2.0 : Introduction

Q.1. Define scalars and vectors with examples.

Ans: i. Scalars:

Physical quantities which have magnitude only and are specified completely by a number and unit are called scalars.

Example: Length, mass, time, volume, temperature, speed, work, energy etc.

ii. Vectors:

Physical quantities which have magnitude as well as direction are called vectors.

Example: Displacement, velocity, acceleration, force, momentum, impulse etc.

Q.2. Distinguish between Scalars and Vectors.

Ans:

No.	Scalars	Vectors
i.	It has magnitude only.	It has magnitude as well as direction.
ii.	Scalars can be added or subtracted according to the rules of algebra.	Vectors are added or subtracted by geometrical (graphical) method or vector algebra.
iii.	It has no proper symbol.	It is represented by symbol (→) arrow.
iv.	The division of a scalar by another scalar is valid.	The division of a vector by another vector is not valid.
v.	Example: Length, mass, time, volume, etc.	Example: Displacement, velocity, acceleration, force, etc.

Q.3. State for each of the following physical quantities, if it is a scalar or a vector: Volume, mass, speed, acceleration, density, number of moles, velocity, angular frequency, displacement, angular velocity. (NCERT)

Ans: Scalars:

Volume, mass, speed, density, number of moles, angular frequency.

Vectors:

Acceleration, velocity, displacement, angular velocity.

Q.4. Justify with the help of an example that mass is a scalar quantity.

Ans: i. Scalar quantities are the physical quantities which have only magnitude and are specified completely by a number and a unit.

ii. When we say that the mass of a stone is 6 kg, it indicates that the stone is 6 times heavier than the standard unit kilogram.

iii. Therefore, number 6 is the magnitude of unit of mass kilogram which gives us a complete idea about the mass of the stone.

iv. Thus, mass is a scalar quantity.

Q.5. Explain representation of a vector graphically and symbolically.

Ans: i. Graphical representation:

A vector is graphically represented by a directed line segment or an arrow.

e.g. displacement of a body from P to Q is represented as P $\longrightarrow$ Q.

ii. Symbolic representation:

Symbolically a vector is represented by a single letter with an arrow above it, such as $\vec{A}$. The magnitude of the vector $\vec{A}$ is denoted as |A| or $|\vec{A}|$ or A.

Q.6. A vector has both magnitude and direction. Does it mean that anything that has magnitude and direction is necessarily a vector? (NCERT)

Ans: i. For a physical quantity, only having magnitude and direction is not a sufficient condition to be a vector.

ii. A physical quantity also has to obey vectors law of addition to be termed as vector.

iii. Example: Though current has definite magnitude and direction, it is not a vector.

Q.7. Define the terms:

i. Parallel vectors

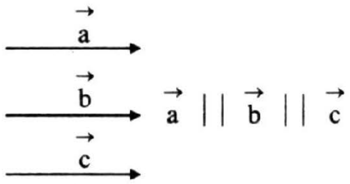
ii. Antiparallel vectors

iii. Collinear vectors

iv. Negative vectors

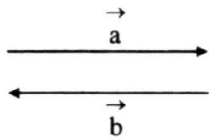
Ans: i. Parallel vectors:

Two or more vectors are said to be parallel, if they act in the same direction.



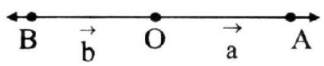
Parallel vectors

ii. Anti-parallel vectors:
Two vectors are said to be anti-parallel, if their directions are opposite to one another.



Anti-parallel vectors

iii. Collinear vectors:
Two or more vectors are said to be collinear if they are either parallel or antiparallel or lie along the same line.

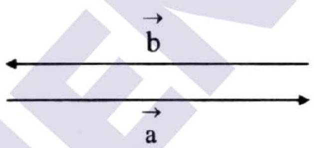


$\vec{a}$ and $\vec{b}$ are collinear vectors

$\vec{a}$ and $\vec{b}$ are collinear vectors

iv. Negative vectors:
A vector is said to be negative if its magnitude is same as that of the given vector but its direction is opposite.
Negative vectors are antiparallel vectors.

In figure, $\vec{b} = -\vec{a}$



Negative vector

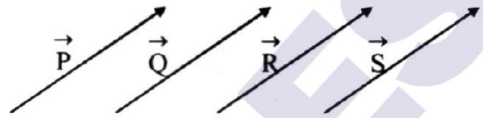
Q.8. Define the terms.
i. Zero (Null) vector
ii. Equal vectors
iii. Position vector

Ans: i. Zero vector (Null vector) :
A vector which has zero magnitude and arbitrary direction is called zero vector.

It is denoted as $\vec{0}$.

Example: Velocity vector of stationary particle, acceleration vector of a body moving with uniform velocity.

ii. Equal vectors:
Two or more vectors representing the same physical quantity, having the same magnitude and direction irrespective of their positions in space are called equal vectors.

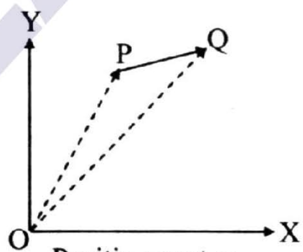


Equal vectors

In the given figure

$|\vec{P}| = |\vec{Q}| = |\vec{R}| = |\vec{S}|$

iii. Position vector:
A vector which gives the position of a particle at a point with respect to the origin of chosen co-ordinate system is called position vector.



Position vector

In the given figure, $\overline{OP}$ represents position vector of $\vec{P}$ with respect to O and $\overline{OQ}$ represents position vector of $\vec{Q}$ with respect to O.

Q.9. Define the following terms.
i. Resultant vector
ii. Composition of vectors

Ans: i. Resultant vector:
The resultant of two or more vectors is defined as that single vector, which produces the same effect as produced by all the vectors together .

ii. Composition of vectors:
The process of finding the resultant of two or more vectors is called composition of vectors.

Q.10. Whether the resultant of two vectors of unequal magnitude be zero?

Ans: The resultant of two vectors of different magnitude cannot give zero resultant.

2.1 : Addition and subtraction of vectors :

Q.11. Explain addition of vectors.

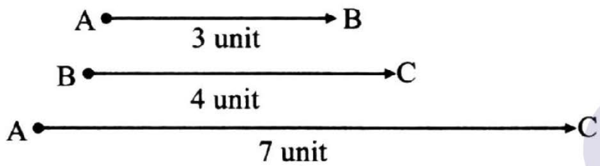
Ans: i. The addition of two or more vectors of same type gives rise to a single vector such that the effect of this single vector is the same as the net effect of the original vectors.

ii. It is important to note that only the vectors of the same type (physical quantity) can be added.

iii. For example, if two vectors, $\vec{a}$ = 3 unit and $\vec{b}$ = 4 units are acting along the same line, then they can be added as,

$|\vec{R}| = |\vec{a}| + |\vec{b}|$

$|\vec{R}| = 3 + 4 = 7$ unit

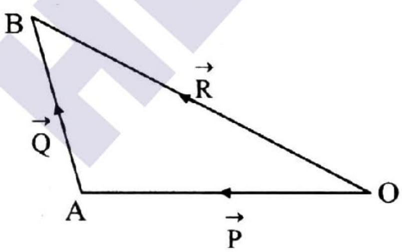


Note: When vectors are not in the same direction, then they can be added by using triangle law of vector addition.

Q.12. What is triangle law of vector addition?

Ans: Triangle law of vector addition:

If two vectors of the same type are represented in magnitude and direction, by the two sides of a triangle taken in order, then their resultant is represented in magnitude and direction by the third side of the triangle drawn from the starting point (tail) of the first vector to the end point (head) of the second vector.



Let $\vec{P}$ and $\vec{Q}$ be the two vectors of same type taken in same order as shown in figure.

$\therefore$ Resultant vector will be given by third side taken in opposite order.

i.e., $\overline{OA} + \overline{AB} = \overline{OB}$

$\therefore \quad \vec{R} = \vec{P} + \vec{Q}$

Q.13. Using triangle law of vector addition, explain the process of adding two vectors which are not lying in a straight line.

Ans: i. Two vectors in magnitude and direction are drawn in a plane as shown in figure (a)

Let these vectors be $\vec{P}$ and $\vec{Q}$

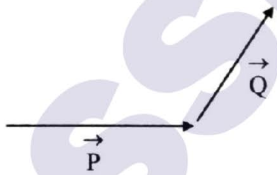


Figure (a)

ii. Join the tail of $\vec{Q}$ to head of $\vec{P}$ in the given direction. The resultant vector will be the line which is obtained by joining tail of $\vec{P}$ to head of $\vec{Q}$ as shown in figure (b).

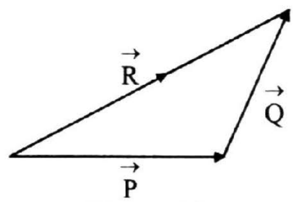


Figure (b)

iii. If R is the resultant vector of $\vec{P}$ and $\vec{Q}$ then using triangle law of vector addition, we have,

$\vec{R} = \vec{P} + \vec{Q}$

Note: The vector sum remains unchanged, even if the order of vectors is interchanged.

Q.14. Explain, how two vectors are subtracted to find their resultant by using triangle law of vector addition.

Ans: i. Let $\vec{P}$ and $\vec{Q}$ be the two vectors in a plane as shown in figure (a).

We have to find $\vec{P} - \vec{Q}$

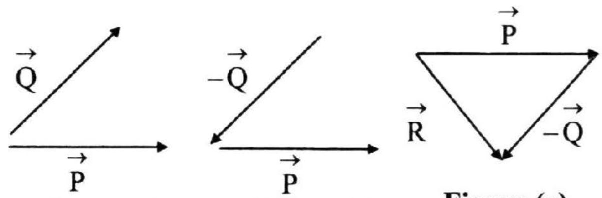


Figure (a) Figure (b) Figure (c)

ii. To subtract $\vec{Q}$ from $\vec{P}$, vector $\vec{Q}$ is reversed so that we get the vector $-\vec{Q}$ as shown in figure (b).

iii. The resultant vector $(\vec{R})$ is obtained by jommg tail of $\vec{P}$ to head of $-\vec{Q}$ as shown in figure (c).

