Projectile Motion

EXERCISE

3.0 : Introduction

Q.1. What is mechanics? State different types of motion in mechanics.

Ans: Mechanics is a branch of physics which deals with the study of motion of a particle.

Different types of motion in mechanics are as follows:

i. Translational motion: Motion of a particle from one point to another is called translational motion. If path is a straight line, then it is called rectilinear motion whereas if path is curved, then it is called curvilinear motion

ii. Vibrational motion: Periodic motion in which a particle moves back and forth about a mean position over the same path is called vibrational motion.

iii. Rotational motion: Motion of a particle is said to be rotational, if it describes concentric circles about the axis of rotation.

Q.2. Explain the following terms:

A. Displacement

B. Path length

Ans: A. Displacement:

i. Displacement of a particle is the change in its position in a particular direction.

ii. Let $\vec{x}_1$ and $\vec{x}_2$ be the position vectors of a particle at time t_1 and t_2 respectively. Then the displacement $\Delta\vec{x}$ in time $\Delta t = (t_2 - t_1)$ is given by,
$\Delta\vec{x} = \vec{x}_2 - \vec{x}_1$.

iii. If $\vec{x}_2 > \vec{x}_1$ then displacement is positive but if $\vec{x}_2 > \vec{x}_1$ then displacement is negative.

iv. It is a vector quantity.

v. S.L unit of displacement is metre.

B. Path length:

i. Path length is the actual distance travelled by the particle during its motion.

ii. For the motion of a particle from one point to another, the displacement may be zero but corresponding path length may not be zero.

iii. The displacement and the path length are equal only if the object doesn't change its direction during the course of motion.

iv. It is a scalar quantity.

v. S.L unit of path length is metre.

Q.3. Define position. Explain in brief why the position at all time instants must be specified to describe the motion of a particle.

Ans: A. The position of a point object at a given instant of time is a point in space at which the object exists at that instant of time.

B. Explanation:

i. To describe the motion of a particle, its position at all the time instants must be specified.

ii. Consider a particle moving along a straight line say, along X-axis. A suitable point on this line can be considered as the origin 0 as shown in figure (a)

iii. The position of a particle (x) can be described in terms of distance from the origin O. By cartesian sign convention, the distances measured to the right of O are considered as positive and those measured to the left of O are considered as negative.

iv. Suppose that a particle starts moving from origin O, then at t = 0, its position co-ordinate (x) is zero at point O.

v. During its motion, if the particle is at point P, then its position co-ordinate is 150 m. If it is at Q, then its position co-ordinate will be 200 m. If it is at R, then its position co-ordinate will be (–25 m.)

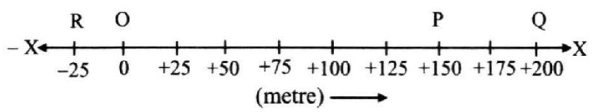


Figure (a) Horizontal line for position measurement

vi. By cartesian sign convention, the distances measured above origin O are taken as positive and those measured below O are taken as negative as shown in figure (b).

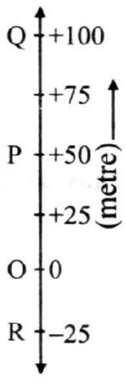


Figure (b) Vertical line for position measurement

vii. If a particle starts moving from origin O then, at t = 0, its position co-ordinate (y) is zero.

viii. During its motion if the particle is at point P, Q and R then its position co-ordinate will be 50 m, 100 m and –25 m respectively.

Q.4. Discuss the rules of sign convention in uniformly accelerated motion.

Ans: Rules of sign convention in uniformly accelerated motion:

i. In case of motion in the vertical direction, the vectors directed in vertically upward direction are considered to be positive.

ii. The vectors directed vertically downwards are considered to be negative.

iii. In case of motion in horizontal direction, vectors directed towards right are considered to be positive.

iv. Vectors directed towards left are considered to be negative.

Q.5. A. State main characteristics of displacement.

B. Under which condition are path length and displacement equal?

Ans: A. Main characteristics of displacement:

i. The magnitude of the displacement gives the shortest distance between the two points.

ii. The displacement may be zero, positive or negative and it is independent of the origin of coordinate system.

iii. The magnitude of displacement is less than or equal to the path length.

iv. Path length is scalar while displacement is a vector quantity.

v. The displacement is measured in the units of length.

