

# Work, Power & Energy

## EXERCISE

### 1.0 : INTRODUCTION

We use the term 'work' in everyday conversation to mean many different things. We talk about going to work, doing homework, working in class. Physicists mean something very specific when they talk about work.

In Physics we use the term work to describe the process of transferring energy from object or system to another or converting energy from one form to another.

You will learn that work and energy are closely related to Newton's laws of motion. You shall see that the energy of an object is its capacity to do work and doing work is the process of transferring energy from one object or form to another by means of a force.

In other words,

n an object with lots of energy can do lots of work.

n when object A transfers energy to object B, the energy of object A decreases by the same amount as the energy of object B increases, we say that object A does work on object B.

Lifting objects or throwing them requires that you do work on them. Even making an electrical current flow requires that something do work. Objects or systems must have energy to be able to do work on other objects or systems by transferring some of their energy.

However, their meaning differ from the meaning we get from scientific definitions.

In physics, work is defined as a force causing the movement or displacement of an object.

Power is the rate of doing work or of transferring heat, i.e. the amount of energy transferred or converted per unit time

Energy is the ability to do work.

**Q.1. The sign of work done by a force on a body is important to understand. State carefully if the following quantities are positive or negative:**

**(a) work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket.**

**(b) work done by the gravitational force in the above case,**

**(c) work done by friction on a body sliding down an inclined plane,**

**(d) work done by an applied force on a body moving on a rough horizontal plane with uniform velocity,**

**(e) work done by the resistive force of air on a vibrating pendulum in bringing it to rest**

**Ans:** (a) It is clear that the direction of both the force and the displacement are the same and thus the work done on it is positive.

(b) It can be noted that the displacement of the object is in an upward direction whereas, the force due to gravity is in a downward direction. Hence, the work done is negative.

(c) It can be observed that the direction of motion of the object is opposite to the direction of the frictional force. So, the work done is negative.

(d) The object which is moving in a rough horizontal plane faces the frictional force which is opposite to the direction of the motion. To maintain a uniform velocity, a uniform force is applied on the object. So, the motion of the object and the applied force are in the same direction. Thus, the work done is positive.

(e) It is noted that the direction of the bob and the resistive force of air which is acting on it are in opposite directions. Thus, the work done is negative.

**Q.2. A body of mass 2 kg initially at rest moves under the action of an applied horizontal force of 7 N on a table with the coefficient of kinetic friction = 0.1. Compute the**

**(a) work done by the applied force in 10 s,**

**(b) work done by friction in 10 s,**

(c) work done by the net force on the body in 10 s,

(d) change in kinetic energy of the body in 10 s,

Ans: The mass of the body = 2 kg

Horizontal force applied = 7 N

Coefficient of kinetic friction = 0.1

Acceleration produced by the applied force,  $a_1 = F/m = 7/2 = 3.5 \text{ m/s}^2$

Force of friction,  $f = \mu R = \mu mg = 0.1 \times 2 \times 9.8$

Retardation produced by friction,  $a_2 = -f/m = -1.96/2 = -0.98$

Net acceleration with which the body moves

$a = a_1 + a_2 = 3.5 - 0.98 = 2.52$

Distance moved by the body in 10 seconds,

$s = ut + (1/2)at^2 = 0 + (1/2) \times 2.52 \times (10)^2 = 126 \text{ m}$

(a) The time at which work has to be determined is  $t = 10 \text{ s}$

Work = Force x displacement

$= 7 \times 126 = 882 \text{ J}$

(b) Work done by the friction in 10 s

$W = -f \times s = -1.96 \times 126$

(c) Work done by the net force in 10 s

$W = (F - f)s = (7 - 1.96) \times 126 = 635 \text{ J}$

(d) From  $v = u + at$

$v = 0 + 2.52 \times 10 = 25.2 \text{ m/s}$

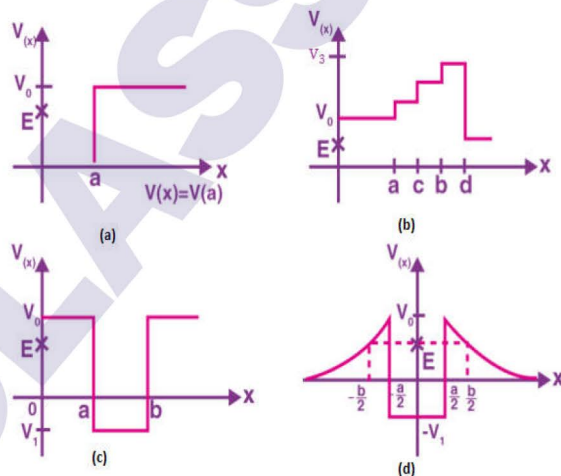
Final Kinetic Energy =  $(1/2)mv^2 = (1/2) \times 2 \times (25.2)^2 = 635 \text{ J}$

Initial Kinetic Energy =  $(1/2)mu^2 = 0$

Change in Kinetic energy =  $635 - 0 = 635 \text{ J}$

The work done by the net force is equal to the final kinetic energy

**Q.3. Given in Figure, are examples of some potential energy functions in one dimension. The total energy of the particle is indicated by a cross on the ordinate axis. In each case, specify the regions, if any, in which the particle cannot be found for the given energy. Also, indicate the minimum total energy the particle must have in each case. Think of simple physical contexts for which these potential energy shapes are relevant.**



Ans: The total energy is given by  $E = \text{K.E.} + \text{P.E.}$

$\text{K.E.} = E - \text{P.E.}$

Kinetic energy can never be negative. The particle cannot exist in the region, where K.E. would become negative.

(a) For the region  $x = 0$  and  $x = a$ , potential energy is zero. So, kinetic energy is positive. For,  $x > a$ , the potential energy has a value greater than  $E$ . So, kinetic energy becomes zero. Thus the particle will not exist in the region  $x > a$ .

The minimum total energy that the particle can have in this case is zero.

