \frac{1}{1.027} \times 10^7$$

$\bar{v}$ = **9.74 × 10^6 m^{-1}**

Ans: i. The energy radiated by the electron in the hydrogen atom during its transition from the third Bohr orbit to the first Bohr orbit is **19.328 × 10^{-19} J.**

ii. The frequency of the corresponding spectral line is **2.92 × 10^{15} Hz.**

iii. The wavelength of the corresponding spectral line is **1027 Å**.

iv. The wave number of the corresponding spectral line is **9.74 × 10^6 m^{-1}**.

Example 28

A radioactive isotope has a half–life of T years. How long will it take to reduce to 3.125% of its original value? (NCERT)

Solution:

$$\frac{N}{N_0} = \frac{3.125}{100} = \frac{1}{32}$$

As $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$

$\therefore \frac{N}{N_0} = \left(\frac{1}{2}\right)^n$

$\therefore$ n = 5

Now t = nT
= 5T

Ans: It will take **5 T** years to reduce the radioactive isotope to 3.125% of its original value.

Example 31

Obtain the binding energy in MeV of a nitrogen nucleus ($^{14}_{7}N$). [Given: m ($^{14}_{7}N$) = 14.00307 u] (NCERT)

Solution:

HENRY CLASSES

Z = 7

A = 14

A – Z = 14 – 7 = 7

∴ Mass defect

$\Delta m = [Z.m_H + (A - Z) m_n - m_N]$ u

$= (7 \times 1.00783 + 7 \times 1.00867 - 14.00307)$ u

$= (7.0548 + 7 \times 06069 - 14.00307)$ u

$= 0.11243$ u

Since, 1 u = 931.5 MeV/c^2

∴ B.E of $^{14}_{7}N$ = 0.11243 × 931.5 MeV

∴ B.E = **104.7 MeV**

Ans: The binding energy of the nitrogen nucleus is **104.7 MeV**.

Example 32

Suppose India has a target of producing by 2020 A.D., 2 × 10^5 MW of electric power, ten percent of which is to be obtained from nuclear power plants. Suppose we are given that on an average, the efficiency of utilization (i.e., conversion to electrical energy) of thermal energy produced in a reactor is 25%. How much amount of fissionable uranium will our country need per year? 'fake the heat energy per fission of U^{235} to be about 200 MeV. (NCERT)

Solution:

Total targeted power = 2 × 10^5 MW

Total Nuclear Power = 10% of 2 × 10^5 MW

= 2 × 10^4 MW.

Energy produced/fission = 200 MeV

Efficiency of–power plant = 25%

Energy converted into electrical energy per fission $= \frac{25}{100} \times 200$

= 50 MeV

= 50 × 1.6 × 10^{-13} joule

Total electrical energy to be produced

= 2 × 10^4 MW

= 2 × 10^4 × 10^6 watt

= 2 × 10^{10} joule/s

= 2 × 10^{10} × 60 × 60 × 24 × 365 joule/year

Number of fissions in one year

$$= \frac{2 \times 10^{10} \times 60 \times 60 \times 24 \times 365}{50 \times 1.6 \times 10^{-13}}$$

$$= 2 \times \frac{36 \times 24 \times 365}{8} \times 10^{24}$$

Mass of 6.023 × 10^{23} atoms of

U^{235} = 235 g = 235 × 10^{-3} kg

Mass of $\frac{2 \times 36 \times 24 \times 365}{8} \times 10^{24}$ atoms

$$= \frac{235 \times 10^{-3}}{6.023 \times 10^{23}} \times \frac{2 \times 36 \times 24 \times 365 \times 10^{24}}{8}$$

≈ 3.08 × 10^4 kg.

Ans: The amount of fissionable uranium the country needs per year is **3.08 × 10^4 kg.**

Example 33

Calculate the de Broglie wavelength associated with an electron moving with a speed of 5 × 10^6 m/s. [m_e = 9.1 × 10^{-31} kg]

Solution:

Given : v = 5 × 10^6 m/s,

m_e = 9.1 × 10^{-31} kg

To find : de Broglie wavelength of electron (λ)

Formula : $\lambda = \frac{h}{mv}$

Calcualtion : From formula,

$$\lambda = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 5 \times 10^{6}}$$

= 0.1457 × 10^{-9} m

λ = **1.457 Å**

Ans: The de Broglie wavelength associated with an electron is **1.457 Å**.

Example 34

Find momentum of the electron having de Broglie wavelength of 0.5 Å.

Solution:

Given : λ = 0.5 Å = 0.5 × 10^{-10} m

To find : Momentum (p)

Formula : $\lambda = \frac{h}{p}$

Calculation : From formula,

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{0.5 \times 10^{-10}}$$

∴ p = 13.26 × 10^{-24} kg ms^{-1}

Ans: The momentum of the electron is **13.26 × 10^{-24} kg ms^{-1}**.