B. The path length and displacement are equal only if the body doesn't change its direction during the course of motion.

e.g., A car moving along a straight path.

Q.6. Distinguish clearly between distance and displacement of a particle.

Ans:

No.	Distance	Displacement
i.	Distance is the total path length of a particle in a given time.	Displacement is the shortest distance between the initial point and final point.
ii.	Distance is a scalar quantity.	Displacement is a vector quantity.
iii.	Magnitude of distance is greater or equal to distance of particle.	Magnitude of displacement is less or equal to distance of particle.
iv.	Distance is always positive.	Displacement may be negative at some instant.

Q.7. Explain the terms:

A. Speed and **B. Velocity**

Ans: A. Speed:

i. Distance travelled by a moving body per unit time is called speed.

ii. It is a scalar quantity.

iii. $\text{Speed} = \frac{\text{distance travelled}}{\text{time taken}}$

iv. S.I. unit: m/s

v. Dimensions: $[M^0L^1T^{-1}]$

B. Velocity:

i. Distance travelled by a moving body per unit time in a given direction is called velocity.

ii. It is a vector quantity.

iii. Velocity $= \frac{\text{displacement}}{\text{time}} = \frac{\Delta\vec{x}}{\Delta t}$

iv. S.I. unit: m/s

v. Dimensions: $[M^0L^1T^{-1}]$

Q.8. Explain uniform motion of a particle.

OR

Write a short note on uniform motion.

Ans: i. Motion of a particle is said to be uniform, if it covers equal distances in equal intervals of time along a straight line.

ii. When a particle is in uniform motion, neither magnitude nor the direction of the velocity changes. Hence, the velocity is constant in uniform motion.

iii. The distance and displacement of the particle is same, hence the speed and the velocity of the body are of same value.

iv. e.g., A car moving along a straight line with constant speed.

Q.9. Explain the terms:

a. Average velocity

b. Instantaneous velocity

c. Uniform velocity

d. Average speed

e. Instantaneous speed

Ans: a. Average velocity:

i. Average velocity $(\vec{v}_{avg})$ of a particle is defined as the displacement $\Delta\vec{x}$ of the particle divided by the time interval Δt in which the displacement occurs.

ii. Average velocity $= \frac{\text{Displacement}}{\text{Time interval}}$

$$\vec{v}_{avg} = \frac{\vec{x}_2 - \vec{x}_1}{t_2 - t_1} = \frac{\Delta\vec{x}}{\Delta\vec{t}}$$

iii. Average velocity is a vector quantity. Its SI unit is m/s and dimension is $[M^0L^1T^{-1}]$

iv. The average velocity is zero for a particle at rest. For a particle in motion, the average velocity may be positive or negative.

b. Instantaneous velocity:

i. Velocity of a particle at any instant of time or any position in its path is called instantaneous velocity.

ii. The instantaneous velocity $\vec{v}$ is defined as the limit of the average velocity as the time interval 'Δt' becomes extremely small (i.e. infinitesimal).

iii. $\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{x}}{\Delta t} = \frac{d\vec{x}}{dt}$

iv. The instantaneous velocity may be positive, negative or zero.

v. For a particle moving with constant velocity, the instantaneous velocity at any instant is equal to the average velocity of the particle.

c. Uniform velocity:

i. A body is said to be moving with a uniform velocity if it covers equal displacements in equal intervals of time.

ii. In such a case, acceleration is zero.

iii. Uniform velocity is independent of time interval.

iv. No force acts on the body when it is moving with uniform velocity.

d. Average speed:

i. Average speed of a particle is defined as the total distance or path length travelled by the particle divided by total time interval during which the motion has taken place.

ii. Average speed $= \frac{\text{Total path length}}{\text{Total time interval}}$

iii. If a body covers equal distances with different speeds (say, v_1 and v_2), then the average speed (v_{av}) is equal to the harmonic mean of individual speeds.