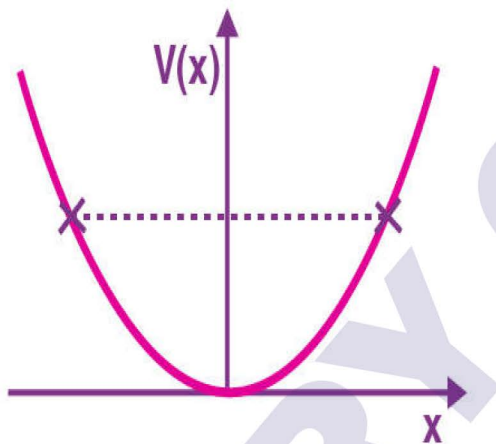
(b) For the entire  $x$ -axis,  $\text{P.E.} > E$ , the kinetic energy of the object would be negative. Thus the

particle will not exist in this region.

(c) Here  $x = 0$  to  $x = a$  and  $x > b$ , the P.E. is greater than  $E$ , so the kinetic energy is negative. The object cannot exist in this region.

(d) For  $x = -b/2$  to  $x = -a/2$  and  $x = a/2$  to  $x = b/2$ . Kinetic energy is positive and the P.E.  $< E$ . The particle is present in this region.

**Q.4. The potential energy function for a particle executing linear simple harmonic motion is given by  $V(x) = kx^2/2$ , where  $k$  is the force constant of the oscillator. For  $k = 0.5 \text{ N m}^{-1}$ , the graph of  $V(x)$  versus  $x$  is shown in Fig. 6.12. Show that a particle of total energy 1 J moving under this potential must 'turn back' when it reaches  $x = \pm 2 \text{ m}$ .**



**Ans:** Particle energy  $E = 1 \text{ J}$

$$K = 0.5 \text{ N m}^{-1}$$

$$\text{K.E} = \frac{1}{2} mv^2$$

Based on law of conservation of energy:

$$E = V + K$$

$$1 = \frac{1}{2} kx^2 + mv^2$$

Velocity becomes zero when it turns back

$$1 = \frac{1}{2} kx^2$$

$$\frac{1}{2} \times 0.5x^2 = 1$$

$$x^2 = 4$$

$$x = \pm 4$$

Thus, on reaching  $x = \pm 2 \text{ m}$ , the particle turns back.

**Q.5. Answer the following:**

(a) **The casing of a rocket in flight burns up due to friction. At whose expense is the heat energy required for burning obtained? The rocket or the atmosphere?**

**Ans.** When the casing burns up due to the friction, the rocket's mass gets reduced.

As per the law of conservation of energy:

Total energy = kinetic energy + potential energy

$$= mgh + \frac{1}{2} mv^2$$

There will be a drop in total energy due to the reduction in the mass of the rocket. Hence, the energy which is needed for the burning of the casing is obtained from the rocket.

(b) **Comets move around the sun in highly elliptical orbits. The gravitational force on the comet due to the sun is not normal to the comet's velocity in general. Yet the work done by the gravitational force over every complete orbit of the comet is zero. Why?**

**Ans.** The force due to gravity is a conservative force. The work done on a closed path by the conservative force is zero. Hence, for every complete orbit of the comet, the work done by the gravitational force is zero.

(c) **An artificial satellite orbiting the earth in a very thin atmosphere loses its energy gradually due to dissipation against atmospheric resistance, however small. Why then does its speed increase progressively as it comes closer and closer to the earth?**

**Ans.** The potential energy of the satellite revolving the Earth decreases as it approaches the Earth and since the system's total energy should remain constant, the kinetic energy increases. Thus, the satellite's velocity increases. In spite of this, the



total energy of the system is reduced by a fraction due to the atmospheric friction.

(d) In Figure the man walks 2 m carrying a mass of 15 kg on his hands. In Fig., he walks the same distance pulling the rope behind him. The rope goes over a pulley, and a mass of 15 kg hangs at its other end. In which case is the work done greater?



**Ans: Scenario I:**

$$m = 20 \text{ kg}$$

Displacement of the object,  $s = 4 \text{ m}$

$$W = Fs \cos \theta$$

$\theta =$  It is the angle between the force and displacement

$$Fs = mgs \cos \theta$$

$$W = mgs \cos \theta = 20 \times 4 \times 9.8 \cos 90^\circ$$

$$= 0 \quad (\cos 90^\circ = 0)$$

**Scenario II:**

Mass = 20 kg

$S = 4 \text{ m}$

The applied force direction is same as the direction of the displacement.

$$\theta = 0^\circ$$

$$\cos 0^\circ = 1$$

$$W = Fs \cos \theta$$

$$= mgs \theta$$

$$20 \times 4 \times 9.8 \cos 0^\circ$$

$$= 784 \text{ J}$$

Thus, the work done is more in the second scenario.

**Q.6. Underline the correct alternative :**

(a) When a conservative force does positive work on a body, the potential energy of the body increases/decreases/remains unaltered.

**Ans.** Decreases

When a body is displaced in the direction of the force, positive work is done on the body by the conservative force due to which the body moves to the centre of force. Thus, the separation between the two decreases and the potential energy of the body decreases.

(b) Work done by a body against friction always results in a loss of its kinetic/potential energy.

**Ans.** Kinetic energy

The velocity of the body is reduced when the work done is in the direction opposite to that of friction. Thus, the kinetic energy decreases.

(c) The rate of change of total momentum of a many-particle system is proportional to the external force/sum of the internal forces on the system

**Ans.** External force

Change in momentum cannot be produced by internal forces, irrespective of their directions. Thus, the change in total momentum is proportional to the external force on the system.

(d) In an inelastic collision of two bodies, the quantities which do not change after the collision is the total kinetic energy/total linear momentum/total energy of the system of two bodies

**Ans.** Total linear momentum

Irrespective of elastic collision or an inelastic collision, the total linear momentum remains the same.

**Q.7.** State if each of the following statements is true or false. Give reasons for your answer

**(a) In an elastic collision of two bodies, the momentum and energy of each body is conserved.**

**Ans.** False

The momentum and the energy of both the bodies are conserved and not individually.

**(b) The total energy of a system is always conserved, no matter what internal and external forces on the body are present.**

**Ans.** False.

The external forces on the system can do work on the body and are able to change the energy of the system.

$$\frac{1}{2}mv^2 + 0 = \frac{1}{2}mv^2$$

**(c) Work done in the motion of a body over a closed loop is zero for every force in nature.**

**Ans.** False.

The work done by the conservative force on the moving body in a closed loop is zero.