Example35

Find the wavelength of a proton accelerated by a potential difference of 50 V.
[Given: m_p = 1.673 × 10^{-27} kg]

HENRY CLASSES

Solution:

Given: $v = 50$ volt, $m_p = 1.673 \times 10^{-27}$ kg

To find: Wavelength (λ)

Formula: $$\lambda = \frac{h}{\sqrt{2m_p eV}}$$

Calculation: From formula,

$$\lambda = \frac{6.63\times10^{-34}}{\sqrt{2\times1.673\times10^{-27}\times1.6\times10^{-19}\times50}}$$

$$= \frac{6.63\times10^{-34}}{\sqrt{1.673\times1.6\times10^{-44}}}$$

$$= \frac{6.63\times10^{-34}\times10^{22}}{\sqrt{1.673\times1.6}}$$

$\therefore$ $\lambda = 0.04052 \times 10^{-10}$ m

$\therefore$ $\lambda =$ **0.04052 Å**

Ans: Wavelength of the proton is **0.04052 Å**.

Example 36

Calculate the de Broglie wavelength of waves associated with a beam of neutrons of energy 0.025 eVe [h = 6.63 × 10^{-34} Js, m, = 1.67 × 10^{-27} kg]

Solution:

Given: $E = 0.025$ eV $= 0.025 \times 1.6 \times 10^{-19}$ J

$= 0.04 \times 10^{-19}$ J,

$h = 6.63 \times 10^{-34}$ Js,

$m_n = 1.67 \times 10^{-27}$ kg

To find: de Broglie wavelength (λ)

Formula: $$\lambda = \frac{h}{\sqrt{2mE}}$$

Calculation: From formula,

$$\lambda = \frac{6.63\times10^{-34}}{\sqrt{2\times1.67\times10^{-27}\times0.04\times10^{-19}}}$$

$$= \frac{6.63\times10^{-34}}{0.3656\times10^{-23}} = 1.814 \times 10^{-10}\text{ m}$$

$\lambda =$ **1.814 Å**

Ans: The de Broglie wavelength of waves associated with the beam of neutrons is **1.814 Å**.

Example 37

Calculate the de Broglie wavelength of an electron moving with 1/3rd of the speed of light in vaccum. (Neglect relativistic effect) (Planck's constant: h = 6.63 × 10^{-34} Js, Mass of electron: m = 9 .11 × 10^{-28} g)

[Oct 14]

Solution:

Given: $$v = \frac{c}{3} = \frac{3\times10^8}{3} = 10^8\text{ m/s}$$

$h = 6.63 \times 10^{-34}$ Js,

$m = 9.11 \times 10^{-28}$

$g = 9.11 \times 10^{-31}$ kg

To find: de Broglie wavelength (λ)

Formula: $$\lambda = \frac{h}{mv}$$

Calculation: From formula,

$$\lambda = \frac{6.63\times10^{-34}}{9.11\times10^{-31}\times10^{8}}$$

$$= \frac{66.3}{9.11} \times 10^{-35} \times 10^{31} \times 10^{-8}$$

$$= \frac{66.3}{9.11} \times 10^{-12}$$

$\lambda =$ **7.278 × 10^{-12} m**

Ans: The de Broglie wavelength of electron is **7.278 × 10^{-12} m**.

Example 41

An electron in hydrogen atom stays in rd orbit for 10^{-8} s. How many revolutions will it make till it jumps to the ground state?

Solution:

Given: $t = 10^{-8}$ s

To find: Number of revolutions

Formula: $\Delta E = hv$

Calculation: From formula,

$$v = \frac{\Delta E}{h} = \frac{E_2 - E_1}{h}$$

$$= \frac{-3.4-(-13.6)}{6.63\times10^{-34}}$$

$$= \frac{10.2\times1.6\times10^{-19}}{6.63\times10^{-34}}$$

$\therefore$ $v = 2.46 \times 10^{15}$ Hz

$\therefore$ It means that electron takes 1s to complete **2.46 × 10^{15}** revolutions.

$\therefore$ In 10^{-8} s electron will complete **2.46 × 10^{15} × 10^{-8} i.e. 2.46 × 10^{7}** revolutions

Ans: The electron in hydrogen atom will make **2.46 × 10^{7}** revolutions till it jumps to the ground state.