i.e., $v_{av} = \frac{2v_1v_2}{v_1 + v_2}$

iv. If a body travels with speeds $v_1, v_2, v_3, ... ,v_N$ etc. during equal time intervals, then the average speed is equal to the arithmetic mean of individual speeds.

i.e., $v_{av} = \frac{v_1 + v_2 + v_3 + ... + v_N}{N}$

v. Average speed is a scalar quantity. Its S.I. unit is mls and dimension is $[M^0L^1T^{-1}]$

e. Instantaneous speed:

i. The magnitude of instantaneous velocity at any instant is called instantaneous speed at that instant.

ii. Using the language of calculus, it is the first derivative of distance with respect to time.

iii. For a body in uniform motion, the instantaneous speed is equal to its uniform speed.

iv. In a car, the speedometer measures its instantaneous speed.

Q.10. Explain the terms:

a. Acceleration

b. Average acceleration

c. Instantaneous acceleration

d. Uniform acceleration

e. Retardation

Ans: a. Acceleration:

i. Acceleration is defined as the rate of change of velocity with respect to time.

ii. If a particle moves with constant velocity, its acceleration is zero.

iii. S.I. unit: m/s^2

iv. Dimension: $[M^0L^1T^{-2}]$

b. Average acceleration:

i. Average acceleration is defined as the ratio of the change in velocity to the time taken to undergo the change in velocity.

OR

The average acceleration is defined as the change in velocity divided by the total time required for the change.

ii. If $\vec{v}_1$ and $\vec{v}_2$ are the velocities of the particle at time t_1 and t_2 respectively, the change in velocity is $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$ and time required for this change is $\Delta t = t_2 - t_1$

iii. The slope of straight line joining two points on velocity - time graph gives the average acceleration of the body between these two points.

iv. Average acceleration can be positive or negative depending upon the sign of slope of velocity - time graph and is zero when the change in velocity of the body in the given time interval is zero.

c. Instantaneous acceleration:

i. The acceleration of a particle at a particular instant of time is called the instantaneous acceleration $(\vec{a})$

ii. It is the limiting value of the average acceleration as time interval becomes infinitesimal.

$$\vec{a}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

$$= \frac{d\vec{v}}{dt}$$

iii. Generally, when the term acceleration is used, it is instantaneous acceleration.

iv. In terms of calculus, the instantaneous acceleration of an object is equal to the time derivative of velocity at a given instant.

v. It is also the slope of the tangent to the velocity - time graph at a position corresponding to given instant of time.

d. Uniform acceleration:

i. The acceleration of a particle is said to be uniform, if its velocity increases by equal amount in equal intervals of time.

ii. When a particle moves with uniform acceleration, its average acceleration measured for different time intervals is constant. Thus, in uniformly accelerated motion of a particle, instantaneas acceleration is equal to its average acceleration.

iii. A freely falling body is an example of uniform acceleration.

e. Retardation (deceleration):

i. A body is said to have retardation or deceleration if its velocity decreases with time.

ii. Negative acceleration may also be termed as retardation (deceleration).

iii. In retarted motion, velocity of the body gradually decreases.

iv. Its unit is m/s^2 in S.I. system and cm/s^2 in CGS system

v. Dimensions : $[M^0L^1T^{-2}]$.

Q.11. Distinguish between uniformly accelerated motion and non-uniform motion.

Ans:

No.	Uniformly accelerated motion	Non-uniform motion
i.	A body covers equal distance in equal intervals of time.	A body covers unequal distance in equal intervals of time.
ii.	Magnitude of acceleration remains same with time.	Magnitude of acceleration varies with time.
iii.	Instantaneous velocity is always equal to average velocity.	Instantaneous velocity is not equal to average velocity.
iv.	v–t graph is straight line inclined to time axis.	v–t graph is not a straight line. It cannot be straight line parallel to t-axis.
v.	x–t graph is parabolic or straight line.	x–t graph is never a straight line.

Q.12. A. What do you mean by accelerated motion?

B. State three kinematical equations for uniformly accelerated motion in vector form.

Ans: A. Accelerated motion:

i. When velocity of a body increases with time, the body is said to be in accelerated motion.

ii. If velocity of a body decreases, the body is said to be decelerated or retarded.