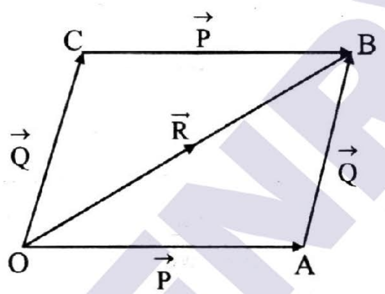
iv. From triangle law of vector addition,

$\vec{R} = \vec{P} + (-\vec{Q}) = \vec{P} - \vec{Q}$

Q.15. Prove that: addition of two vectors obey commutative law.

Ans: Proof:

i. Let two vectors $\vec{P}$ and $\vec{Q}$ be represented in magnitude and direction by two sides $\overline{OA}$ and $\overline{AB}$ respectively. To show that $\vec{P}+\vec{Q}=\vec{Q}+\vec{P}$.



ii. Complete a parallelogram OABC such that $\overline{OA}=\overline{CB}=\vec{P}$ and $\overline{AB}=\overline{OC}=\vec{Q}$ Join OB.

iii. In ΔOAB, $\overline{OA}=\overline{AB}=\overline{OB}$
(By triangle law of vector addition) ,

$\therefore$ $\vec{P}+\vec{Q}=\vec{R}$ .... (1)

In ΔOCB, $\overline{OC}+\overline{CB}=\overline{OB}$
(By triangle law of vector addition)

$\therefore$ $\vec{Q}+\vec{P}=\vec{R}$ .... (2)

iv. From equation (1) and (2),

$\vec{P}+\vec{Q}=\vec{Q}+\vec{P}$

Hence, addition of two vectors obey commutative law.

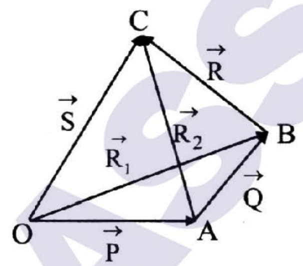
Q.16. Prove the associative property of vector addition.

Ans: Associative property of vector addition:

According to associative property,

For three vectors $\vec{P}$, $\vec{Q}$ and $\vec{R}$

$(\vec{P}+\vec{Q})+\vec{R}=\vec{P}+(\vec{Q}+\vec{R})$



Proof:

i. Let $\overline{OA}=\vec{P}$, $\overline{AB}=\vec{Q}$, $\overline{BC}=\vec{R}$

To prove

$(\vec{P}+\vec{Q})+\vec{R}=\vec{P}+(\vec{Q}+\vec{R})$

ii. Join OB and AC

In ΔOAB,

$\overline{OA}+\overline{AB}=\overline{OB}$ (From triangle law of vectors addition)

$\therefore$ $\vec{P}+\vec{Q}=\vec{R}_1$ ....(i)

In ΔOBC,

$\overline{OB}+\overline{BC}=\overline{OC}$ (From triangle law of vector addition)

$\therefore$ $\vec{R}_1+\vec{R}=\vec{S}$

[From equation (1)]

$(\vec{P}+\vec{Q})+\vec{R}=\vec{S}$ ....(2)

iii. In ΔABC,

$\overline{AB}+\overline{BC}=\overline{AC}$

$\therefore$ $\vec{P}+\vec{R}=\vec{R}_2$ .... (3)

iv. In ΔOAC,

$\overline{OA}+\overline{AC}=\overline{OC}$

$\therefore$ $\vec{P}+\vec{R}_2=\vec{S}$

From equation (3)

$$\vec{P}+\left(\vec{Q}+\vec{R}\right)=\vec{S} \quad ....(4)$$

On Comparing, equation (2) and (4), we get,

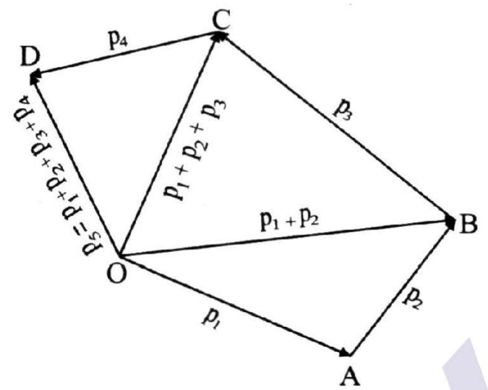
$$\left(\vec{P}+\vec{Q}\right)+\vec{R}=\vec{P}+\left(\vec{Q}+\vec{R}\right)$$

Hence, associative law is proved.

Q.17. What is polygon law of vectors?

Ans: Polygon law of vector addition:

If a number of vectors are represented in magnitude and direction by the sides of an incomplete polygon taken in order, then resultant is represented in magnitude and direction by the remaining side of the polygon, directed from the starting point (tail) of the first vector to the end point (head) of last vector.



In ΔOAB,

$\overline{OB} = \overrightarrow{p_1} + \overrightarrow{p_2}$

In ΔOBC,

$\overline{OC} = \overrightarrow{p_1} + \overrightarrow{p_2} + \overrightarrow{p_3}$

ΔOCD

$\overline{OD} = \overrightarrow{p_1} + \overrightarrow{p_2} + \overrightarrow{p_3} + \overrightarrow{p_4}$

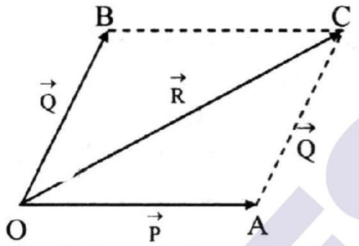
Thus, $\overrightarrow{p_5}$ is the resultant vector for $\overrightarrow{p_1} + \overrightarrow{p_2} + \overrightarrow{p_3} +$ and $\overrightarrow{p_4}$.

Q.18. State and prove parallelogram law of vectors addition and determine magnitude and direction of resultant vector.

Ans: a. Parallelogram law of vector addition:

If two vectors of same type starting from the same point, are represented in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant vector is given in magnitude and direction, by the diagonal of the parallelogram starting from the same point.

b. Proof:



Parallelogram law of vectors

i. Consider two vectors $\vec{P}$ and $\vec{Q}$ of the same type, with their tails at the point 'O' and 'θ' is the angle between $\vec{P}$ and $\vec{Q}$.

ii. Join BC and AC to complete the parallelogram OACB, with $\overline{OA} = \vec{P}$ and $\overline{AB} = \vec{Q}$ as the adjacent sides. We have to prove that diagonal $\overline{OC} = \vec{R}$, the resultant of sum of the two given vectors.

iii. By the triangle law of vector addition, we have

$$\overline{OA} + \overline{AC} = \overline{OC} \quad ....(1)$$

$\therefore$ AC is parallel to OB

where $|\overline{AC}| = AC$

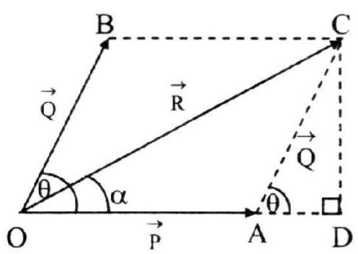
$$\therefore \quad \overline{AC} = \overline{OB} = \vec{Q} \quad ....(2)$$

Substituting $\overline{OA}$ and $\overline{OC}$ in equation (1) we have,

$$\vec{P} + \vec{Q} = \vec{R}$$

Hence proved.

c. Magnitude of resultant vector:



i. To find the magnitude of resultant

vector $\vec{R} = \overline{OC}$, draw a perpendicular from C to meet OA extended at D.

$\therefore$ $\angle CAD = \angle BOA = \theta$ and AC = OB = Q

ii. In right angle triangle ADC,

$$\cos\theta = \frac{AD}{AC}$$

$\therefore$ $AD = AC\cos\theta = Q\cos\theta$ ....(3)

and

$$\sin\theta = \frac{DC}{AC}$$

$\therefore$ $DC = AC\sin\theta = Q\sin\theta$ ....(4)

iii. Using Pythagoras theorem in right angle triangle ODC

$(OC)^2 = (OD)^2 + (DC)^2$
$= (OA + AD)^2 + (DC)^2$

$(OC)^2 = (OA^2) + 2(OA).(AD) + (AD^2) + (DC)^2$ ....(5)

iv. From right angle triangle ADC,

$AD^2 + DC^2 = AC^2$ ....(6)

v. From equation (5) and (6), we get

$OC^2 = (OA)^2 + 2(OA)(AD) + (AC)^2$ ....(7)

vi. Using (3) and (7), we get

$OC^2 = (OA)^2 + (AC)^2 + 2(OA)(AC)\cos\theta$

$\therefore$ $R^2 = P^2 + Q^2 + 2\,P.Q.\cos\theta$

$\therefore$ $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$ ....(8)

Equation (8) gives the magnitude of resultant vector $\vec{R}$.

d. Direction of resultant vector:

To find the direction of resultant vector $\vec{R}$, let $\vec{R}$ make an angle α with $\vec{P}$.

In ΔODC, $\tan\alpha = \frac{DC}{OD}$

$$\tan\alpha = \frac{DC}{OA + AD}$$

From equations (3) and (4), we get

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta} \quad ....(9)$$

$\therefore$ $\alpha = \tan^{-1}\left(\frac{Q\sin\theta}{P + Q\cos\theta}\right)$ ....(10)

Equation (10) represents direction of resultant vector.

e. Special cases:

Case 1 :

When $\theta = 0°$, i.e., $\vec{P}$ and $\vec{Q}$ are in the same direction, then from equation (8),

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos 0^\circ}$$

$$= \sqrt{P^2 + Q^2 + 2PQ} = \sqrt{(P+Q)^2}$$

$\therefore$ R = P + Q

From equation (9),

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$$

$\therefore$ $\alpha = 0$

Case 2:

When $\theta = 90°$, i.e., when $\vec{P}$ and $\vec{Q}$ are mutually perpendicular to each other, then from equation (8)

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos 90^\circ}$$

$R = \sqrt{P^2 + Q^2}$ $[\because \cos 90° = 0]$

From equation (9),

$$\alpha = \tan^{-1}\left(\frac{Q\sin 90^\circ}{P + Q\cos 90^\circ}\right)$$

$$= \tan^{-1}\left(\frac{Q\times 1}{P + Q\times 0}\right)$$

$\therefore$ $\alpha = \tan^{-1}\left(\frac{Q}{P}\right)$

Case 3:

When $\theta = 180°$, i.e., when $\vec{P}$ and $\vec{Q}$ are in the opposite directions, then from equation (8),

$$R = \sqrt{P + Q + 2PQ\cos 180^\circ}$$

$\therefore$ $R = \sqrt{P^2 + Q^2 - 2PQ}$

$[\because \cos 180^\circ = -1]$

$\therefore$ $R = \sqrt{(P-Q)^2}$

$\therefore$ R = P - Q

From equation (9),

$$\alpha = \tan^{-1}\left(\frac{Q\sin 180^\circ}{P + Q\cos 180^\circ}\right) = 0$$

$\alpha = 0$ $[\because \sin 180° = 0]$

Q.19. Define unit vector and give its physical significance.