HENRY CLASSES

Exercise :

Section A: Practice Problems

1. Calculate the radius of the second Bohr's orbit of the electron in the hydrogen atom.

2. The radius of the fifth Bohr's orbit in hydrogen atom is 132.5×10^{-11} m. What is the radius of the third orbit?

3. The speed of the electron in the first orbit is 2.182×10^{6} m/s. Find the speed of the electron in the third orbit.

4. Find the angular speed of the electron in the first orbit in a hydrogen atom. (h, e and ε_0 are given)

5. Find the period of revolution of the electron in the third orbit in a hydrogen atom.

6. Find the frequency of revolution of the electron in the third orbit in a hydrogen atom.

7. Find the wavelength of the photon emitted when the electron jumps from a state of energy –0.85 eV to another state of energy –10.2 eV.

8. The energy of the electron in the first orbit in a hydrogen atom is –13.6 eV. Find the energy of the photon emitted when the electron jumps from the fourth orbit to first orbit.

9. A frequency of H, line in the Balmer series in Hydrogen spectrum is 6.914×10^{14} Hz. Find the frequency of the H, line in the Balmer senes.

10. Find the de Broglie wavelength of a proton of energy 1 eV.

    [Mass of Proton = 1.673×10^{-27} kg]

11. Calculate de Broglie wavelength for a beam of neutrons of energy 0.10 eV.

    [$h = 6.62 \times 10^{-34}$ Js, $m = 1.67 \times 10^{-27}$ kg]

12. If the radius of third Bohr's orbit in Hydrogen atom is 47.7×10^{-11} m. What will be the radius of the first Bohr's orbit?

13. Find the longest wavelength in the P fund senes.

    [Given that Rydberg constant is 1.097×10^{7} m^{-1}]

14. The·energy of the electron in a hydrogen atom in its ground state is –13.6 eV. Calculate Rydberg's constant and the wavelength of the series limit of the Pashcen series.

15. Calculate the de Broglie wavelength of' electrons accelerated through a p.d. of 25 kV.

    [$e = 1.6 \times 10^{-19}$ C, $m_e = 9.1 \times 10^{-31}$ kg,

    $h = 6.63 \times 10^{-34}$ Js]

16. Calculate the wavelength and frequency of the first line of the Lyman series.

17. Rydberg's constant for hydrogen is 1.097×10^{7} m^{-1}. Calculate the energy of the electron in the ground state and the wavelength of the second line of the Balmer senes.

18. Determine the angular momentum and the linear momentum of an electron in the second Bohr's orbit of a hydrogen atom.

19. A radioative isotope × has a half life of 6 seconds. At t = 0, a given sample of this isotope contains 16000 atoms. Calculate (i) its decay constant (ii) average life (iii) the time t, when, 103 atoms of the isotope X remain in the sample (iv) number of decays/see in the sample at t = t' sec.

Section B: Theoretical Board Questions

1. State the postulates of Bohr's theory of hydrogen atom. **[Mar 96]**

2. Derive an expression for the energy of the electron in the nth orbit of a hydrogen atom, on the basis of Bohr's theory. **[Oct 96]**

3. Rutherford model of the atom cannot be accepted due to certain drawbacks. State any two of them. Derive an expression for the radius of the nth Bohr's orbit in Hydrogen atom. Hence show that the radius of the orbit is directly proportional to the square of the principal quantum number.

   [Oct 97]

4. Show that the radius of the Bohr's .orbit is directly proportional to the square of the principal quantum number ..Also show that the energy of an electron in a stationary orbit is inversely proportional to the radius of the orbit. **[Mar'99, Oct 04]**

5. Draw the energy level diagram for the Hydrogen atom and hence explain the different series in the Hydrogen atom spectrum. **[Mar 2000]**

6. Derive an expression for the energy of the electron revolving in the stationary orbit. Draw the energy level diagram of hydrogen atom. Show Balmer and Paschen series in it.

   [Oct 2000]

HENRY CLASSES

Exercise :

Section A: Practice Problems

1. Calculate the radius of the second Bohr's orbit of the electron in the hydrogen atom.

2. The radius of the fifth Bohr's orbit in hydrogen atom is 132.5×10^{-11} m. What is the radius of the third orbit?

3. The speed of the electron in the first orbit is 2.182×10^{6} m/s. Find the speed of the electron in the third orbit.

4. Find the angular speed of the electron in the first orbit in a hydrogen atom. (h, e and ε_0 are given)

5. Find the period of revolution of the electron in the third orbit in a hydrogen atom.

6. Find the frequency of revolution of the electron in the third orbit in a hydrogen atom.

7. Find the wavelength of the photon emitted when the electron jumps from a state of energy –0.85 eV to another state of energy –10.2 eV.

8. The energy of the electron in the first orbit in a hydrogen atom is –13.6 eV. Find the energy of the photon emitted when the electron jumps from the fourth orbit to first orbit.

9. A frequency of H, line in the Balmer series in Hydrogen spectrum is 6.914×10^{14} Hz. Find the frequency of the H, line in the Balmer senes.