**(d) In an inelastic collision, the final kinetic energy is always less than the initial kinetic energy of the system.**

**Ans.** True

**Q.8.** Answer carefully, with reasons :

**(a) In an elastic collision of two billiard balls, is the total kinetic energy conserved during the short time of collision of the balls (i.e. when they are in contact)?**

**Ans.** The initial and the final kinetic energy is equal in an elastic collision. When the two balls collide, there is no conservation of kinetic energy. It gets

converted into potential energy.

**(b) Is the total linear momentum conserved during the short time of an elastic collision of two balls?**

**Ans.** The total linear momentum of the system is conserved in an elastic collision.

**(c) What are the answers to (a) and (b) for an inelastic collision?**

**Ans.** There will be a loss of kinetic energy in an inelastic collision. The K.E after the collision is always less than the K.E before the collision.

The total linear momentum of the system is conserved in an inelastic collision also.

**(d) If the potential energy of two billiard balls depends only on the separation distance between their centres, is the collision elastic or inelastic? (Note, we are talking here of potential energy corresponding to the force during a collision, not gravitational potential energy).**

**Ans.** It is an elastic collision as the forces involved are conservative forces. It depends on the distance between the centres of the billiard balls.

**Q.9.** A body is initially at rest. It undergoes one-dimensional motion with constant acceleration. The power delivered to it at time  $t$  is proportional to

(i)  $t^{\frac{1}{2}}$

(ii)  $t^{\frac{3}{2}}$

(iii)  $t^2$

(iv)  $t$

**Ans.** body mass =  $m$

Acceleration =  $a$

According to Newton's second law of motion:

$$F = ma \text{ (constant)}$$

We know that  $a = \frac{dv}{dt} = \text{constant}$

$$dv = dt \times \text{constant}$$

On integrating

$$v = \alpha t \rightarrow 1$$

The relation of power is given by:

$$P = F.v$$

From equation 1 & 2

$$p \propto t$$

Thus, from the above, we conclude that power is proportional to time.

**Q.10. A body is moving unidirectionally under the influence of a source of constant power. Its displacement in time  $t$  is proportional to**

(i)  $t^{\frac{1}{2}}$

(ii)  $t^{\frac{3}{2}}$

(iii)  $t^2$

(iv)  $t$

**Ans.** We know that the power is given by:

$$\begin{aligned} P &= Fv \\ &= mav = mv \frac{dv}{dt} \\ &= k \text{ (constant)} \end{aligned}$$

$$v dv = \frac{k}{m} dt$$

On integration:

$$\frac{v^2}{2} = \frac{k}{m} dt$$

$$v = \sqrt{\frac{2kt}{m}}$$

To get the displacement:

$$v = \frac{dx}{dt} = \sqrt{\frac{2k}{m}} t^{\frac{1}{2}}$$

$$dx = k^1 t^{\frac{1}{2}} dt$$

where  $k^1 = \sqrt{\frac{2k}{m}}$

$$x = \frac{2}{3} k^1 t^{\frac{3}{2}}$$

Hence, from the above equation it is shown that

$$x \propto t^{\frac{3}{2}}$$

**Q.11. A body constrained to move along the z-axis of a coordinate system is subject to a constant force  $F$  given by**

$$F = -\hat{i} + 2\hat{j} + 3\hat{k} \text{ N}$$

where  $\hat{i}$ ,  $\hat{j}$ ,  $\hat{k}$ , are unit vectors along the x- y- and z-axis of the system respectively. What is the work done by this force in moving the body at a distance of 4 m along the z-axis?

**Ans.** The body is displaced by 4 m along z-axis

$$\vec{S} = 0\hat{i} + 0\hat{j} + 4\hat{k}$$

$$\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$$

Work done,

$$W = \vec{F} \cdot \vec{S} = (0\hat{i} + 0\hat{j} + 4\hat{k}) \cdot (-\hat{i} + 2\hat{j} + 3\hat{k})$$

$$= 12(\hat{k} \cdot \hat{k}) \text{ Joule} = 12 \text{ Joule}$$



**Q.12.** An electron and a proton are detected in a cosmic ray experiment, the first with kinetic energy 10 keV, and the second with 100 keV. Which is faster, the electron or the proton ? Obtain the ratio of their speeds.

(electron mass =  $9.11 \times 10^{-31}$  kg, proton mass =  $1.67 \times 10^{-27}$  kg, 1 eV =  $1.60 \times 10^{-19}$  J)

**Ans.** Electron mass,  $m_e = 9.11 \times 10^{-31}$  kg

Proton mass,  $m_p = 1.67 \times 10^{-27}$  kg

Electron's kinetic energy

$$\begin{aligned} E_{ke} &= 20 \text{ keV} = 20 \times 10^3 \text{ eV} \\ &= 20 \times 10^3 \times 1.60 \times 10^{-19} \\ &= 3.2 \times 10^{-15} \text{ J} \end{aligned}$$

Proton's kinetic energy,

$$\begin{aligned} E_{kp} &= 200 \text{ keV} = 2 \times 10^5 \text{ eV} \\ &= 3.2 \times 10^{-14} \text{ J} \end{aligned}$$

To find the velocity of electron  $v_e$ , kinetic energy is used.

$$\begin{aligned} E_{ke} &= \frac{1}{2} m v_e^2 \\ V_e &= \sqrt{\frac{2 \times E_{ke}}{m}} \\ &= \sqrt{\frac{2 \times 3.2 \times 10^{-15}}{9.11 \times 10^{-31}}} \\ &= 8.38 \times 10^7 \text{ m/s} \end{aligned}$$

To find the velocity of proton  $v_p$ , the kinetic energy is used.

$$\begin{aligned} E_{kp} &= \frac{1}{2} m v_p^2 \\ V_p &= \sqrt{\frac{2 \times E_{kp}}{m}} \\ V_p &= \sqrt{\frac{2 \times 3.2 \times 10^{-14}}{1.67 \times 10^{-27}}} \\ &= 6.19 \times 10^6 \text{ m/s} \end{aligned}$$

Thus, electron moves faster when compared with proton.