B. Three kinematical equations for uniformly accelerated motion:

i. $\vec{v} = \vec{u} + \vec{a}t$

ii. $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

iii. $\vec{v}^2 = \vec{u}^2 + 2\vec{a}.\vec{s}$

where $\vec{u}$ = initial velocity of the body

$\vec{v}$ = final velocity of the body

$\vec{a}$ = acceleration of the body

$\vec{s}$ = displacement

t = time interval

Q.13. Derive the following relations for accelerated motion.

i. $\vec{v} = \vec{u} + \vec{a}t$ **ii.** $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

iii. $\vec{v}^2 = \vec{u}^2 + 2\vec{a}.\vec{s}$

where symbols have their usual meaning.

OR

Derive the three kinematical equations for a body moving with uniform acceleration.

Ans: i. $\vec{v} = \vec{u} + \vec{a}t$

Consider a body moving with uniform acceleration $\vec{a}$.

Let, $\vec{u}$ = initial velocity of body

$\vec{v}$ = final velocity of body

By definition

$$\vec{a} = \frac{\Delta\vec{v}}{t}$$

But $\Delta\vec{v}$ = final velocity - initial velocity

$= \vec{v} - \vec{u}$

Q.14. What is acceleration due to gravity?

OR

Explain the term: Acceleration due to gravity.

Ans: i. Acceleration due to gravity is defined as the acceleration set up in a body while falling freely under the effect of gravity alone.

ii. In case of vertical motion of a body, if distance covered is small compared to radius of the earth, the acceleration can be assumed to be constant through out the fall.

iii. This constant acceleration is referred to as acceleration due to gravity.

iv. Its value is approximately 9.8 m/s^2 near the earth's surface.

Q.15. A. Define the term relative velocity.

B. Explain with the help of one example.

Ans: A. Relative velocity:

Relative velocity is the time rate of change of relative position of one object with respect to another.

B. Explanation:

i. If $\vec{v}_A$ and $\vec{v}_B$ be the velocities of two bodies then relative velocity of A with respect to B is given by $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

ii. To explain relative velocity, consider two trains moving on two parallel tracks with same speed in same direction.

iii. Passenger sitting in one of such trains observes that he is in motion with respect to the earth i.e. buildings, trees, electric poles, etc. But he observes that the other train does not seem to move at all.

iv. This implies that both the trains appear to be stationary.

v. In fact, it is not so, both the trains are moving with same speed in same direction for a person on the earth.

vi. This shows that relative velocity of one train with respect to observer in other train is zero.

Q.16. Explain the variation of acceleration, velocity and distance with time for an object under free fall.

Ans:

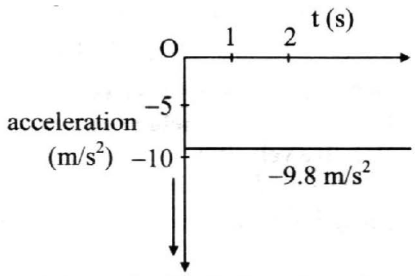


Figure (a) Variation of acceleration with time

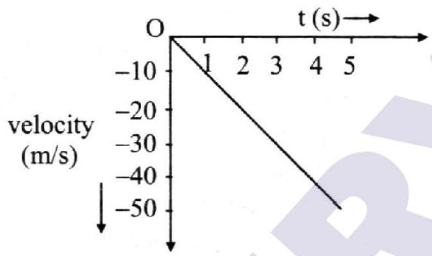


Figure (b) Variation of velocity with time

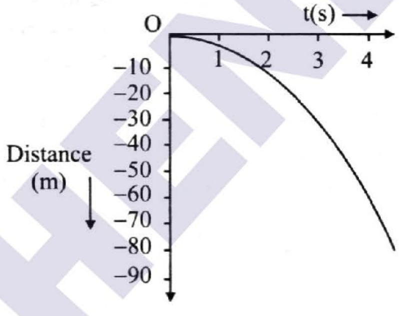


Figure (c) Variation of distance with time

i. For a free falling object, considering the downward direction as negative, the object is released from rest.