10. Find the de Broglie wavelength of a proton of energy 1 eV.

    [Mass of Proton = 1.673×10^{-27} kg]

11. Calculate de Broglie wavelength for a beam of neutrons of energy 0.10 eV.

    [$h = 6.62 \times 10^{-34}$ Js, $m = 1.67 \times 10^{-27}$ kg]

12. If the radius of third Bohr's orbit in Hydrogen atom is 47.7×10^{-11} m. What will be the radius of the first Bohr's orbit?

13. Find the longest wavelength in the P fund senes.

    [Given that Rydberg constant is 1.097×10^{7} m^{-1}]

14. The·energy of the electron in a hydrogen atom in its ground state is –13.6 eV. Calculate Rydberg's constant and the wavelength of the series limit of the Pashcen series.

15. Calculate the de Broglie wavelength of' electrons accelerated through a p.d. of 25 kV.

    [$e = 1.6 \times 10^{-19}$ C, $m_e = 9.1 \times 10^{-31}$ kg,

    $h = 6.63 \times 10^{-34}$ Js]

16. Calculate the wavelength and frequency of the first line of the Lyman series.

17. Rydberg's constant for hydrogen is 1.097×10^{7} m^{-1}. Calculate the energy of the electron in the ground state and the wavelength of the second line of the Balmer senes.

18. Determine the angular momentum and the linear momentum of an electron in the second Bohr's orbit of a hydrogen atom.

19. A radioative isotope × has a half life of 6 seconds. At t = 0, a given sample of this isotope contains 16000 atoms. Calculate (i) its decay constant (ii) average life (iii) the time t, when, 103 atoms of the isotope X remain in the sample (iv) number of decays/see in the sample at t = t' sec.

Section B: Theoretical Board Questions

1. State the postulates of Bohr's theory of hydrogen atom. **[Mar 96]**

2. Derive an expression for the energy of the electron in the nth orbit of a hydrogen atom, on the basis of Bohr's theory. **[Oct 96]**

3. Rutherford model of the atom cannot be accepted due to certain drawbacks. State any two of them. Derive an expression for the radius of the nth Bohr's orbit in Hydrogen atom. Hence show that the radius of the orbit is directly proportional to the square of the principal quantum number.

   [Oct 97]

4. Show that the radius of the Bohr's .orbit is directly proportional to the square of the principal quantum number ..Also show that the energy of an electron in a stationary orbit is inversely proportional to the radius of the orbit. **[Mar'99, Oct 04]**

5. Draw the energy level diagram for the Hydrogen atom and hence explain the different series in the Hydrogen atom spectrum. **[Mar 2000]**

6. Derive an expression for the energy of the electron revolving in the stationary orbit. Draw the energy level diagram of hydrogen atom. Show Balmer and Paschen series in it.

   [Oct 2000]

HENRY CLASSES

7. For the innermost orbit of an electron, the binding energy is maximum. Explain. **[Oct 03]**

8. State the postulates of Bohr's theory in H–spectrum. Show that the linear velocity of the electron is inversely proportional to the principal quantum number. **[Oct 03]**

9. Derive Bohr's formula for hydrogen atom. State the spectral regions of Lyman and Paschen series in hydrogen spectrum. **[Oct 04]**

10. Derive an expression for energy of electron in stationary orbit in hydrogen atom. **[Mar 05]**

11. Explain Lyman, Balmer and Paschen series with the help of energy level diagram. **[Feb 06]**

12. From the expression for total energy of the electron in hydrogen atom, obtain the expression for wave number of the spectral line of hydrogen atom. From above expression explain the emission of Balmer's series. **[Oct 06]**

13. State de–Broglie hypothesis of matter waves and derive an expression for de–Broglie wavelength. **[Mar 09]**

14. State Bohr's third postulate for hydrogen atom and hence derive Bohr's formula for wave number. **[Mar 10]**

15. Draw a neat labelled energy level diagram of first four series for hydrogen atom. **[Oct 11]**

Section C : Numerical Board Problems

1. The shortest wavelength line in Lyman series is 911.3 Å. Cafculate the wavelength of the shortest wavelength line in Balmer and Paschen series. **[Mar 95, Feb 03]**

2. The radius of 2nd Bohr's orbit of an electron in H–atom is 2.12 Å. Calculate the radius of 3rd orbit of the same atom. **[Mar 97]**

3. The energy of the electron in the first Bohr's orbit is –13.6 eV. Calculate

   i. Rydberg's constant and

   ii. Energy of the electron in the third orbit. **[Oct 98]**

4. Calculate the smallest wavelength of a line in Lyman series. **[Mar 99]**

5. Calculate the energy of the electron in the ground state of hydrogen atom. **[Oct 02]**

6. Find the mass of a photon of the radiation having wavelength of 5000 Å. **[Feb 04]**

7. Calculate the change in the angular momentum of the electron when it jumps from the fourth orbit to the first orbit in a hydrogen atom. **[Oct 04]**

8. Find the energy of the electron in second Bohr orbit of hydrogen atom [Energy of an electron in the first Bohr orbit = –13.6 eV] **[Mar 08]**

9. Calculate the de–Broglie wavelength of proton if it ,is moving with a speed of 2×10^5 m/s. **[Oct 09]**

10. The wavelength of H, line of Balmer series of Hydrogen spectrum is 6563 Å. Find the

    (a) wavelength of H, line of Balmer series.