The speed ratios are:

$$\frac{v_e}{v_p} = \frac{8.38 \times 10^7}{6.19 \times 10^6} = 13.53$$

**Q.13.** A raindrop of radius 2 mm falls from a height of 500 m above the ground. It falls with decreasing acceleration (due to viscous resistance of the air) until at half its original height, it attains its maximum (terminal) speed, and moves with uniform speed thereafter. What is the work done by the gravitational force on the drop in the first and second half of its journey? What is the work done by the resistive force in the entire journey if its speed on reaching the ground is  $10 \text{ ms}^{-1}$ ?

**Ans.**

Radius of the raindrop,  $r = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$

$$V = \frac{4}{3} \pi r^3$$

Density of water,  $\rho = 10^3 \text{ kg m}^{-3}$

Mass of the rain drop,  $m = \rho V$

$$= \frac{4}{3} \times 3.14 \times (2 \times 10^{-3})^3 \times 10^4 \text{ kg}$$

Gravitational force,  $F = mg$

$$= \frac{4}{3} \times 3.14 \times (2 \times 10^{-3}) \times 10^3 \times 9.8 \text{ N}$$

The work that is done by the gravitational force on the drop in the first half of its journey

$$W_1 = F_s$$

$$= \frac{4}{3} \times 3.14 \times (2 \times 10^{-3}) \times 10^3 \times 9.8 = 0.082 \text{ J}$$

This is the same amount of work that is done by the gravitational force on the second half of its journey,

i.e.;  $W_{H2} = 0.082 \text{ J}$

Under the energy conservation law, if there is no resistive force, the total energy of the raindrop

energy will remain the same.

Total energy at the top:  $E_T = mgh + 0$

$$= \frac{4}{3} \times 3.14 \times (2 \times 10^{-3})^3 \times 10^3 \times 9.8 = 0.1643 \text{ J}$$

The drop hits the ground with a velocity equal to 10 m/s because of the presence of a resistive force.

Total energy at the ground:

$$E_G = \frac{1}{2} mv^2 + 0$$

$$= \frac{1}{2} \times \frac{4}{3} \times 3.14 \times (2 \times 10^{-3}) \times 10^3 \times 9.8 \times (10)^2$$

$$= 1.675 \times 10^{-3} \text{ J}$$

$$\text{Resistive force} = E_G - E_T = -0.162 \text{ J}$$

**Q.14. A molecule in a gas container hits a horizontal wall with speed  $200 \text{ m s}^{-1}$  and angle  $30^\circ$  with the normal, and rebounds with the same speed. Is momentum conserved in the collision? Is the collision elastic or inelastic?**

**Ans.** Momentum is always conserved for a elastic or inelastic collision.

The molecule approaches and rebounds with the same speed of  $200 \text{ m/s}$ .

$$u = v = 200 \text{ m s}^{-1}$$

Therefore, Initial kinetic energy

$$= (1/2) mu^2 = (1/2)m(200)^2$$

$$\text{Final kinetic energy} = (1/2) mv^2 = (1/2)m(200)^2$$

Therefore, kinetic energy is also conserved

**Q.15. A pump on the ground floor of a building can pump up water to fill a tank of volume  $30 \text{ m}^3$  in 15 min. If the tank is 40 m above the ground, and the efficiency of the pump is 30%, how much electric power is consumed by the pump?**

**Ans.** Volume of the tank =  $30 \text{ m}^3$

time taken to fill the tank = 15 min =  $15 \times 60 = 900 \text{ s}$

height of the tank above the ground,  $h = 40 \text{ m}$

Efficiency of the pump,  $\eta = 30\%$

Density of water,  $\rho = 103 \text{ kg m}^{-3}$

Mass of water pumped,

$$m = \text{volume} \times \text{density} = 30 \times 10^3 \text{ kg}$$

Power consumed or output power

$$P_{\text{output}} = W/t = mgh/t$$

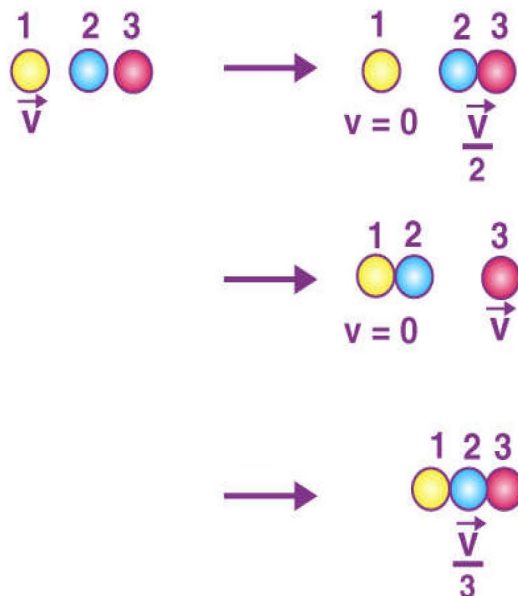
$$= \frac{30 \times 10^3 \times 9.8 \times 40}{900} = 13066 \text{ watt}$$

$$\text{Efficiency, } \eta = \frac{P_{\text{output}}}{P_{\text{input}}}$$

$$P_{\text{input}} = \frac{P_{\text{output}}}{\eta} = \frac{13066}{\frac{30}{100}} = \frac{1306600}{30}$$

$$= 43553 \text{ W} = 43.6 \text{ kW}$$

**Q.16. Two identical ball bearings in contact with each other and resting on a frictionless table is hit head-on by another ball bearing of the same mass moving initially with a speed  $V$ . If the collision is elastic, which of the following figure is a possible result after collision?**





The mass of the ball bearing is  $m$ ,

Before the collision, Total K.E. of the system

$$\frac{1}{2}mv^2 + 0 = \frac{1}{2}mv^2$$

After the collision, Total K.E. of the system is

**Case I,**

The final Kinetic Energy is

$$T_f = \frac{1}{2}mo^2 + \frac{1}{2}m\left(\frac{V}{2}\right)^2 + \frac{1}{2}m\left(\frac{V}{2}\right)^2 = \frac{1}{4}mV^2$$