Ans: Unit vector: A vector having unit magnitude in a given direction is called a unit vector.

If $\vec{p}$ is a non zero vector $(P \neq 0)$ then the unit vector up in the direction of $\vec{p}$ is given by, .,

$$\hat{u}_p = \frac{\vec{P}}{P}$$

$$\therefore \quad \vec{P} = \hat{u}_p P$$

Significance of unit vector:

i. The unit vector gives the direction of a given vector.

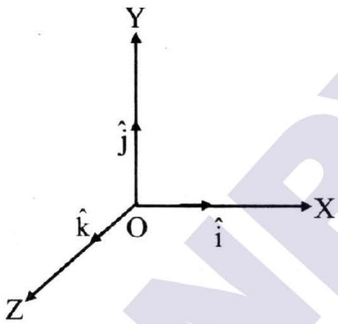
ii. Unit vector along X, Y and Z direction of a cartesian coordinate is represented by $\hat{i}, \hat{j}$ and $\hat{k}$ respectively.

Q.20. What are rectangular unit vectors? How are they denoted?

OR

Explain rectangular unit vectors.

Ans: i. The unit vectors along the positive directions of the axes of cartesian co-ordinate systems are called rectangular unit vectors.



ii. The unit vector along the positrve direction of the X-axis is denoted by $\hat{i}$, the unit vector along the positive direction of the Y-axis is denoted by $\hat{j}$ and the unit vector along the positive direction of the Z-axis is denoted by $\hat{k}$.

iii. These unit vectors may not be located at the origin. They can be translated anywhere ill space provided their directions with respect to the respective axis remain same.

iv. These unit vectors are also called base vectors.

Q.21. Define Components of a vector.

Ans: i. The number of vectors whose combined effect is same as that of the given vector are called components of the given vector.

ii. The component of a vector in a given direction gives the measure of the effect of that vector in that direction.

Note: The magnitude of a vector is a scalar while each component of a vector is always a vector.

Q.22. Explain what is resolution of vector.

Ans: i. The process of finding the components (or component vectors) of a given vector is called resolution of vectors.

ii. Resolution of vector is equal to replacing the original vector with the sum of the component vectors.

iii. Consider vector $\vec{R}$ which is to be resolved into two vector components, one along direction of $\vec{A}$ and another along $\vec{B}$

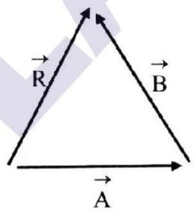


Figure (a)

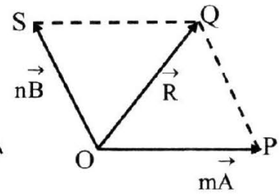


Figure (b)

iv. Let $\overline{OQ} = \vec{R}$, then from O, draw a line parallel to $\vec{A}$ and from Q, draw a line parallel to $\vec{B}$.

v. Using triangle law of vector addition, from figure (b), we can say,

$$\overline{OQ} = \overline{OP} + \overline{PQ}$$

i.e., $\vec{R} = m\vec{A} + n\vec{B}$, where m and n are real numbers.

vi. Thus, $\overline{OP}$ and $\overline{PQ}$ are the two vector components of $\vec{R}$ in the direction of $\vec{A}$ and $\vec{B}$.

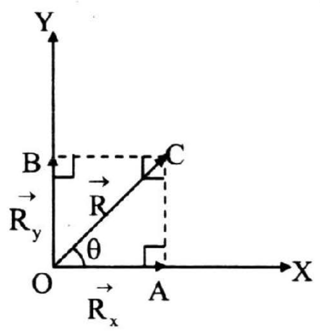
vii. Most commonly for resolution of vector, in three dimensions, a set of three vectors is chosen such that one vector is parallel to the X-axis, second parallel to the Y-axis and third parallel to the Z-axis.

Q.23. What are rectangular components of vectors? Explain their uses.

Ans: i. Rectangular components of a vector: If components of a given vector are at right angles to each other then they are called rectangular components of that vector.

ii. Rectangular components help us to find the magnitude and direction of a vector when they are resolved into two or more components.

iii. Consider a vector $\vec{R} = \overline{OC}$ originating from the origin 'O' of a rectangular coordinate system as shown in figure.



Two dimensional rectangular components

iv. Draw CA ⊥ OX and CB ⊥ OY.

Let component of $\vec{R}$ along X-axis = $\vec{R}_x$,

and component of $\vec{R}$ along Y-axis = $\vec{R}_y$

By parallelogram law of vectors,

$$\vec{R} = \vec{R}_x + \vec{R}_y \qquad (\because \overline{AC} = \overline{OB} = \vec{R}_y)$$

or, $\vec{R} = \hat{i}R_x + \hat{j}R_y$

where $\hat{i}$ and $\hat{j}$ are unit vectors along positive direction of X and Y axes respectively.

v. If θ is angle made by $\vec{R}$ with X-axis, then

$$\cos\theta = \frac{OA}{OC} = \frac{R_x}{R}$$

∴ $R_x = R\cos\theta$ ....(1)

Similarly,

$$\sin\theta = \frac{R_y}{R}$$

∴ $R_y = R\sin\theta$ ....(2)

vi. Squaring and adding equation (1) and (2) we get,

$$R_x^2 + R_y^2 = R^2\cos^2\theta + R^2\sin^2\theta$$

$$= R^2(\cos^2\theta + \sin^2\theta)$$

∴ $R_x^2 + R_y^2 = R^2$

∴ $R = \sqrt{R_x^2 + R_y^2}$ ...(3)

Equation (3) gives the magnitude of $\vec{R}$.

vii. Direction of $\vec{R}$ can be found out by dividing equation (2) by (1),

i.e. $\frac{R_y}{R_x} = \tan\theta$

$$\theta = \tan{-1}\left(\frac{R_y}{R_x}\right) \qquad ...(4)$$

Equation (4) gives direction of $\vec{R}$.

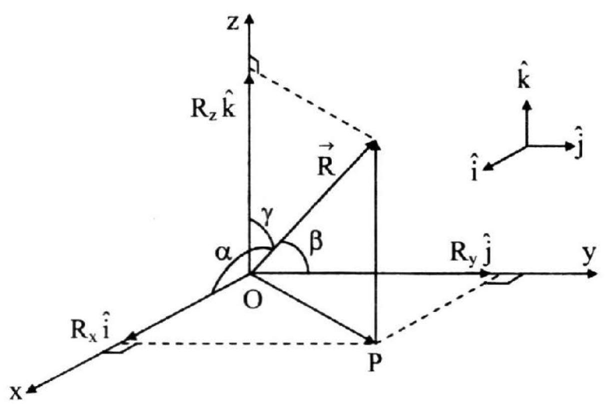
viii. When there are noncoplanar vectors, it becomes necessary to use the third dimension. If $\vec{R}_x$, $\vec{R}_y$ and $\vec{R}_z$, are three rectangular components of $\vec{R}$ along X, Y and Z axes of a three dimensional rectangular cartesian co-ordinate system then,

$$\vec{R} = \vec{R}_x + \vec{R}_y + \vec{R}_z$$

or $\vec{R} = \hat{i}R_x + \hat{j}R_y + \hat{k}R_z$

Magnitude of R is given by,

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} \quad \left[\because \hat{i}^2 = \hat{j}^2 = \hat{k}^2 = 1\right]$$



Note: If α, β and γare angles made by R with R_x, R_y and R_z then direction cosine of vector is given by $\cos\alpha = \frac{R_x}{R}$, $\cos\beta = \frac{R_y}{R}$ and cos

$$\gamma = \frac{R_z}{R}.$$

Q.24. Whether it. is possible to add two vectors representing physical quantities having different dimensions?

Ans: It is not possible to add two vectors representing physical quantities having different dimensions.

Q.25. Is it possible to add two velocities using triangle law?

Ans: Yes, It is possible to add two velocities using triangle law.

Q.26. Whether the subtraction of given vectors is commutative or associative?

Ans: The subtraction of given vectors is neither commutative nor associative.

Q.27. The diagonal of the parallelogram made by two vectors as adjacent sides is not passing through common point of two vectors. What does it represent?

Ans: The diagonal of the parallelogram made by two vectors as adjacent sides is not passing through common point of two vectors. This represents triangle law of vector addition.

Q.28. If $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$ then what can be the angle between $\vec{A}$ and $\vec{B}$?

Ans: Let θ be the angle between $\vec{A}$ and $\vec{B}$, then

$|\vec{A} + \vec{B}|^2 = A^2 + B^2 + 2AB\cos\theta$

Also the angle between $\vec{A}$ and $-\vec{B}$ is $(180° - \theta)$

Hence $|\vec{A} + \vec{B}|^2 = A^2 + B^2 + 2AB\cos(180° - \theta)$

$= A^2 + B^2 - 2AB\cos\theta$

As $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$, we can equate above two equations, $2AB\cos\theta = -2AB\cos\theta$

$\Rightarrow 4AB\cos\theta = 0$

Assuming $\vec{A}$ and $\vec{B}$ as nonzero vector,

we get, $\cos\theta = 0$

$\Rightarrow \theta = 90°$

Thus, if $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$, then vectors $\vec{A}$ and $\vec{B}$ must be at right angles to each other.

Q.29.What are (i) dimensions and (ii) units of a unit vector?