    (b) shortest wavelength of Brackett series. **[Mar 11]**

11. The shortest wavelength for Lyman series is 912 A.U. Find shortest wavelength for Paschen and Brackett series in Hydrogen atom. **[Mar 12]**

HENRY CLASSES

Multipal Choice Questions

1. The radius of hydrogen atom in the second excited state is
a) 2.12 Å b) 4.77 Å
c) 1.59 Å d) 3.18 Å

2. If the orbital velocity of the electron in the first orbit of H–atom is 2.2×10^6 m/s, then its orbital velocity in the second orbit is given by
a) 1.1×10^6 m/s b) 4.4×10^6 m/s
c) $\sqrt{2.2\times10^6}$ m/s d) 1.1×10^3 m/s

3. An electron jumps from the 4th orbit to the 2nd orbit of hydrogen atom. Given the Rydberg's constant R = 10^5 cm^{-1}, the frequency in Hz of the emitted radiation will be
a) $\frac{3}{16} \times 10^5$ b) $\frac{3}{16} \times 10^{15}$
c) $\frac{9}{16} \times 10^{15}$ d) $\frac{3}{4} \times 10^{15}$

4. The de–Broglie wavelength of a particle of K.E 'K' is λ. What would be the wavelength of the particle, if its K.E was K/4?
a) $\frac{\lambda}{4}$ b) 2λ
c) 4λ d) $\frac{\lambda}{2}$

5. The period of revolution of electron in the third orbit in a H–atom is 4.132×10^{-15} s. Hence the period in the fourth orbit is
a) 9.794×10^{-15} s b) 9.794×10^{-14} s
c) 9.974×10^{-15} s d) 9.974×10^{-14} s

6. In Davisson and Germer experiment, the function of nickel crystal is
a) to absorb the incident beam of electrons.
b) to diffract the incident beam of electrons.
c) to interfere the incident beam of electrons.
d) to refract the incident beam of electrons.

7. Let 'p' and 'E' denote the linear momentum and energy of emitted photon respectively. If the wavelength of incident radiation is increased,
a) both p and E increase
b) p increase's and E decreases
c) p decreases and E increases
d) both p and E decrease

8. The angular momenta of electrons in an atom produces
a) magnetic moment b) Zeeman effect
c) light d) nuclear fission

9. Angular speed of an electron in Bohr's orbit is given by
a) $\omega = \frac{\pi m e^4}{2\varepsilon_0^2 n^3 h^3}$ b) $\omega = \frac{4\varepsilon_0^2 n^3 h^3}{m e^4}$
c) $\omega = \frac{m e^4}{4\varepsilon_0^2 n^3 h^3}$ d) all of these

10. What will be the ratio of the de Broglie wavelengths associated with an α–particle and proton accelerated through the same potential difference?
a) 1 : 2 b) 1 :2$\sqrt{2}$
c) 2 : 1 d) 2$\sqrt{2}$: 1

11. The linear momentum of the electron in the ground state of H–atom is 2×10^{-24} kg m/s, its linear momentum in the 8th orbit is
a) 2.5×10^{-25} kg m/s
b) 5.2×10^{-25} kg m/s
c) 5.2×10^{-15} kg m/s
d) 2.5×10^{-15} kg m/s

12. If the wavelength of the first line of Balmer series of hydrogen atom is 6561 Å, the wavelength of the second line of the series will be
a) 3575 Å b) 3860 Å
c) 4500 Å d) 4860 Å

13. The idea of matter waves was given by
a) Davisson and Germer
b) de–Broglie
c) Einstein
d) Planck

14. An electron is accelerated from rest between two points at which the potentials are 20 V and 40 V respectively. The de–Broglie wavelength associated with the electron will be
a) 7.5 Å b) 2.75 Å
c) 0.275 Å d) 0.75 Å

15. The shortest wavelength of spectral line in Lyman senes is 912 Å. The shortest wavelength of the spectral line of the Paschen series is

a) 8208 Å b) 8028 Å
c) 8828 Å d) 8820 Å

16. The wavelength of de–Broglie waves is 2 μm, then its momentum is ($h = 6.63 \times 10^{-34}$ Js)
a) 3.315×10^{-28} kg m/s
b) 1.66×10^{-28} kg m/s
c) 4.97×10^{-28} kg m/s
d) 9.9×10^{-28} kg m/s