**Case II,**

The final Kinetic Energy is

$$T_f = \frac{1}{2}mo^2 + \frac{1}{2}mo^2 + \frac{1}{2}mV^2 = \frac{1}{2}mV^2$$

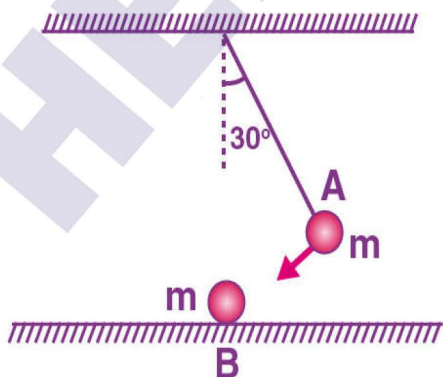
**Case III,**

The final Kinetic Energy is  $T_f =$

$$\frac{1}{2}m\left(\frac{V}{3}\right)^2 + \frac{1}{2}m\left(\frac{V}{3}\right)^2 + \frac{1}{2}m\left(\frac{V}{3}\right)^2 = \frac{1}{6}mV^2$$

Thus, case II is the only possibility since K.E. is conserved in this case.

**Q.17. A ball A which is at an angle  $30^\circ$  to the vertical is released and it hits a ball B of same mass which is at rest. Does the ball A rises after collision? The collision is an elastic collision.**



**Ans.** In an elastic collision when the ball A hits the ball B which is stationary, the ball B acquires the velocity of the ball A while the ball A comes to rest immediately after the collision. There is a transfer of momentum to the moving body from the stationary body. Thus, the ball A comes to rest after collision and ball B moves with the velocity of ball A.

**Q.18. The bob of a pendulum is released from a horizontal position. If the length of the pendulum is 1.5 m, what is the speed with which the bob arrives at the lowermost point, given that it dissipated 5% of its initial energy against air resistance?**

**Ans.**

As provided, the length of the pendulum,  $l = 1.5\text{m}$

The mass of the bob =  $m$

Energy dissipated =  $5$

The law of conservation of momentum states the linear momentum of a system is unchanged in the absence of external force.

In the horizontal position

The PE of the bob is  $E_p = mgl$ ,

while the KE is  $E_k = 0$

and hence the total energy is  $E = mgl$

While at the lowermost point or the mean position, the potential energy is  $E_p = 0$ ,

the KE is  $E_k = \frac{1}{2}mv^2$

and the total energy is  $E = \frac{1}{2}mv^2$

But the bob loses 5% of its energy.

Hence the total energy at the lowermost point is 95% of the total energy at the higher/horizontal position

$$\Rightarrow \frac{1}{2}mv^2 = \frac{95}{100}mgl$$

$$\Rightarrow v = 5.28\text{m/s}$$

Hence the bob arrives with 5.28m/s at the lowest point.

**Q.19.** A trolley of mass 300 kg carrying a sandbag of 25 kg is moving uniformly with a speed of 27 km/h on a frictionless track. After a while, the sand starts leaking out of a hole on the floor of the trolley at the rate of  $0.05 \text{ kg s}^{-1}$ . What is the speed of the trolley after the entire sandbag is empty?

**Ans.** As the trolley carrying the sand bag is moving without friction, which means, there is no friction which can act as an external force on the system. Hence the law of conservation of momentum is applicable.

The given initial mass of the bag  $m_s(t = 0) = 25 \text{ kg}$  and of the trolley is  $m_T = 300 \text{ kg}$  hence the total initial mass is  $M_{\text{Tot}}(T = 0) = 325 \text{ kg}$ .

And the mass drop rate is  $\frac{dM}{dt} = 0.05 \text{ kg/s}$

The initial velocity of the trolley is

$$v(t = 0) = 27 \text{ km/h} = 7.5 \text{ m/s}$$

The momentum at any given time  $t$  is

$$p = (325 - 0.05t)v(t) \text{ and at } t = 0, \text{ the momentum } p(t = 0) = 325v(t = 0) = 325 \times 7.5 = 2437.5 \text{ kg m/s}$$

The time at which the sandbag empties,

$$t_{\text{final}} = \frac{25}{0.05} = 500 \text{ s}$$

Thus the momentum at  $t_{\text{final}}$  can be written as

$$2437.5 = (325 - 0.05 \times 500) \cdot v(t_{\text{final}} = 500)$$

$$\Rightarrow 2437.5 = 300 \cdot v(t_{\text{final}})$$

$$\Rightarrow v(t_{\text{final}}) = 8.125 \text{ m/s}$$

Hence the final velocity when the bag is emptied is **8.125 m/s**.

**Q.20.** A body of mass  $m = 0.5 \text{ kg}$  travels in a straight line with velocity  $v = ax^{\frac{3}{2}}$  where

$a = 5 \text{ m}^{\frac{1}{2}} \text{ s}^{-1}$  What is the work done by the net force during its displacement from  $x = 0$  to  $x = 2$  ?

**Ans.** The given mass of the body is  $m = 0.5 \text{ kg}$

and is moving with velocity  $v = ax^{\frac{3}{2}}$

where  $a = 5 \text{ m}^{\frac{1}{2}} \text{ s}^{-1}$

The kinetic energy at  $x = 0$  is

$$E_K(x = 0) = \frac{1}{2} \cdot 0.5 \times 0 \text{ and at } x = 2 \text{ is}$$

$$E_K(x = 2) = \frac{1}{2} \cdot 0.5 \times (5 \times 2^{\frac{3}{2}})^2 = 50 \text{ J}$$

The difference in these two values is equal to the work done.

$$W = E_K(x = 2) - E_K(x = 0) = 50 \text{ J}$$

Hence the work done is 50J.

**Q.21.** The windmill sweeps a circle of area  $A$  with their blades. If the velocity of the wind is perpendicular to the circle, find the air passing through it in time  $t$  and also the kinetic energy of the air. 25 % of the wind energy is converted into electrical energy and  $v = 36 \text{ km/h}$ ,  $A = 30 \text{ m}^2$  and the density of the air is  $1.2 \text{ kg m}^{-3}$ . What is the electrical power produced?