Ans: Unit vector does not have any dimensions and Unit.Unit vector is used to specify direction only.

Q.30. If the frame of reference is rotated or displaced, then what happens to the vector and its components?

Ans: If the frame of reference is rotated or displaced, the vector will not change since it is independent of choice of frame of reference. However, if the frame of reference is displaced, the magnitude of components of vector will change with the co-ordinate axes. This is because when the frame of reference is rotated or displaced, co-ordinate axes changes. If the frame of reference is rotated, the components as well as direction cosines of the vector change.

2.3 : Product of vectors :

Q.31. Define scalar product of two vectors. Give suitable examples.

Ans: Scalar product of two vectors:

i. The scalar product of two non-zero vectors is defined as the product of the magnitude of the two vectors and cosine of the angle θ between the two vectors.

ii. The dot sign is used between the two vectors to be multiplied therefore scalar product is also called dot product.

iii. The scalar product of two vectors $\vec{P}$ and $\vec{Q}$ is given by,

$\vec{P}.\vec{Q} = PQ\cos\theta$

where, P = magnitude of $\vec{P}$

Q = magnitude of $\vec{Q}$

θ = angle between $\vec{P}$ and $\vec{Q}$

iv. Examples of scalar product:

a. Work is a scalar product of force ($\vec{F}$) and displacement ($\vec{s}$).

$\therefore$ $W = \vec{F}\cdot\vec{s}$

b. Power ($\vec{P}$) is a scalar product of force ($\vec{F}$) and velocity ($\vec{v}$)

$\therefore$ $P = \vec{F}\cdot\vec{v}$

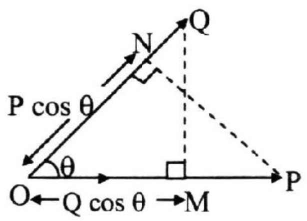
c. Magnetic flux (ϕ) linked with a surface is a scalar product of magnetic induction ($\vec{B}$) and the area vector ($\vec{A}$)

$\therefore$ $\phi = \vec{B}\cdot\vec{A}$

Q.32. Give the geometrical meaning of dot product of two vectors.

Ans: i. The scalar product of two vectors is equivalent to the product of magnitude of one vector with component of the other in the direction of the first.

ii. Let $\vec{P}$ and $\vec{Q}$ be two vectors with an angle θ between them as shown in figure.



iii. From definition of dot product,

$\vec{P} \cdot \vec{Q} = P\,Q \cos\theta$

$= P\,(Q \cos\theta)$

$= P$ (component of $\vec{Q}$ in direction of $\vec{P}$)

Similarly, $\vec{Q} \cdot \vec{P} = Q\,P \cos\theta$

$= Q\,(P\cos\theta)$

$= Q$ (Component of $\vec{P}$ in direction of $\vec{Q}$)

Q.33. Define scalar product of two vectors. State the characteristics of scalar product.

Ans: The scalar product of two non-zero vectors is defined as the product of the magnitude of the two vectors and cosine of the angle θ between the two vectors.

The scalar product of two vectors $\vec{P}$ and $\vec{Q}$ is given by, $\vec{P} \cdot \vec{Q} = PQ\cos\theta$

Characteristics of the scalar product of two vectors:

i. The scalar product of two vectors obeys the commutative law of multiplication.

$\vec{P} \cdot \vec{Q} = PQ\cos\theta = QP\cos\theta = \vec{Q} \cdot \vec{P}$

$\vec{P} \cdot \vec{Q} = \vec{Q} \cdot \vec{P}$

ii. The scalar product obeys the distributive law of multiplication.

$\vec{P} \cdot (\vec{Q} + \vec{R}) = \vec{P} \cdot \vec{Q} + \vec{P} \cdot \vec{R}$

iii. The scalar product of a vector with itself (i.e., self dot product) is equal to the square of its magnitude.

$\vec{P} \cdot \vec{P} = PP\cos 0^\circ = P^2$

iv. The scalar product of two mutually, perpendicular vectors is equal to zero.

$\vec{P} \cdot \vec{Q} = PQ\cos 90^\circ = 0$

The converse is also true i.e., if the scalar product of two non zero vectors is zero, then the vectors are perpendicular to each other.

v. The scalar product of two parallel vectors is equal to their product of magnitudes.

If $\vec{P}$ and $\vec{Q}$ are parallel, $\theta = 0^\circ$ and $\cos 0^\circ = 1$

$\therefore$ $\vec{P} \cdot \vec{Q} = PQ\cos 0^\circ = PQ$

vi. For two antiparallel vectors $\vec{A}$ and $\vec{B}$ ($\theta = 180^\circ$), $\vec{A} \cdot \vec{B} = -AB$

vii. The scalar product of two vectors is equivalent to the product of magnitude of one vector with component of the other in the direction of the first. This gives the geometrical meaning of scalar product.

$\vec{A} \cdot \vec{B} = |\vec{A}|$ (component of $\vec{B}$ in the direction of $\vec{A}$)

$= |\vec{B}|$ (component of $\vec{A}$ in the direction of $\vec{B}$)

viii. Scalar product of rectangular unit vectors:

$\hat{i}.\hat{i} = |\hat{i}||\hat{i}|\cos 0^\circ = 1$

Similarly, $\hat{j}.\hat{j} = 1$ and $\hat{k}.\hat{k} = 1$

Thus, $\hat{i}.\hat{j} = |\hat{i}|.|\hat{j}|\cos 90^\circ = 0$

$(\because \hat{i} \perp \hat{j}, \theta = 90^\circ)$

Similarly, $\hat{j}.\hat{k} = \hat{k}.\hat{i} = \hat{j}.\hat{i} = \hat{k}.\hat{j} = \hat{i}.\hat{k} = 0$

Tabular form of dot product of unit vector is given below:

	i	j	k
i	1	0	0
j	0	1	0
k	0	0	1

Q.34. Derive an expression for scalar product of vectors in terms of their scalar components.

Ans: Expression for scalar product of two vectors:

i. Let two vectors $\vec{P}$ and $\vec{Q}$ are represented in magnitude by,

$$\vec{P} = \hat{i}P_x + \hat{j}P_y + \hat{k}P_z$$

$$\vec{Q} = \hat{i}Q_x + \hat{j}Q_y + \hat{k}Q_z$$

ii. Scalar product of $\vec{P}$ and $\vec{Q}$ is given by,

$$\vec{P}\cdot\vec{Q} = (\hat{i}P_x + \hat{j}P_y + \hat{k}P_z).(\hat{i}Q_x + \hat{j}Q_y + \hat{k}Q_z)$$

$$= P_xQ_x(\hat{i}.\hat{i}) + P_xQ_y(\hat{i}.\hat{j}) + P_xQ_z(\hat{i}\cdot\hat{k})$$

$$+ P_yQ_x(\hat{j}.\hat{i}) + P_yQ_y(\hat{j}.\hat{j}) + P_yQ_z(\hat{j}.\hat{k})$$

$$+ P_zQ_x(\hat{k}.\hat{i}) + P_zQ_y(\hat{k}.\hat{j}) + P_zQ_z(\hat{k}.\hat{k})$$

iii. Since, $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$ and

$$\hat{j}.\hat{k} = \hat{k}.\hat{i} = \hat{j}.\hat{i} = \hat{k}.\hat{j} = \hat{i}.\hat{k} = 0$$

$$\vec{P}\cdot\vec{Q} = P_xQ_x + P_yQ_y + P_zQ_z$$

Q.35.A. Define vector product of two vectors.

B. Explain vector product of two vectors with suitable examples.

Ans: Statement:

i. The vector product of two vectors is a third vector whose magnitude is equal to the product of magnitude of the two vectors and sine of the smaller angle θ between the two vectors.

ii. Vector product is also called cross product of vectors because cross sign is used to represent vector product.

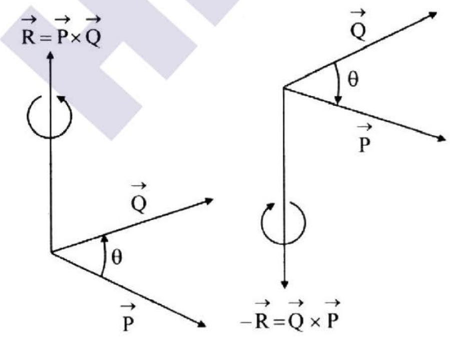
Explanation:

iii. The vector product of two vectors $\vec{P}$ and $\vec{Q}$, is a third vector $\vec{R}$ and is written as,

$$\vec{R} = \vec{P} \times \vec{Q}$$

iv. The magnitude of $\vec{R} = |\vec{R}| = PQ\sin\theta$

Where $0 \le \theta \le \pi$.



Vector product of two vectors

v. The direction of the resultant vector $\vec{R}$ is perpendicular to the plane formed by $\vec{P}$ and $\vec{Q}$.

The direction of $\vec{R}$ can be found by the right hand screw rule.

vi. If rotation is anticlockwise then resultant vector is positive, whereas if rotation is clockwise then resultant vector is negative.

vii. Examples of vector product:

a. Moment of a force or torque ($\vec{\tau}$) is the vector product of the position vector ($\vec{r}$) and the force ($\vec{F}$).

$$\vec{\tau} = \vec{r} \times \vec{F}$$

b. The angular momentum ($\vec{L}$) of the particle is the vector product of the position vector ($\vec{r}$) and the linear momentum ($\vec{p}$) of the particle.

$$\vec{L} = \vec{r} \times \vec{p}$$

c. The instantaneous velocity ($\vec{v}$) of a particle is equal to the cross product of its angular velocity ($\vec{w}$) and its position vector ($\vec{r}$).

$$\vec{v} = \vec{r} \times \vec{\omega}$$

d. The torque ($\vec{\tau}$) acting on a bar magnet freely suspended in a uniform magnetic induction ($\vec{B}$) is given by, $\vec{\tau} = \vec{M} \times \vec{B}$

Where, $\vec{M}$ is the magnetic dipole moment of the bar magnet.

d. The torque ($\vec{\tau}$) acting on a bar magnet freely suspended in a uniform magnetic induction ($\vec{B}$) is given by, $\vec{\tau} = \vec{M} \times \vec{B}$

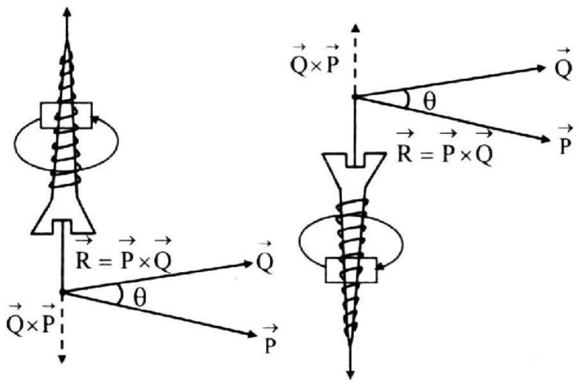
Where, $\vec{M}$ is the magnetic dipole moment of the bar magnet.