17. In the hydrogen atom spectrum, the series which lies in ultraviolet region is
a) Lyman series b) Balmer series
c) Paschen series d) Brackett series

18. Davisson and Germer experiment proved
a) wave nature of light
b) particle nature of light
c) matter wave of proton
d) matter wave of electron

19. The radius of Bohr orbit r is proportional to
a) principal quantum number (n)
b) square of principle quantum number (n^2)
c) 1/n
d) $1/n^2$

20. The energy of an electron in nth Bohr orbit is proportional to
a) n^2 b) n
c) $\frac{1}{n}$ d) $\frac{1}{n^2}$

21. Energy of the lowest level of hydrogen atom is –13.6 eV. The energy of the photon emitted in the transition–from n = 3 to n = 1 is
a) –27 eV b) –9 eV
c) –3 eV d) –12.09 eV

22. What is the momentum of a photon in radiation of frequency 10^{14} Hz?
a) 3×10^{-20} kg m/s
b) 2.21×10^{-28} kg m/s
c) 6.63×10^{-30} kg m/s
d) 3.315×10^{-28} kg m/s

23. The ground state energy of H–atom is 13.6 eV. The energy needed to ionise H–atom from its second excited state is
a) 1.51 eV b) 3.4 eV
c) 13.6 eV d) 12.1 eV

24. The wavelength of second member of Balmer series of H–atom has a wavelength of 4860 Å. What will be the wavelength of fourth member of the series?
a) 3509 Å b) 4522 Å
c) 4101 Å d) 3961 Å

25. The ratio of longest wavelength and the shortest wavelength as observed in the five spectral series or emission spectrum of hydrogen is
a) $\frac{4}{3}$ b) $\frac{525}{376}$
c) 25 d) $\frac{900}{11}$

26. The wavelength λ of de Broglie waves associated with an electron (mass m, charge e) accelerated through a P.D of V is
a) $\lambda = \frac{h}{mV}$ b) $\lambda = \frac{h}{\sqrt{meV}}$
c) $\lambda = \frac{h}{\sqrt{2meV}}$ d) $\lambda = \frac{h}{2meV}$

27. According to Bohr's theory, the relation between the period of revolution of electron and principle quantum number is
a) $T \propto 1/n^2$ b) $T \propto 1/n^3$
c) $T \propto n^2$ d) $T \propto n^3$

28. de Broglie wavelength of a particle having momentum p is given by
a) $h \times p$ b) h/p
c) $h\sqrt{p}$ d) $h/\sqrt{p}$

29. The radioactive nucleus may emit
a) all the three α, β and γ radiations simultaneously.
b) all the three α, β and γ one after the other.
c) only α, β simultaneously.
d) only one among α, β or γ at a time.

30. If ω is the angular speed of electron in the n^{th} orbit of the hydrogen atom, then
a) $\omega \propto n^{1/2}$ b) $\omega \propto \frac{1}{n}$
c) $\omega \propto \frac{1}{n^2}$ d) $\omega \propto \frac{1}{n^3}$

31. If the radius of the first orbit of the hydrogen atom is 5.29×10^{-11} metre, the radius of the second orbit will be
a) 21.16×10^{-11} m b) 15.87×10^{-11} m

HENRY CLASSES

c) 10.58×10^{-11} m d) 2.64×10^{11} m

32. The radius of first Bohr orbit is 0.53 Å and radius of n^{th} Bohr orbit is 212 Å The value of 'n' is

a) 2 b) 12
c) 20 d) 400

33. The angular momentum of the electron in the second Bohr's orbit of a hydrogen atom is 'l'. Its angular momentum in the third Bohr's orbit is

a) $\frac{2}{3}l$ b) $\frac{3}{2}l$
c) $3\,l$ d) $\frac{4}{3}l$

34. Which of the following quantities has same units and dimensions as that of Planck's constant?

a) Moment of inertia
b) Angular momentum
c) Linear momentum
d) Rate of change of linear momentum

35. The longest wavelength in Lyman series IS 1240 Å, the highest frequency emitted In Balmer series is

a) 8×10^{14}Hz b) 3×10^{14}Hz
c) 8×10^{12}Hz d) 3×10^{15}Hz

36. The series limit wavelength of the Lyman series for the hydrogen atom is given by

a) $\frac{1}{R}$ b) $\frac{4}{R}$
c) $\frac{9}{R}$ d) $\frac{16}{R}$

37. In Davisson and Germer's experiment, a crystal which diffracts a beam of electrons is of

a) Sodium chloride b) Nickel
c) Silver d) Calcium chloride

38. The Lyman transitions involve

a) largest changes of energy
b) smallest changes of energy
c) largest changes of potential energy
d) smallest changes in potential energy