∴ initial velocity $u = 0$;
$a = -g = -9.8 \text{ m/s}^2$

∴ The kinematical equations become,
$v = u + at = 0 - gt = -gt = -9.8\,t$

$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}(-g)\,t^2 = -\frac{1}{2}gt^2$$

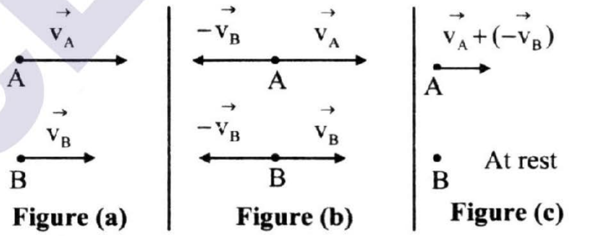
$$v^2 = u^2 + 2as = 0 + 2(-g)s$$
$$= -2gs$$
$$= -2 \times 9.8s$$
$$= -19.6s$$

ii. These equations give the velocity and the distance travelled as a function of time and also the variation of velocity with distance.

iii. The variation of acceleration, velocity and distance with the time is as shown in figure a, band c respectively.

Q.17. Obtain an expression for relative velocity when two objects are moving along a straight line in the same direction.

Ans: i. Let A and B be two objects moving in same direction with velocities $\vec{v}_A$ and $\vec{v}_B$ respectively.



ii. The relative velocity of object A with respect to object B can be found out. To get relative velocity bring object B to rest by superimposing velocity $\vec{v}_B$ on it, so that the velocity of object B will not affect motion of object A with respect to B. Now, object A is also superimposed with velocity $-\vec{v}_B$ so that the system remains same.

iii. The figure (a) shows objects A and B before superimposition, while figure (b) shows objects A and B when superimposed with velocity $-\vec{v}_B$'

iv. After superimposition, the result is as shown in figure (c), object B may be treated as object at rest while object A may be treated as moving with velocity

$$\left[\vec{v}_A + \left(-\vec{v}_B\right)\right]$$

v. $\vec{v}_R = \vec{v}_A + (-\vec{v}_B)$

The object B being at rest, relative velocity of object A with respect to object B is

$\vec{v}_R = \vec{v}_A + (-\vec{v}_B) = \vec{v}_R = \vec{v}_A + (-\vec{v}_B)$

vi. Both the objects are moving in same direction along a straight line, hence we can drop vector sign.

$\vec{v}_R = \vec{v}_A + (-\vec{v}_B)$

vii. When two objects A and B are moving in same direction along a straight line, magnitude of relative velocity of the object A with respect to object B is equal to magnitude of velocity of object A minus magnitude of velocity of object B.

Q.18. Obtain an expression for relative velocity when two objects are moving along a straight line in opposite direction.

Ans: i. Let A and B be two objects moving in opposite direction with velocities $\vec{v}_A$ and $\vec{v}_B$ respectively. The relative velocity of object A with respect to object B can be found out. For that bring object B at rest by superimposing velocity $-\vec{v}_B$ on it, so that velocity of B will not affect the motion of object A.

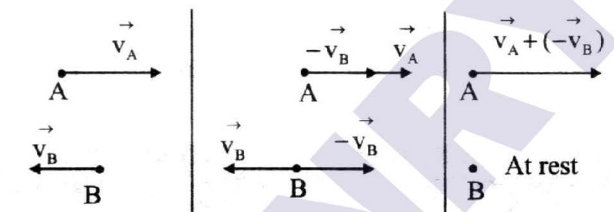


Figure (a) Figure (b) Figure (c)

ii. Now for object A, superimpose velocity $-\vec{v}_B$ on it.

iii. Figure (a) shows objects before superimposition.

Figure (b) shows objects when superimposition is applied.

Figure (c) shows result of superimposition on both the objects.

iv. The result is that object B may be treated as at rest, while object A is treated as moving with velocity

$\vec{v}_R = \vec{v}_A + (-\vec{v}_B)$

v. The object B is at rest, so the relative velocity of object A with respect to B is

$\vec{v}_R = \vec{v}_A + (-\vec{v}_B) - \vec{v}_R = \vec{v}_A + (-\vec{v}_B)$

vi. Since both the objects are moving in the opposite direction, along a straight line, we can drop the vector sign.