Q.36. Explain right handed screw rule.

Ans: i. Statement of Right handed screw rule:

Hold a right handed screw with its axis perpendicular to the plane containing vectors and the screw rotated from first vector to second vector through a small angle, the direction in which the screw tip would advance is the direction of the vector product of two vectors.

ii. **Diagramatical representation:**



Right handed screw rule

If R is the magnitude of resultant vector product, then $\vec{R} = \vec{P} \times \vec{Q} = PQ \sin\theta\ \hat{n}$ where, fi is a unit vector in the direction of $\vec{R}$.

Q.37. State the characteristics of the vector product (cross product) of two vectors.

Ans: Characteristics of the vector product (cross product):

i. The vector product of two vectors does not obey the commutative law of multiplication.

$\vec{P} \times \vec{Q} \neq \vec{Q} \times \vec{P}$ because $\vec{P} \times \vec{Q} = PQ \sin\theta$ and $\vec{Q} \times \vec{P} = PQ \sin(-\theta) = -PQ \sin\theta$

$\therefore$ $\vec{P} \times \vec{Q} = -(\vec{Q} \times \vec{P})$

ii. The vector product obeys the distributive law of multiplication.

$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$

iii. The vector product of a vector with itself (i.e., self cross product) is equal to zero.

If $\vec{P} = \vec{Q}$ then,

$|\vec{P} \times \vec{Q}| = |\vec{P} \times \vec{P}| = PP \sin 0° = 0$

iv. The vector product of two non zero perpendicular vectors is equal to the product of their magnitudes.

If $\vec{P}$ and $\vec{Q}$ are perpendicular, then θ = 90° and $\sin\theta = \sin 90° = 1$

$\therefore$ $|\vec{P} \times \vec{Q}| = PQ \sin\theta = PQ$

v. Vector product of two parallel or antiparallel vectors is equal to zero.

If $\vec{P}$ and $\vec{Q}$ are parallel, $\theta = 0°$ and $\sin 0 = 0$

$\therefore$ $|\vec{P} \times \vec{Q}| = PQ \sin\theta° = 0$

If $\vec{P}$ and $\vec{Q}$ are antiparallel, $\theta = 180°$ and $\sin 180° = 0$

$|\vec{P} \times \vec{Q}| = PQ \sin 180° = 0$

vi. Vector product of rectangular unit vectors is given by cycle rule.
According to this rule :

a. If rotation is in anticlockwise direction then,

$\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$

$\hat{i}\times\hat{j} = \hat{k}, \hat{j}\times\hat{k} = \hat{i}, \hat{k}\times\hat{i} = \hat{j}$

b. If rotation is in clockwise direction then,

$\hat{j}\times\hat{i} = -\hat{k}, \hat{k}\times\hat{j} = -\hat{i}, \hat{i}\times\hat{k} = -\hat{j}$

Cross product of unit vectors can also be remembered by using table given below

	$\hat{i}$	$\hat{j}$	$\hat{k}$
$\hat{i}$	0	$\hat{k}$	$-\hat{j}$
$\hat{j}$	$-\hat{k}$	0	$\hat{i}$
$\hat{k}$	$\hat{j}$	$-\hat{i}$	0

vii. The vector product of two vectors can be expressed in terms of their components.

If $\vec{P} = \hat{i}P_x + \hat{j}P_y + \hat{k}P_z$

$\vec{Q} = \hat{i}Q_x + \hat{j}Q_y + \hat{k}Q_z$

$\vec{P} \cdot \vec{Q} = (\hat{i}P_x + \hat{j}P_y + \hat{k}P_z).(\hat{i}Q_x + \hat{j}Q_y + \hat{k}Q_z)$

$+ P_xQ_z(\hat{i}\cdot\hat{k})$

Q.38. Derive an expression for cross product of two vectors and express it in determinant form.

Ans: Expression for cross product of two vectors:

i. Let two vectors $\vec{P}$ and $\vec{Q}$ be represented in magnitude and direction by,

If $\vec{P} = \hat{i}P_x + \hat{j}P_y + \hat{k}P_z$ and

$\vec{Q} = \hat{i}Q_x + \hat{j}Q_y + \hat{k}Q_z$

ii. Cross product of vector $\vec{P}$ and $\vec{Q}$ is given by,

$$\vec{P}\cdot\vec{Q} = (\hat{i}P_x + \hat{j}P_y + \hat{k}P_z).(\hat{i}Q_x + \hat{j}Q_y + \hat{k}Q_z)$$

$$= P_xQ_x(\hat{i}.\hat{i}) + P_xQ_y(\hat{i}.\hat{j}) + P_xQ_z(\hat{i}\cdot\hat{k})$$

$$+ P_yQ_x(\hat{j}.\hat{i}) + P_yQ_y(\hat{j}.\hat{j}) + P_yQ_z(\hat{j}.\hat{k})$$

$$+ P_zQ_x(\hat{k}.\hat{i}) + P_zQ_y(\hat{k}.\hat{j})$$

$$+ P_zQ_z(\hat{k}.\hat{k})$$

$$\therefore \quad \vec{P}\cdot\vec{Q} = (\hat{i}P_x + \hat{j}P_y + \hat{k}P_z).(\hat{i}Q_x + \hat{j}Q_y + \hat{k}Q_z) + P_xQ_z(\hat{i}\cdot\hat{k})$$

$$\therefore \quad \vec{P}\cdot\vec{Q} = (\hat{i}P_x + \hat{j}P_y + \hat{k}P_z).(\hat{i}Q_x + \hat{j}Q_y + \hat{k}Q_z) + P_xQ_z(\hat{i}\cdot\hat{k})$$

iii. Determinant form of cross product of two vectors $\vec{P}$ and $\vec{Q}$ is given by,

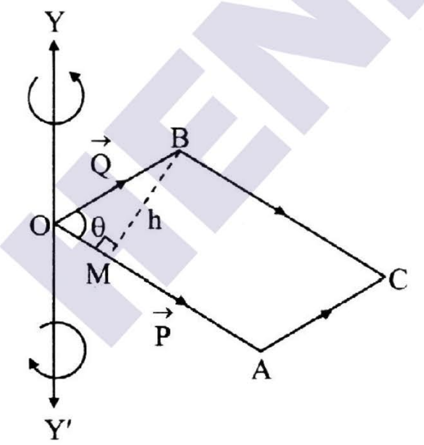
$$\vec{P}\times\vec{Q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

Q.39. Show that magnitude of vector product of two vectors is numerically equal to the area of a parallelogram formed by the two vectors.

Ans: Suppose OACB is a parallelogram of adjacent sides, $\vec{OA} = \vec{P}$ and $\vec{OB} = \vec{Q}$.

$\angle AOB = \theta$ as shown in figure.

We have to prove that area of parallelogram OACB $= \vec{P}\times\vec{Q}$



In right angled ΔOBM,

$$\sin\theta = \frac{BM}{OB} = \frac{h}{OB}$$

$$\therefore \quad h = OB\sin\theta$$

$$= Q\sin\theta$$

Now,

Area of parallelogram,

OACB = Base × height

$= OA \times h = P\,(Q\sin\theta) = PQ\sin\theta$

Now, since $PQ\sin\theta = |\vec{P}\times\vec{Q}|$

$\therefore$ Area of parallelogram OACB $= |\vec{P}\times\vec{Q}|$

= magnitude of the vector product.

Note: If $\hat{n}$ is a unit vector perpendicular to the plane then, $\vec{P}\times\vec{Q} = PQ\sin\theta\,\hat{n}$.

$$\therefore \quad \hat{n} = \frac{\vec{P}\times\vec{Q}}{PQ\sin\theta}$$

Q.40. Distinguish between scalar product (dot product) and vector product (cross product).

Ans:

No	Scalar product	Vector product
i.	The magnitude of a scalar product is equal to the product of the magnitudes of the two vectors and the cosine of the angle between them. $\vec{P}\cdot\vec{Q} = PQ\cos\theta$.	The magnitude of a vector product is equal to the product of the magnitude of the two vectors and sine of small angle (θ) between them. $\lvert\vec{P}\rvert\times\lvert\vec{Q}\rvert = PQ\sin\theta$.
ii.	It has no direction.	Its direction is perpendicular to the plane of the two vectors, i.e. in the sense of advancement of a right–handed screw.
iii.	It obeys the commutative law of vector multiplication	It does not obey the commutative law of vector multiplication.
iv.	It is zero if the two vectors are mutually perpendicular to each other.	It is zero if the two vectors are parallel or antiparallel to each other.
v.	The self dot-product of a vector is equal to the square of its magnitude.	The self cross-product of a vector is zero.

Q.41. Can you associate vectors with (a) the length of wire bent into a loop, (b) a plane area? (NCERT)

Ans: a. A vector can be associated with the length of the wire bent into a loop such that corresponding resultant vector will be a null vector.

b. Area $\vec{A} = \vec{l} \times \vec{b}$ where $\vec{l}$ and $\vec{b}$ is length and breadth of area respectively. Their cross product will result in a vector, which represents area of that plane.