39. In the nucleus of $_{11}N^{23}$, the number of protons, neutrons and electrons is

a) 11,12,11 b) 23,12,11
c) 12, 11, 0 d) 23, 11, 12

40. The H_β line of the Balmer series has a wavelength 4860 Å. The wavelength of H_α , line will be

a) 6561 Å b) 6400 Å
c) 5960 Å d) 5800 Å

41. Wavelength of radiations emitted, when an electron jumps from a state A to C is 2000 Å and it is 6000 Å when the electron jumps from state B to state C. Wavelength of the radiations emitted, when an electron jumps from state A to B will be

a) 2000 Å b) 3000 Å
c) 4000 Å d) 6000 Å

42. When electron in hydrogen atom jumps from second orbit to first orbit, the wavelength of radiation emitted is λ. When electron jumps from third orbit to first orbit, the wavelength of emitted radiation would be _______

a) $\frac{27}{32}\lambda$ b) $\frac{32}{27}\lambda$
c) $\frac{2}{3}\lambda$ d) $\frac{3}{2}\lambda$

43. The energy in the ground state of hydrogen atom is –13.6 eV, then energy of 1^{st} excited state is

a) –6.5 eV b) – 3.4 eV
c) –1.51 eV d) – 0.27 eV

44. If the energy of an electron in the ground state of hydrogen atom is –13.6 eV. Its energy in 2^{nd} and 4^{th} orbit respectively will be

a) – 6.8 eV and – 3.4 eV
b) – 3.4 eV and – 0.85 eV
c) – 27.2 eV and – 54.4 eV
d) – 54.4 eV and – 217.2 eV

45. Energy of an electron in the third Bohr orbit is –1.51 eV. Its energy in the second Bohr orbit is

a) –1.51 eV b) + 1.51 eV
c) –3.4 eV d) + 3.4 eV

46. Velocity of the electron in the first Bohr orbit of hydrogen atom is 2.18×10^6 m/s. Its velocity in the third Bohr's orbit is

a) 7.27×10^5 m/s b) 2.4×10^5 m/s
c) 6.54×10^6 m/s d) 1.962×10^5 m/s

47. The time taken by an electron to complete one revolution in the first orbit of hydrogen atom of radius 0.53 A.u. with speed 2.18×10^6 m/s will be

a) 1.527×10^{-15}s b) 1.527×10^{-16}s
c) 1.527×10^{-17}s d) 1.527×10^{-18}s

HENRY CLASSES

48. The de Broglie wavelength of an electron beam accelerated through a P.D. of 60 V is
a) 1.58 Å b) 3.58 Å
c) 15.8 Å d) 12.26 Å

49. If the accelerating potential in Davisson and Germer experiment is 54 V, the de–Broglie wavelength of the electron is
a) 0.65 Å b) 1.67 Å
c) 2.65 Å d) 0.165 Å

50. Bohr's atom model is not based on
a) Rutherford's nuclear atom model
b) Planck's quantum theory
c) Photoelectric effect
d) Coulomb's law for positive and negative charges

51. Which of the following radiations will have maximum penetrating power?
a) X–rays b) β–rays
c) α–rays d) γ–rays

52. The composition of an α–particle can be expressed as
a) 1p + 1n b) 1p + 2n
c) 2p + 1n d) 2p + 2n

53. If 10% of a radioactive substance decays in 5 days, then the amount of the original materials left after 20 days is about
a) 60% b) 65%
c) 70% d) 75%

54. The half life of radium is 1600 years. The fraction of a sample of radium that would remain after 6400 year is
a) $\frac{1}{2}$ b) $\frac{1}{4}$
c) $\frac{1}{8}$ d) $\frac{1}{16}$

55. The half life of an element is 5d. How much time is required for the decay of 7/8 th of the sample?
a) 5 d b) 10 d
c) 15 d d) 35/8 d

56. An electron is accelerated under a potential difference of 54 V. The speed of electron will be
a) 4×10^5 m/s b) 6.46×10^5 m/s
c) 4.36×10^6 m/s d) 4.36×10^5 m/s

57. Bohr's atomic model is the modification of Rutherford's atomic model by the application of
a) Newton's theory
b) Huygen's theory
c) Maxwell's theory
d) Planck's quantum theory

58. What is the de–Broglie wavelength of α–particle accelerated through a potential difference 'V'?
a) $\frac{0.287}{\sqrt{V}}$Å b) $\frac{12.27}{\sqrt{V}}$Å
c) $\frac{0.101}{\sqrt{V}}$Å d) $\frac{0.202}{\sqrt{V}}$Å

59. The de–Broglie wavelength of 1 mg grain of sand blown by a wind at the speed of 20 m/s is _____ [h = 6.63×10^{-34} s^{-1} Unit]
a) 33.15×10^{-36} m b) 33.15×10^{-33} m
c) 33.15×10^{-30} m d) 33.15×10^{30} m