**Ans.** Area =  $A$

Velocity =  $V$

Density =  $\rho$

**(a)**

Volume of the wind through the windmill per sec

$$= Av$$

$$\text{Mass} = \rho Av$$

Mass  $m$  through the windmill in time  $t = \rho Avt$

**(b)**

$$\text{kinetic energy} = \frac{1}{2} mv^2$$

$$= \frac{1}{2} (\rho Avt)v^2 = \frac{1}{2} \rho Av^3 t$$

(c)

$$\text{Area} = 30 \text{ m}^2$$

$$\text{Velocity} = 36 \text{ km/h}$$

$$\text{Density of air } \rho = 1.2 \text{ kg m}^{-3}$$

Electric energy = 25 % of wind energy

$$= \frac{25}{100} \times \text{kinetic energy}$$

$$= \frac{1}{8} \rho A v^3 t$$

$$\text{power} = \frac{\text{Electric energy}}{\text{Time}}$$

$$= \frac{1}{8} \frac{\rho A v^3}{t} = \frac{1}{8} \rho A v^3$$

$$= \frac{1}{8} \times 1.2 \times 30 \times 10^3$$

$$= 4.5 \times 10^3 \text{ W} = 4.5 \text{ kW}$$

**Q.22. A person trying to lose weight (dieter) lifts a 10kg mass, one thousand times, to a height of 0.5m each time. Assume that the potential energy lost each time she lowers the mass is dissipated.**

**(a) How much work does she do against the gravitational force?**

**Ans:** Mass of the dumbbell  $m = 10 \text{ kg}$

The height up to which it is lifted  $h = 0.5 \text{ m}$

Hence the work done with every lift is  $mgh$

Number of times the dumbbell is lifted  $n = 1000$

Hence the total work done against gravity is

$$= n mgh = 49 \text{ kJ.}$$

**(b) Fat supplies  $3.8 \times 10^7 \text{ J}$  of energy per kilogram which is converted to mechanical energy with a 20 efficiency rate. How much fat will the dieter use up?**

**Ans:** Energy supplied with every kilogram of fat is  $3.8 \times 10^7 \text{ J}$

The efficiency is 20

Hence the mechanical energy supplied by the body

$$= \frac{20}{100} 3.8 \times 10^7 \text{ J} = 7.6 \times 10^6 \text{ J}$$

And the equivalent fat loss is  $6.45 \times 10^{-3} \text{ kg}$

Hence the body loses  $6.45 \times 10^{-3} \text{ kg}$  of fat.

**Q.23. A family uses 8kW of power.**

**(a) Direct solar energy is incident on the horizontal surface at an average rate of 200W**

**per square meter. If 20 of this energy can be converted to useful electrical energy, how large an area is needed to supply 8kW?**

**Ans:** The given power requirement of a family is

$$P = 8 \text{ kW} = 8 \times 10^3 \text{ W}$$

The solar energy per sq mts  $\frac{P_{\text{solar}}}{A} = 200 \text{ W}$

Efficiency in converting solar to electrical energy

$$\eta = 20$$

Let the area needed to generate the required electricity be  $A$

Total solar power is  $P_{\text{solar}} = \frac{P_{\text{solar}}}{A} A$

And the expected electrical power output can be written as  $P = 20$

$$\Rightarrow A = 200 \text{ m}^2$$

Hence the area required is  $A = 200 \text{ m}^2$ .

**(b) Compare this area to that of the roof of a typical house.**

**Ans:** A typical house can have a roof of the dimensions  $15 \times 15 \text{ m}^2$ ,

hence the area is  $\geq 200 \text{ m}^2$ .

And this is more than the required area.



**Q.24.** A bullet of mass 0.012 kg and horizontal speed 70 m s<sup>-1</sup> strikes a block of wood of mass 0.4 kg and instantly comes to rest with respect to the block. The block is suspended from the ceiling by means of thin wires. Calculate the height to which the block rises. Also, estimate the amount of heat produced in the block.

**Ans.**

Mass of the bullet,  $m_1 = 0.012$  kg

Initial speed of the bullet,  $u_1 = 70$  m/s

Mass of the wooden block,  $m_2 = 0.4$  kg

Initial speed of the wooden block,  $u_2 = 0$

Final speed of the system of the bullet and the block =  $v$  m/s

Applying the law of conservation of momentum:

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

$$(0.012 \times 70) + (0.4 \times 0) = (0.012 + 0.4)v$$

$$v = \frac{0.84}{0.412}$$

$$= 2.04 \text{ m/s}$$

Let  $h$  be the height to which the block rises

Applying the law of conservation of energy to this system:

Potential energy of the combination = Kinetic energy of the combination

$$(m_1 + m_2)gh = \frac{1}{2}(m_1 + m_2)v^2$$

$$h = \frac{v^2}{2g}$$

$$= \frac{(2.04)^2}{2 \times 9.8}$$

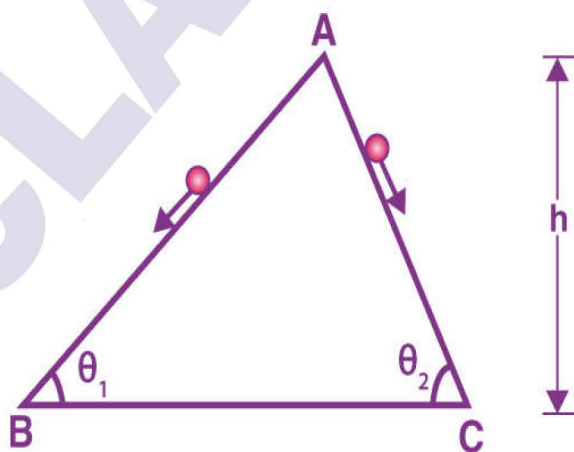
$$= 0.212 \text{ m}$$

The wooden block will rise to a height of 0.212 m

**The heat produced = Initial kinetic energy of the bullet – final kinetic energy of the combination**

$$\begin{aligned} &= \frac{1}{2}m_1u_1^2 - \frac{1}{2}(m_1 + m_2)v^2 \\ &= \frac{1}{2} \times 0.010 \times 70^2 - \frac{1}{2} \times (0.012 + 0.4) \times 2.04^2 \\ &= 29.4 - 0.857 = 28.54 \text{ J} \end{aligned}$$

**Q.25.** Two inclined frictionless tracks, one gradual and the other steep meet at A from where two stones are allowed to slide down from rest, one on each track Fig. Will the stones reach the bottom at the same time? Will they reach there with the same speed? Explain. Given  $\theta_1 = 30^\circ$ ,  $\theta_2 = 60^\circ$ , and  $h = 10$  m, what are the speeds and times taken by the two stones?