Q.42. State with reasons, whether the foUowing algebraic operations with scalar and vector physical quantities are meaningful.

a. Adding any two scalars,

b. Adding a scalar to a vector of the same dimensions,

c. Multiplying any vector by any scalar,

d. Multiplying any two scalars,

e. Adding any two vectors. (NCERT)

Ans: a. Not any two scalars can be added. To add two scalars it is essential that they represent same physical quantity.

b. This operation is meaningless. Only a vector can be added to another vector.

c. This operation is possible. When a vector is multiplied with a dimensional scalar, the resultant vector will have different dimensions.

eg. acceleration vector is multiplied with mass (a dimensional scalar), the resultant vector has the dimensions of force.

When a vector is multiplied with nondimensional scalar, it will be a vector having dimensions as that of the given vector.

eg : $\vec{A} \times 3 = 3\vec{A}$

d. This operation is possible. Multiplication of non-dimensional scalars is simply algebraic multiplication. Multiplication of non dimensional scalars will result into scalar with different dimensions.

eg. Volume × density = mass.

e. Not any two vectors can be added. To add two vectors it is essential that they represent same physical quantity .

Q.43. Resultant of the product of two vectors always gives a vector, true or false? Explain.

Ans: It is false. Resultant of the product of two vectors does not always give vector. The dot product of two vectors is a scalar. For example work, $W = \vec{F} \cdot \vec{s}$ and power, $P = \vec{F} \cdot \vec{v}$.

Formulae :

1. Resultant of addition of two vectors $\vec{P}$ and $\vec{Q}$:

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

2. Direction of resultant vector :

$$\alpha = \tan^{-1}\left[\frac{Q\sin\theta}{P + Q\cos\theta}\right]$$

Where $\vec{P}$ and $\vec{Q}$ are two adjacent vectors.

3. Commutative law of vector addition:

$$\vec{P} + \vec{Q} = \vec{Q} + \vec{P}$$

4. Associative law of vector addition:

$$\vec{P} + (\vec{Q} + \vec{R}) = (\vec{P} + \vec{Q}) + \vec{R}$$

5. Distributive law of multiplication over addition:

$$\vec{P} \times (\vec{Q} + \vec{R}) = \vec{P} \times \vec{Q} + \vec{P} \times \vec{R}$$

6. Distributive law of multiplication over subtraction:

$$\vec{P} \times (\vec{Q} - \vec{R}) = \vec{P} \times \vec{Q} - \vec{P} \times \vec{R}$$

7. Magnitude of resolution of a vector along two rectangular component:

$$R = \sqrt{R_x^2 + R_y^2}$$

8. Resultant of three components:

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

9. Angle of inclination of resultant with positive direction of X-axis:

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

10. Scalar (dot) product of two vectors:

i. $\vec{P}.\vec{Q} = PQ\cos\theta$

ii. $\cos\theta = \dfrac{\vec{P}.\vec{Q}}{|P|.|Q|}$

iii. $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$

iv. $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$

11. Vector (cross) product of two vectors:

i. $\vec{P} \times \vec{Q} = PQ\sin\theta$

ii. $\sin\theta = \dfrac{\vec{P} \times \vec{Q}}{|P|.|Q|}$

iii. Unit vector perpendicular to the cross product,

$$\hat{n} = \frac{\vec{P} \times \vec{Q}}{PQ\sin\theta}$$

iv. Cross product of unit vectors,

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$$

v. $\hat{i} \times \hat{j} = \hat{k}, \hat{j} \times \hat{k} = \hat{i}, \hat{k} \times \hat{i} = \hat{j}$

12. Direction cosine of a vector:

i. $\cos\alpha = \dfrac{R_x}{R}$

ii. $\cos\beta = \dfrac{R_y}{R}$

iii. $\cos\gamma = \dfrac{R_z}{R}$

13. Area of parallelogram: cross product of two vectors representing its adjacent sides

14. Area of triangle:

$\dfrac{1}{2} \times$ [cross product of two adjacent sides]

Solved Examples :

Type 1: Problems based on addition and subtraction of vectors

Example 1

Find the vector drawn from the point (–4, 10, 7) to the point (3, –2, 1). Also find its magnitude.

Solution:

If $\vec{A}$ is a vector drawn from the point (x_1, y_1, z_1) to the point (x_2, y_2, z_2), then

$\vec{A} = \hat{i}\,(x_2 - x_1) + \hat{j}\,(y_2 - y_1) + \hat{k}\,(z_2 - z_1)$

Here,

$x_1 = -4,\ y_1 = 10,\ z_1 = 7,\ x_2 = 3,\ y_2 = -2,\ z_2 = 1$

$\therefore\quad \vec{A} = \hat{i}\,[3-(-4)] + \hat{j}\,(-2-10) + \hat{k}\,(1-7)$

$\therefore\quad \vec{A} = 7\hat{i} - 12\hat{j} - 6\hat{k}$

If $\vec{A} = \hat{i}A_x + \hat{j}A_y + \hat{k}A_z$, then magnitude of $\vec{A}$ is given by,

$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

Here $A_x = 7,\ A_y = -12,\ A_z = -6$

$\therefore\quad A = \sqrt{7^2 + (-12)^2 + (-6)^2}$

$= \sqrt{49 + 144 + 36} = \sqrt{229}$

= 15.13 units

Ans: $\vec{A}$ is $7\hat{i} - 12\hat{j} - 6\hat{k}$ and its magnitude is **15.13 units.**

Example 2

In a cartesian co-ordinate system, the co-ordinates of two points P and Q are (2, 4, 4) and (–2, –3, 7) respectively, find $\overline{PQ}$ and its magnitude.

Solution:

Given : Position vector of P = (2, 4, 4)

$\overline{OP} = 2\hat{i} + 4\hat{j} + 4\hat{k}$

Position vector of Q = (–2, –3, 7)

$\overline{OQ} = -2\hat{i} - 3\hat{j} + 7\hat{k}$

To find : $\overline{PQ}$, its magnitude ($|\overline{PQ}|$)

Formula : $\overline{PQ} = \overline{OQ} - \overline{OP}$

Calculation : From formula,

$\therefore\quad \overline{PQ} = (-2\hat{i} - 3\hat{j} + 7\hat{k}) - (2\hat{i} + 4\hat{j} + 4\hat{k})$

$\therefore\quad \overline{PQ} = -4\hat{i} - 7\hat{j} + 3\hat{k}$

$|\overline{PQ}| = \sqrt{(-4)^2 + (-7)^2 + (3)^2}$

$= \sqrt{16 + 49 + 9} = \sqrt{74}$

$\therefore\quad |\overline{PQ}| =$ **8.6 unit**

Ans: $\overline{PQ}$ is $-4\hat{i} - 7\hat{j} + 3\hat{k}$ and its magnitude is **8.6 units.**

Example 3

If $\vec{P} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{Q} = 2\hat{i} - 5\hat{j} + 2\hat{k}$. Find

i. $\vec{P} + \vec{Q}$ **ii.** $3\vec{P} - 2\vec{Q}$

Solution:

Given : $2\hat{i} + 3\hat{j} - \hat{k}$, $\vec{Q} = 2\hat{i} - 5\hat{j} + 2\hat{k}$

To find : i. $\vec{P} + \vec{Q}$ ii. $3\vec{P} - 2\vec{Q}$

Calculation :

i. $\vec{P} + \vec{Q} = (2+2)\hat{i} + (3-5)\hat{j} + (2-1)\hat{k}$

$= 4\hat{i} - 2\hat{j} + \hat{k}$

ii. $3\vec{P} = 3(2\hat{i} + 3\hat{j} - \hat{k}) = 6\hat{i} + 9\hat{j} - 3\hat{k}$

$2\vec{Q} = 2(2\hat{i} - 5\hat{j} + 2\hat{k}) = 4\hat{i} - 10\hat{j} + 4\hat{k}$

$\therefore\quad 3\vec{P} - 2\vec{Q} = (6\hat{i} + 9\hat{j} - 3\hat{k}) - (4\hat{i} - 10\hat{j} + 4\hat{k})$

$= 6\hat{i} + 9\hat{j} - 3\hat{k} - 4\hat{i} + 10\hat{j} - 4\hat{k}$

$\therefore\quad 3\vec{P} - 2\vec{Q} = 2\hat{i} + 19\hat{j} - 7\hat{k}$

Ans: $\vec{P} + \vec{Q}$ is $4\hat{i} - 2\hat{j} + \hat{k}$

$3\vec{P} - 2\vec{Q} = 2\hat{i} + 19\hat{j} - 7\hat{k}$

Example 4

Two forces, F_1 and F_2, each of magnitude 5 N are inclined to each other at 60°. Find the magnitude and direction of their resultant force.

Solution:

Given : $F_1 = 5$ N, $F_2 = 5$ N, $\theta = 60°$

To find : Magnitude of resultant force (R), Direction of resultant force (α)

Formulae :

i. $|R| = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

ii. $\alpha = \tan^{-1}\left(\dfrac{Q\sin\theta}{P + Q\cos\theta}\right)$

Calculation : From formula (i),

$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

$= \sqrt{52 + 52 + 2 \times 5 \times 5 \times \cos 60^\circ}$

$= \sqrt{25 + 25 + 25}$

$= 5\sqrt{3}$ N

$=$ **8.662 N**

From formula (ii),

$$\alpha = \tan^{-1}\left(\frac{F_2 \sin\theta}{F_1 + F_2 \cos\theta}\right)$$

$$\therefore \quad \alpha = \tan^{-1}\left(\frac{5\sin 60^\circ}{5 + 5\cos 60^\circ}\right)$$

$$= \tan^{-1}\left(\frac{\frac{5\sqrt{3}}{2}}{5(1+\cos 60^\circ)}\right)$$

$$= \tan^{-1}\left(\frac{\frac{5\sqrt{3}}{2}}{5 \times 2\cos^2 30^\circ}\right)$$

$$[\because 1 + \cos\theta = 2\cos^2\frac{\theta}{2}]$$

$$= \tan^{-1}\left(\frac{\frac{5\sqrt{3}}{2}}{10 \times \frac{3}{4}}\right)$$

$$= \tan^{-1}\left(\frac{5\sqrt{3}}{2} \times \frac{4}{30}\right)$$

$$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$\therefore \quad \alpha = 30^\circ$

Ans: i. The magnitude of resultant force is **8.66 N**

ii. The direction of resultant force is 30° w.r.t. $\vec{F}_1$.