$\vec{v}_R = \vec{v}_A + (-\vec{v}_B)$

vii. Thus, when two objects A and Bare moving along a straight line in opposite direction, magnitude of relative velocity of object A with respect to object B is equal to the sum of the magnitude of their velocities.

Q.19. Define variable velocity. State the characteristics of motion of a body with variable velocity.

Ans: Variable velocity:

An object is said to be moving with variable velocity if it undergoes unequal displacements in equal intervals of time.

Characteristics of motion with variable velocity:

i. When an object moves with variable velocity, either its speed or direction of motion or both change with time.

ii. The instantaneous velocity of the object at different instants of time during a time interval is not equal to average velocity in that time interval.

iii. v – t graph cannot be a straight line inclined to t-axis. It will not be parallel to the time axis.

iv. The position-time graph is never a straight line inclined to time axis.

Q.20. Discuss the position-time graph for two objects moving in the same direction with

a. equal velocities

b. unequal velocities

Ans: a. Equal velocities:

i. Consider two objects A and B which are initially at position Xo and xo' respectively from the origin. A and B move along a straight line with uniform velocity V_A and V_B respectively.

ii. The velocities of two objects are same i.e, $V_A = V_B$.

iii. The graph of each body is a straight line. Also both the graphs are parallel to each other. i.e. their slopes are equal.

iv. Intercept along position axis gives initial position of every object (see figure.)

v. Figure.(a) represents position-time graph for two objects moving in the same direction with equal velocities.

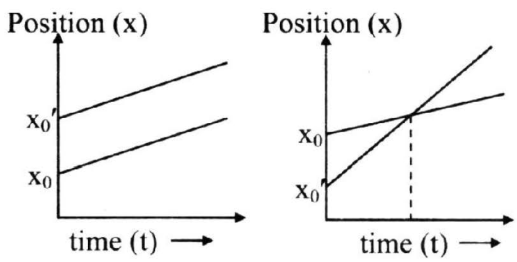


Figure (a)
Equal velocities

Figure (b)
Unequal velocities

b. Unequal velocities:

i. Consider two objects A and B which are initially at position Xo and Xo' respectively from origin. Also let Xo > xo'.

ii. The magnitude of velocities of two objects A and B are v and v' respectively. Also let v' > v.

iii. The graph of each object is straight line and they are not parallel. Hence, they will meet at a point at a certain time for which both the objects are at same position (distance) called time of meeting.

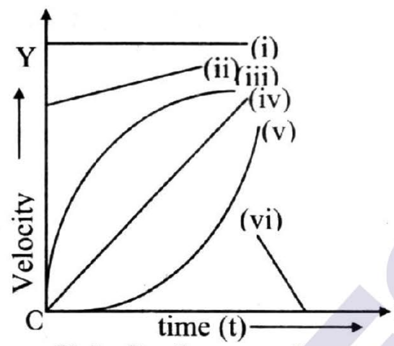
iv. The sign of displacement reverses after the time of meeting.

v. Figure.(b) represents positiontime graph for two objects moving in the same direction with unequal velocities.

Note:

If the velocities of two objects have opposite signs, then magnitudes of their relative velocity is greater than magnitudes of both velocities (i.e. v and v'). Hence, after the time of meeting or crossing, they appear to be moving very fast with respect to each other.

Q.21. Figure shows velocity - time graph for various situations. What does each graph indicate?



Velocity-time graph

Ans: i. Velocity is constant with time. Hence, acceleration is zero.
Initial velocity, u > 0.

ii. Finite initial velocity increasing with time. Acceleration, a > 0 and is constant.

iii. Initial velocity, u = 0. Acceleration a is positive and decreasing in magnitude with time. while velocity is increasing with time.

iv. Constant acceleration starting from rest. Initial velocity, u = 0. Velocity is increasing with time.

v. Initial velocity, u = 0. Acceleration a and velocity is increasing with time.

vi. Initial velocity u > 0, acceleration a < 0. Decreasing velocity ultimately comes to rest.

3.2 : Position - Time (x – t) graph and Velocity - Time (v – t) Graph

Q.22. Discuss the position-time graph of

a. a particle at rest and

b. a particle moving with constant velocity.