**Ans.** In the figure, the sides AB and AC are inclined to the horizontal at  $\angle\theta_1$  and  $\angle\theta_2$  respectively.

According to law of conservation of mechanical energy,

PE at the top = KE at the bottom

$$\therefore mgh = \frac{1}{2}mv_1^2 \quad \text{————— (1)}$$

$$\text{and } mgh = \frac{1}{2}mv_2^2 \quad \text{————— (2)}$$

Since the height of both the sides is the same, therefore, both the stones will reach the bottom at the same speed.

From (1) and (2), we get  $v_1 = v_2$

Hence both the stones will reach the bottom with the same speed.

**For the stone 1**

Net force acting on the stone is given by

$$F = ma_1 = mg \sin\theta_1$$

$$a_1 = g \sin\theta_1$$

**For stone 2**

$$a_2 = g \sin\theta_2$$

As  $\theta_2 > \theta_1$

Therefore,  $a_2 > a_1$

From  $v = u + at = 0 + at$

$$\Rightarrow t = \frac{v}{a}$$

For stone 1,  $t_1 = \frac{v}{a_1}$

For stone 2,  $t_2 = \frac{v}{a_2}$

As  $t \propto \frac{1}{a}$ , and  $a_2 > a_1$

Therefore,  $t_2 < t_1$

Hence, stone 2 will reach faster than stone 1.

By applying the law of conservation of energy we get

$$mgh = \frac{1}{2}mv^2$$

When the height,  $h = 10$  m, the speed of the stones are

$$v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 10} = 14\text{m/s}$$

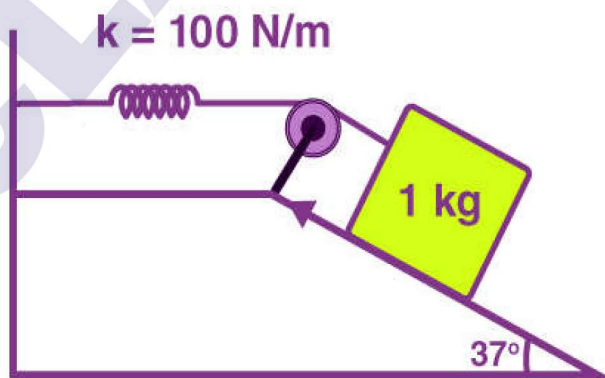
The time taken is given as

$$t_1 = \frac{v}{a_1} = \frac{v}{g \sin\theta_1} = \frac{14}{9.8 \times \sin 30} = \frac{14}{9.8 \times \frac{1}{2}} = 2.86\text{s}$$

$$t_2 = \frac{v}{a_2} = \frac{v}{g \sin\theta_2} = \frac{14}{9.8 \times \sin 30} = \frac{14}{9.8 \times \frac{\sqrt{3}}{2}} = 1.65\text{s}$$

**Q.26. A 1 kg block situated on a rough incline is connected to a spring of spring constant 100**

**N m<sup>-1</sup> as shown in Fig. The block is released from rest with the spring in the unstretched position. The block moves 10 cm down the incline before coming to rest. Find the coefficient of friction between the block and the incline. Assume that the spring has a negligible mass and the pulley is frictionless.**



**Ans.** Mass of the block,  $m = 1$  kg

Spring constant,  $k = 100 \text{ N m}^{-1}$

Displacement in the block,  $x = 10 \text{ cm} = 0.1 \text{ m}$

**At equilibrium:**

Normal reaction,  $R = mg \cos 37^\circ$

Frictional force,  $F = \mu R = mg \sin 37^\circ$

$\mu$  is the coefficient of friction

Net force acting on the block down the incline =  $mg \sin 37^\circ - F$

$$= mg \sin 37^\circ - \mu mg \cos 37^\circ$$

$$= mg(\sin 37^\circ - \mu \cos 37^\circ)$$

At equilibrium

Work done = Potential energy of the stretched string

$$mg(\sin 37^\circ - \mu \cos 37^\circ) x = \frac{1}{2} kx^2$$

$$1 \times 10 \times (\sin 37^\circ - \mu \cos 37^\circ) = \frac{1}{2} \times 100 \times 0.1$$

$$10 (0.602 - \mu (0.798)) = \frac{1}{2} \times 100 \times 0.1$$

$$0.602 - \mu (0.798) = 0.5$$

$$\text{Therefore, } \mu = \frac{0.602 - 0.5}{0.798} = \frac{0.102}{0.798} = 0.127$$

$$\mu = 0.127$$

**Q.27. A bolt of mass 0.3 kg falls from the ceiling of an elevator moving down with a uniform speed of 7 ms<sup>-1</sup>. It hits the floor of the elevator (length of elevator = 3 m) and does not rebound. What is the heat produced by the impact? Would your answer be different if the elevator were stationary?**

**Ans:** The given bolt has a mass of  $m = 0.3\text{kg}$

The elevator moves with speed  $v = 7\text{m/s}$

And the height of the elevator is  $h = 3\text{m}$

The relative velocity of the bolt wrt lift is zero; hence at the time of impact, the change in potential energy is converted to heat.

$$\Delta PE = mgh = 0.3 \times 9.8 \times 3 = 8.82\text{J}$$

The heat produced is dependent on the relative velocity of the bolt wrt the elevator, and not on anything else. Hence the heat generated remains the same even if the elevator is moving.