Example 5

Find unit vector parallel to the resultant of the vectors $\vec{A} = \hat{i} + 4\hat{j} - 2\hat{k}$ and $\vec{B} = 3\hat{i} - 5\hat{j} + \hat{k}$

Solution:

The resultant of $\vec{A}$ and $\vec{B}$ is,

$\vec{R} = \vec{A} + \vec{B} = (\hat{i} + 4\hat{j} - 2\hat{k}) + (3\hat{i} - 5\hat{j} + \hat{k})$

$= (1+3)\hat{i} + (4-5)\hat{j} + (1-2)\hat{k}$

$= 4\hat{i} - \hat{j} - \hat{k}$

$|\vec{R}| = \sqrt{4^2 + (-1)^2 + (-1)^2}$

$= \sqrt{16+1+1} = \sqrt{18} = 3\sqrt{2}$

The unit vector parallel to $\vec{R}$ is,

$$\hat{R} = \frac{\vec{R}}{|\vec{R}|} = \frac{1}{3\sqrt{2}}(4\hat{i} - \hat{j} - \hat{k})$$

Ans: The required unit vector is $\frac{1}{3\sqrt{2}}(4\hat{i} - \hat{j} - \hat{k})$

Example 6

One of the rectangular components of a velocity of 80 kmh^{-1} is 40 kmh^{-1}. Find the other component.

Solution:

Given: $v = 80$ kmh^{-1}, $v_x = 40$ kmh^{-1}

To find: Velocity component (v_y)

Formula: $v = \sqrt{v_x^2 + v_y^2}$

Calculation: From formula,

$\therefore \quad v_y = \sqrt{v^2 - v_x^2} = \sqrt{80^2 - 40^2}$

$= \sqrt{6400 - 1600}$

$= \sqrt{4800}$

$\therefore \quad v_y = 69.28$ kmh^{-1}

Ans: The other rectangular component of velocity is **69.28 kmh^{-1}**.

Example 7

Determine the angles which the vector $\vec{A} = 5\hat{i} + 0\hat{j} + 5\hat{k}$ makes with X, Y and Z axes.

Solution:

$A = |\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$= \sqrt{5^2 + 0^2 + 5^2} = 5\sqrt{2}$

If vector $\overrightarrow{OA}$ makes angles α, β and γ with X, Y and Z–axis respectively, then

$\cos\alpha = \frac{A_x}{A} = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}} \quad \therefore \quad \alpha = 45^\circ$

$\cos\beta = \frac{A_y}{A} = \frac{0}{5\sqrt{2}} = 0 \qquad \therefore \quad \beta = 90^o$

$\cos\gamma = \frac{A_z}{A} = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}} \qquad \therefore \quad \gamma = 45^o$

Ans: The vector makes angles of 45°, 90° and 45° w.r.t. axis X, Y and Z respectively.

Type 11 : Problems based on product of vectors

Example 8

Find the angle between the vectors $\vec{A} = \hat{i}+2\hat{j}-\hat{k}$ and $\vec{B} = -\hat{i}+\hat{j}-2\hat{k}$.

Solution:

Given: $\vec{A} = \hat{i}+2\hat{j}-\hat{k}$, $\vec{B} = -\hat{i}+\hat{j}-2\hat{k}$

To find: Angle between the vectors (θ)

Formula: $\cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}$

Calculation: From formula,

$|\vec{A}| = \sqrt{1^2+2^2+(-1)^2} = \sqrt{6}$

$|\vec{B}| = \sqrt{(-1)^2+1^2+(-2)^2} = \sqrt{6}$

$\vec{A}.\vec{B} = 1 \times (-1) + 2 \times 1 + (-1) \times (-2)$
$= -1 + 2 + 2 = 3$

$\therefore \quad \cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|} = \frac{3}{\sqrt{6}\times\sqrt{6}}$

$\cos\theta = \frac{3}{6} = \frac{1}{2}$

$\therefore \quad \theta = \cos^{-1}\left(\frac{1}{2}\right) = 60^o$

$\therefore \quad \theta = 60°$

Ans: The angle between the vectors is **60°**.

Example 9

1f $\vec{A} = 2\hat{i}-\hat{j}+\hat{k}$ and $\vec{B} = \hat{i}+2\hat{j}-\hat{k}$ are two vectors, find $|\vec{A}\times\vec{B}|$

Solution:

Given: $\vec{A} = 2\hat{i}-\hat{j}+\hat{k}$, $\vec{B} = \hat{i}+2\hat{j}-\hat{k}$

To find: $|\vec{A}\times\vec{B}|$

Formula: $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

Calculation: From formula,

$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 2 & -1 \end{vmatrix}$

$= -\hat{i}+3\hat{j}+5\hat{k}$

$|\vec{A}\times\vec{B}| = \sqrt{(-1)^2+(3)^2+(5)^2}$

$\therefore \quad |\vec{A}\times\vec{B}| = \sqrt{35}$
$= \mathbf{5.91}$

Example 10

A force $\vec{F} = 4\hat{i}+6\hat{j}+3\hat{k}$ acting on a particle produces a displacement of $\vec{s} = 2\hat{i}+3\hat{j}+5\hat{k}$ where F is expressed in newton and s in metre. Find the work done by the force.

Solution:

Given: $\vec{F} = 4\hat{i}+6\hat{j}+3\hat{k}$, $\vec{s} = 2\hat{i}+3\hat{j}+5\hat{k}$

To find: Work done (W)

Formula: $W = \vec{F}.\vec{s}$

Calculation: From formula,

$W = \vec{F}.\vec{s} = (4\hat{i}+6\hat{j}+3\hat{k}).(2\hat{i}+3\hat{j}+5\hat{k})$
$= (4 \times 2) + (6 \times 3) + (3 \times 5)$
$= 8 + 18 + 15$

$\therefore \quad W = \mathbf{41\ J}$

Ans: The work done by the force is **41 J**.

Example 11

One of the rectangular components of a velocity of 80 kmh^{-1} is 40 kmh^{-1}. Find the other component.

Solution:

Given: $v = 80$ kmh^{-1}, $v_x = 40$ kmh^{-1}

To find: Velocity component (v_y)

Formula: $v = \sqrt{v_x^2 + v_y^2}$

Calculation: From formula,

$\therefore \quad v_y = \sqrt{v^2 - v_x^2} = \sqrt{80^2 - 40^2}$
$= \sqrt{6400 - 1600}$
$= \sqrt{4800}$

$\therefore \quad v_y = 69.28$ kmh^{-1}

Ans: The other rectangular component of velocity is **69.28 kmh^{-1}**.

Additional Theory Questions :

Q.1. Define.

A. scalars

Ans: Refer Q.1 point (i)

B. vectors

Ans: Refer Q.1 point (ii)

Q.2. A. State and prove parallelogram law of vectors addition.

Ans: A. Refer Q.18 (a), (b)

Q.3. Using parallelogram law of vectors addition determine

A. agnitude and

B. direction of resultant vector

C. Determine the magnitude and direction of the resultant vector for $\theta = 0°, 90°$ and $180°$.

Ans: A. Refer Q.18 (c)

B. Refer Q.18 (d)

C. Refer Q.18 (e)

Q.4. Define vector product and give two examples.

Ans: Refer Q.35 points (i) and (vii-a.b).

Q.5. State right handed screw rule to give direction of cross product.

Ans: Refer Q.36 point (i).

Practice Problems :

Type I: Problems based on addition and subtraction of vectors

1. If $\vec{a} = 3\hat{i} - 4\hat{j}$ and $\vec{b} = 8\hat{i} + 6\hat{j}$, find the magnitude and direction of

   i. $\vec{a}$ ii. $\vec{b}$

   iii. $\vec{a} + \vec{b}$ iv. $\vec{a} - \vec{b}$

   v. $\vec{b} - \vec{a}$

2. Points, P and Q, have co-ordinates (1, 2, 3) and (4,5,6), respectively. Find PQ.

3. Find the angle between two equal vectors if their resultant is equal to either of them.

4. The velocity of a particle is $\vec{v} = 3\hat{i} + 2\hat{j} + 3\hat{k}$. Find the vector component of the velocity along the line $\hat{i} - \hat{j} + \hat{k}$ and its magnitude.

5. A velocity of 10 ms^{-1} has its Y-component $5\sqrt{2}ms^{-1}$. Calculate its X-component.

6. If $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$, find the angle between $\vec{A}$ and $\vec{B}$.

Type II : Problems based on product of vectors

7. Find $\vec{P}.\vec{Q}$ where

   $\vec{P} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{Q} = \hat{i} - \hat{j} + 2\hat{k}$

8. Find the angle between the vectors

   $\vec{A} = 4\hat{i} + 2\hat{j} - 4\hat{k}$ and $\vec{B} = \hat{i} + 4\hat{j} + 3\hat{k}$

9. If $\vec{A} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{B} = 3\hat{i} + 2\hat{j} + 4\hat{k}$, then find the value of $(\vec{A} + \vec{B}) \times (\vec{A} - \vec{B})$.

10. The diagonals of a parallelogram are given by the vectors $3\hat{i} + \hat{j} + 2\hat{k}$ and $\hat{i} - 3\hat{j} + 4\hat{k}$.

    Find the area of the parallelogram.

11. If $\vec{A} = \hat{i} + 3\hat{j} - \hat{k}$ and $\vec{B} = 2\hat{i} - \hat{j} + \hat{k}$, determine the unit vector parallel to $\vec{A} \times \vec{B}$. Also find the sine of the angle between $\vec{A}$ and $\vec{B}$.

Type III :Miscellaneous

12. Find the components along X and Y axes of a vector 20 unit long when it makes an angle of 30° with the X-axis.

13. If vectors $\vec{P}$, $\vec{Q}$ and $\vec{R}$ have magnitudes 5, 12 and 13 unit respectively and $\vec{P} + \vec{Q} = \vec{R}$, find the angle between $\vec{Q}$ and $\vec{R}$.

14. A body acted upon by a force of 50 N is displaced through a distance of 10m in a direction making an angle of 60° with force . Calculate the work done by the force.