60. The nuclei having same number of protons but different number of neutrons are called
a) isobars b) α – particles
c) isotopes d) γ – particles

61. According to the Bohr's theory the wave number of shortest wavelength of a spectral line in Balmer series is [R = 1.1×10^7 m^{-1}]
a) 5.5×10^5 m^{-1} b) 4.4×10^7 m^{-1}
c) 2.75×10^6 m^{-1} d) 2.75×10^8 m^{-1}

62. Electrons in a certain energy level $n = n_1$, can emit 3 spectral lines. When they are in another energy level, $n = n_2$, they can emit 6 spectral lines. The orbital speed of the electrons in the two orbits are in the ratio.
a) 4 : 3 b) 3 : 4
c) 2 : 1 d) 1 : 2

63. For a Davisson Germer experiment, if the angle of diffraction be 52°, then what is the glancing angle?
a) 26° b) 64°
c) 52° d) 104°

64. The de–Broglie wavelength associated with the particle of mass 'm' moving with velocity 'v' is
a) h/mv b) mv/h
c) mh/v d) m/hv

65. What is the K.E of a neutron if de Broglie wavelength associated with it is 1.4 A?
a) 0.013 eV b) 0.049 eV
c) 0.93 eV d) 0.042 eV

66. The wavelength of the matter wave is independent of
a) mass b) velocity
c) momentum d) charge

HENRY CLASSES

Answer Keys

1. b)	2. a)	3. c)	4. b)	5. a)	6. b)	7. d)	8. a)	9. a)	10. b)
11. a)	12. d)	13. b)	14. b)	15. a)	16. a)	17. a)	18. d)	19. b)	20. d)
21. d)	22. b)	23. a)	24. c)	25. c)	26. c)	27. d)	28. b)	29. d)	30. d)
31. a)	32. c)	33. b)	34. b)	35. a)	36. a)	37. b)	38. a)	39. a)	40. a)
41. b)	42. a)	43. b)	44. b)	45. c)	46. a)	47. b)	48. a)	49. b)	50. c)
51. d)	52. d)	53. b)	54. d)	55. c)	56. c)	57. d)	58. b)	59. c)	60. c)
61. c)	62. a)	63. b)	64. a)	65. d)	66. d)				

Answers :

Section A

1. 2.127 Å

2. 4.77 Å

3. 7.273×10^{5} m/s

4. 4.01×10^{16} rad/s

5. 4.134×10^{-15} s

6. 2.42×10^{14} Hz

7. 1329 Å

8. 12.75.eV

9. 7.316×10^{14} Hz

10. 2.865×10^{-11} m

11. 0.906 Å

12. 5.3×10^{-11}

13. 74580 Å

14. 1.097×10^{7} m^{-1}, 8204 Å

15. 7.77×10^{-12} m

16. 1215 Å, 2.469×10^{15} Hz

17. –13.6 eV, 4862 Å

18. 2.11×10^{-34} kg m^2/s, 9.925×10^{-25} kg m/s

19. i. 0.116 s^{-1}
ii. 8.66 s
iii. 24 s
iv. 116 s^{-1}

Section C

1. 3.654×10^{-7} m, 8.201×10^{-7} m

2. 4.77 Å

3. i. 1.094×10^{7} m^{-1}
ii. –1.511 eV

4. 911 Å

5. $E_1 = -13.6$ eV

6. 3.167×10^{-34} kg m^2/s

7. 4.42×10^{-36} kg

8. –3.4 eV

9. 0.0198 Å

10. 4340.6 Å, 14584.44 Å

11. 8208 Å, 14592 Å

HENRY CLASSES

Q.26. Do the same exercise as above with the replacement of the earlier transformer by a 40,000-220 V step-down transformer (Neglect, as before, leakage losses though this may not be a good assumption any longer because of the very high voltage transmission involved). Hence, explain why high voltage transmission is preferred?

Ans. Therefore, RMS current in the line, $I = \frac{(800 \times 10^3)}{40000}$

$$I = 20\ A$$

Total resistance of the two wire line, $R = 2 \times 15 \times 0.5 = 15\Omega$

(a) Line Power loss $= I^2R = 20^2 \times 15 = 60 \times 10^2\ W = 6kW$
(b) Power supply to the plant= 800 kW + 6 kW= 806 kW
(c) Voltage drop on the line = I R = 20 x 15 = 300 V
The step-up transformer is 440 V – 40 V=300 V. It is clear that percentage power loss is greatly reduced by high voltage transmission. In Question 25, this power loss is (600/1400) × 100 = 43%. Here it is only (6/806) × 100 = 0.74%.

HENRY CLASSES