**Q.28. A trolley of mass 200kg moves with a uniform speed of  $v = 36\text{km}$  on a frictionless track. A child of mass 20kg runs on the trolley from one end to the other (10 m away) with a speed of 4m/s relative to the trolley in a direction opposite to the its motion, and jumps out of the trolley. What is the final speed of the trolley? How much has the trolley moved from the time the child begins to run?**

**Ans.** As provided, the trolley weighs  $M=200\text{kg}$  and is  $l=10\text{m}$  long

It moves with a velocity of  $v = 36\text{km/h} = 10\text{m/s}$

And the mass of the boy is  $m = 20\text{kg}$

The initial momentum of trolley + boy is

$$(M + m) v = 2200\text{kgm/s}$$

Now let the final velocity be  $v'$

And the boy's final velocity wrt ground is  $v' - 4$

Hence the final momentum is

$$Mv' + m(v' - 4) = 220v' - 80$$

Since there is no external force, the final momentum is equal to the initial momentum

$$2200 = 220v' - 80$$

$$\Rightarrow v' = 10.36\text{m/s}$$

The boy's speed is  $v'' = 4\text{m/s}$

The time taken to run across the trolley

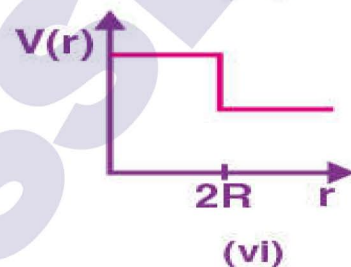
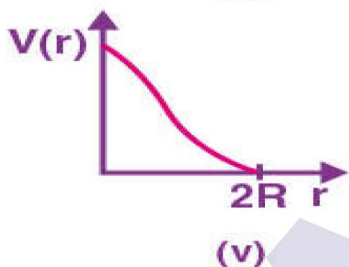
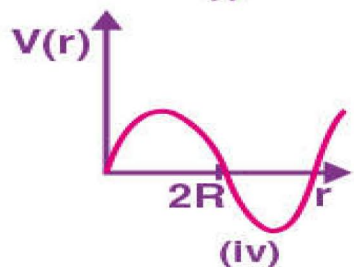
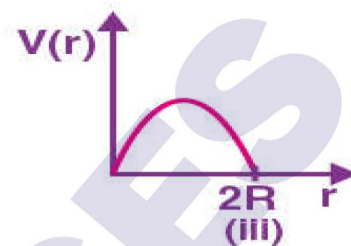
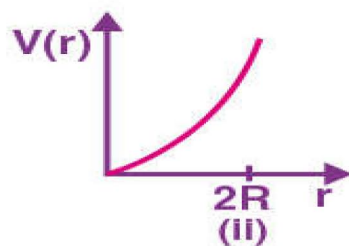
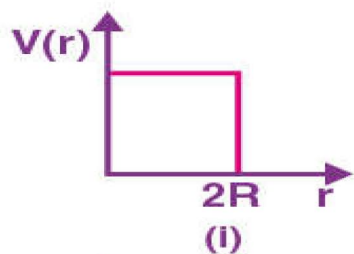
$$t = 10/4 = 2.5\text{s}$$

Hence the distance moved by the trolley is

$$v''t = 25.9\text{m}$$



Q.29. Which of the following does not describe the elastic collision of two billiard balls? Distance between the centres of the balls is  $r$ .



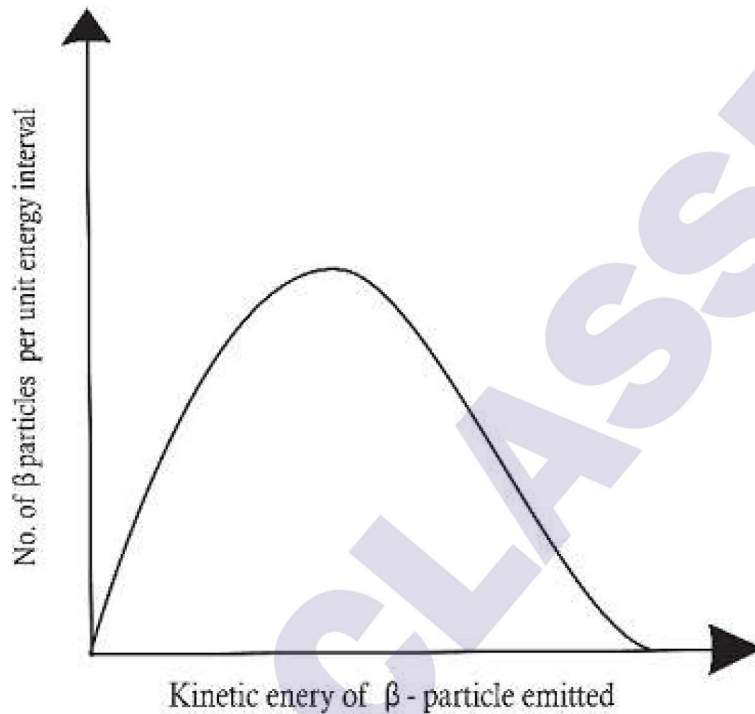
**Ans.** The gravitational potential energy of a system of two masses is inversely related to the separation between the two of them.

However, In the given case, the potential energy of the system of the two balls will decrease as they come closer to each other. It will become zero when the two balls touch each other, i.e., when the distance between the two is  $2R$ .

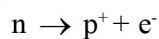
The potential energy curves given in figures (i), (ii), (iii), (iv), and (vi) do not satisfy these two conditions, and hence they can't describe the elastic collisions.

Q.30. Consider the decay of a free neutron at rest:  $n \rightarrow p^+ + e^-$

Show that the two-body decay of this type must necessarily give an electron of fixed energy and, therefore, cannot account for the observed continuous energy distribution in decay the of a neutron or a nucleus (figure).



**Ans.** The given neutron decay at rest can be written as



According to Einstein's mass-energy relation, the mass lost ( $\Delta m$ ) will be taken by the ejected electron as energy  $\Delta mc^2$ . The mass defect ( $\Delta m$ ) is the difference in the mass of the neutron and the sum of the proton masses and electron.

$$\Delta m = m_{\text{neutron}} - (m_{p^+} + m_{e^-})$$

The speed of light is denoted by  $c$

Hence the equation misses showing the change in the mass or the energy and where it went. Adding an energetic neutrino  $\nu$  on the RHS would solve the issue.