Multipal Choice Questions

1. Which of the following is a vector?
   a) speed b) displacement
   c) mass d) time

2. The angle between the vectors $\vec{a} = 2\hat{i} + 3\hat{j}$ and $\vec{b} = 6\hat{i} - 4\hat{j}$ is
   a) 90° b) 60°
   c) 30° d) 0°

3. Two quantities of 5 and 12 unit when added gives a quantity 13 unit. This quantity is
   a) time b) mass
   c) linear momentum d) speed

4. If | $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ land a and b are non zero vectors, then
   a) $\vec{a}$ and $\vec{b}$ are antiparallel
   b) $|\vec{a}| = |\vec{b}|$
   c) $\vec{a}$ and $\vec{b}$ are zero parallel vectors
   d) $\vec{a}$ is $\perp$ to $\vec{b}$

5. The equation $\vec{a} + \vec{a} = \vec{a}$ is
   a) meaningless
   b) always true
   c) may be possible for limited values of 'a'
   d) true only when $\vec{a} = 0$

6. The resultant of two vectors of magnitude $|\vec{p}|$ is also $|\vec{p}|$. They act at an angle
   a) 60° b) 90°
   c) 120° d) 180°

7. Given that $\vec{R} = \vec{P} + \vec{Q}$. Which of the following relations is necessarily valid ?
   a) $P < Q$ b) $P > Q$
   c) $P = Q$ d) None of these

8. The minimum number of numerically equal vectors whose vector sum can be zero is
   a) 4 b) 3
   c) 2 d) 1

9. A force of 60 N acting perpendicular to a force of 80 N, magnitude of resultant force is
   a) 20 N b) 70 N
   c) 100 N d) 140 N

10. A river is flowing at the rate of 6 km h^{-1}, A man swims across it with a velocity of 9 km h^{-1}. The resultant velocity of the man will be
    a) $\sqrt{15}$ km h^{-1} b) $\sqrt{45}$ km h^{-1}
    c) $\sqrt{177}$ km h^{-1} d) $\sqrt{225}$ km h^{-1}

11. The vectors $\vec{A}$ and $\vec{B}$ are such that $\vec{A} + \vec{B} = \vec{C}$ and A^2 + B| = C^2. Angle e between positive directions of $\vec{A}$ and $\vec{B}$ is
    a) $\frac{\pi}{2}$ b) 0
    c) π d) $\frac{2\pi}{3}$

12. If $\vec{A} = \vec{B} + \vec{C}$ and magnitudes of $\vec{A}$, $\vec{B}$ and $\vec{C}$ are 5, 4 and 3 unit respectively, then angle between $\vec{A}$ and $\vec{B}$ is
    a) $\sin^{-1}(3/4)$ b) $\cos^{-1}(4/5)$
    c) $\tan^{-1}(5/3)$ d) $\cos^{-1}(3/5)$

13. Out of the following set of forces, the resultant of which set of forces can never be zero?
    a) 15,15,15 b) 15,30,60
    c) 15,15,30 d) 15,30,30

14. The expression $\frac{1}{\sqrt{2}}(\hat{i} + \hat{j})$ is a _____
    a) unit vector
    b) null vector
    c) vector of magnitude $\sqrt{2}$
    d) scalar

15. What is the angle between $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i}$?
    a) 0° b) $\frac{\pi}{6}$
    c) $\frac{\pi}{3}$ d) none of the above

16. $(\vec{P} + \vec{Q})$ is a unit vector along X-axis. If $\vec{p} = \hat{i} - \hat{j} + \hat{k}$, then $\vec{Q}$ is
    a) $\hat{i} + \hat{j} - \hat{k}$ b) $\hat{j} - \hat{k}$
    c) $\hat{i} + \hat{j} + \hat{k}$ d) $\hat{j} + \hat{k}$

17. The magnitude of scalar product of the vectors $\vec{A} = 2\hat{i} + 5\hat{k}$ and $\vec{B} = 3\hat{i} + 4\hat{k}$ is

a) 20 b) 22
c) 26 d) 29

18. If $\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{B} = 3\hat{i} - 2\hat{j} + \hat{k}$, then the area of parallelogram formed from these vectors as the adjacent sides will be

a) $2\sqrt{3}$ square units b) $4\sqrt{3}$ square units
c) $6\sqrt{3}$ square units d) $8\sqrt{3}$ square units

19. Three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ satisfy the relation $\vec{A} \cdot \vec{B} = 0$ and $\vec{A} \cdot \vec{C} = 0$, then $\vec{A}$ is parallel to

a) $\vec{B}$ b) $\vec{C}$
c) $\vec{B} \times \vec{C}$ d) $\vec{B} \cdot \vec{C}$

20. What vector must be added to the sum of two vectors $2\hat{i} - \hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} - 2\hat{k}$ so that the resultant is a unit vector along Z axis?

a) $5\hat{i} + \hat{k}$ b) $-5\hat{i} + 3\hat{k}$
c) $3\hat{i} + 5\hat{k}$ d) $-3\hat{j} + 2\hat{k}$

21. The maximum value of magnitude of $(\vec{A} - \vec{B})$ is

a) A – B b) A
c) A + B d) $\sqrt{(A^2 + B^2)}$

22. The magnitude of the X and Y components of $\vec{A}$ are 7 and 6. Also the magnitudes of the X and Y components of $\vec{A} + \vec{B}$ are 11 and 9 respectively. What is the magnitude of $\vec{B}$?

a) 5 b) 6
c) 8 d) 9

23. If $\vec{A} = 5\hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{B} = 2\hat{i} + \hat{j} + 2\hat{k}$, then component of $\vec{B}$ along $\vec{A}$ is

a) $\frac{\sqrt{28}}{38}$ b) $\frac{28}{\sqrt{38}}$
c) $\frac{\sqrt{28}}{48}$ d) $\frac{14}{\sqrt{38}}$

24. If $\vec{A} + \vec{B} = \vec{A} - \vec{B}$ then vector $\vec{B}$ must be

a) zero vector b) unit vector
c) Non zero vector d) equal to $\vec{A}$

25. Choose the WRONG statement

a) The division of vector by scalar is valid.
b) The multiplication of vector by scalar is valid.
c) The multiplication of vector by another vector is valid by using vector algebra.
d) The division of a vector by another vector is valid by using vector algebra.

26. A person moves from a point S and walks along the path which is a square of each side 50 m. He runs east, south, then west and finally north. Then the total displacement covered is

a) 200 m b) 100 m
c) 50 $\sqrt{2}$ m d) zero

27. Walking on an inclined plane, the road is an example of ______

a) vectors
b) scalars
c) resolution of Vectors
d) null vector

28. Which of the following is a scalar?

a) Electric field
b) Angular momentum
c) Angular frequency
d) Torque

29. Which of the following is a vector?

a) Pressure
b) Gravitational potential
c) Angle
d) Current density

30. The resultant of two forces of 3 Nand 4 N is 5 N, the angle between the forces is

a) 30° b) 60°
c) 90° d) 120°

31. Let the angle between two non-zero vectors $\vec{A}$ and $\vec{B}$ be 120° and its resultant be $\vec{C}$, then

a) $\vec{C}$ must be equal to |A – B|
b) $\vec{C}$ must be less than |A – B|
c) $\vec{C}$ must be greater than |A – B|
d) $\vec{C}$ must be zero

32. If $\hat{n}$ is the unit vector in the direction of $\vec{A}$ then,

a) $\hat{n} = \frac{\vec{A}}{|\vec{A}|}$ b) $\hat{n} = |\vec{A}|\,|\vec{A}|$

c) $\hat{n} = \frac{|\vec{A}|}{\vec{A}}$ d) $\hat{n} = \hat{n} \times \vec{A}$

33. What is the maximum number of components into which a force can be resolved?

a) Two b) Three
c) Four d) Any number

34. The unit vector along $\hat{i} + \hat{j}$ is

a) $\hat{k}$ b) $\hat{i} + \hat{j}$

c) $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$ d) $\frac{\hat{i} + \hat{j}}{2}$

35. If vectors $\vec{p}$ and $\vec{Q}$ are perpendicular to each other, then

a) $\vec{P}.\vec{Q} = 0$ b) $\vec{P} \times \vec{Q} = 0$

c) $\vec{P} + \vec{Q} = 0$ d) $\vec{P} - \vec{Q} = 0$

36. If $\vec{A}, \vec{B}$ and $\vec{C}$ are non zero vectors and $\vec{A}.\vec{B} = 0$ and $\vec{B}.\vec{C} = 0$ then magnitude of $\vec{A}.\vec{C}$ is

a) A + C b) AC
c) AB d) BC

Answer Keys

1. b)	2. a)	3. c)	4. d)	5. d)	6. c)	7. d)	8. c)	9. c)	10. c)
11. a)	12. b)	13. b)	14. a)	15. d)	16. b)	17. c)	18. d)	19. c)	20. b)
21. c)	22. a)	23. d)	24. a)	25. d)	26. d)	27. a)	28. c)	29. d)	30. c)
31. c)	32. a)	33. d)	34. c)	35. a)	36. b)				

Answers in Paractic Problems :

1. i. $5, \theta = \tan^{-1}\left(\frac{-4}{3}\right)$

ii. $10, \theta = \tan^{-1}\left(\frac{3}{4}\right)$

iii. $5, \sqrt{5}, \theta = \tan^{-1}\left(\frac{2}{11}\right)$

iv. $5, \sqrt{5}, \theta = \tan^{-1}(2)$

v. $5, \sqrt{5}, \theta = \tan^{-1}(2)$

2. $3\left(\hat{i} + \hat{j} + \hat{k}\right)$

3. 120^o

4. $\left(\frac{4}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})\right), \frac{4}{\sqrt{3}}$

5. $5\sqrt{2}\text{ms}^{-1}$

6. 90^o

7. 3

8. 90o

9. $-20\,\hat{i} + 10\,\hat{j} + 10\,\hat{k}$

10. 8.66 sq. units.

11. $\left(\frac{2\hat{i} - 3\hat{j} - 7\hat{k}}{\sqrt{62}}\right), \sqrt{\frac{31}{33}}$

12. $10\sqrt{3}$ unit, 10 unit

13. $\cos^{-1}\left(\frac{12}{13}\right)$

14. 250 J