Ans: a. The position-time graph of a particle at rest:

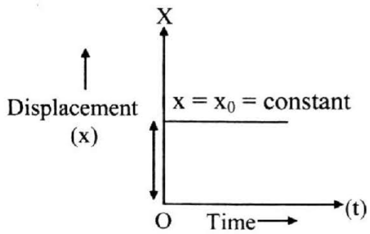
i. For a particle at rest, there is no change in its position with respect to its surrounding in the due course of time.

ii. The position co-ordinate x has constant value (x_0) which is independent of time.

iii. The graph is a straight line parallel to time axis.

iv. Slope of the graph is zero, which indicates that velocity of the particle is zero.

v. Intercept along the position axis gives the distance of stationary particle from the origin 'O'.



b. A particle moving with constant velocity:

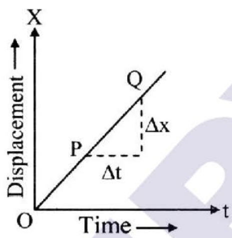
i. When the particle moves, the position of the particle changes with respect to time.

ii. There is equal change in position in equal intervals of time. Therefore, the graph is a straight line with positive slope.

iii. The slope of this graph is given by,

$$\frac{\Delta x}{\Delta t} = \frac{\text{change in position}}{\text{time in which the change takes place}}$$

iv. The slope is constant and it gives the average velocity. In this case, the average velocity and instantaneous velocity of the particle are equal.



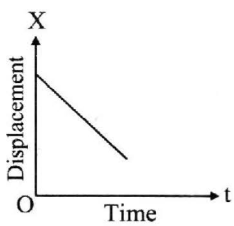
Q.23. Discuss the position-time graph of a particle moving with negative velocity.

Ans: Position-time graph of a particle moving with negative velocity:

i. Displacement decreases with increase in time.

ii. The slope of the graph is negative.

iii. It indicates that the velocity of the particle is negative.



Q.24. Explain the position-time graph of a particle moving with variable velocity.

Ans: i. The particle is said to be moving with variable velocity, if the speed or direction of motion or both change with time.

ii. When the velocity of the particle changes with time, a particle is said to be in accelerated motion. As the velocity of the particle is different at different points (non linear), slope of the graph is different at different points.

[See figure. (a)].

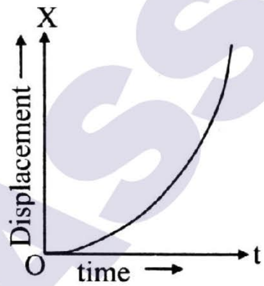


Figure (a)

iii. Slope of position-time graph as shown in figure (b) gives the average velocity of particle. The slope is given by

$$\text{slope} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} = v_{avg}$$

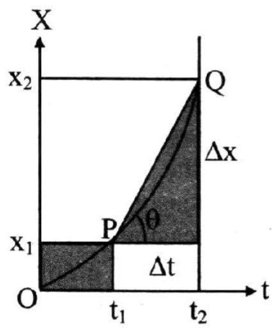


Figure (b)

iv. The slope $\frac{\Delta x}{\Delta t}$ of tangent at a point R, in figure (c) gives the instantaneous velocity of particle.

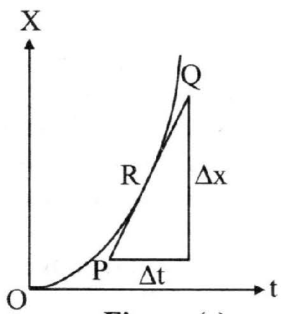


Figure (c)

Q.25. State the characteristics of position-time graph.

Ans: Characteristics of a position-time graph:

i. When the graph is a straight line inclined to time axis, the particle is moving with uniform velocity.

ii. When the particle has variable velocity, the graph is a curve. For constant acceleration, this curve is parabolic.

iii. The slope of a straight line that connects two particular points on the curve, (x_1, t_1) and (x_2, t_2) gives the magnitude of the average velocity during the time interval $\Delta t = t_2 - t_1$.

$$|\vec{v}_{av}| \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

iv. The slope of the tangent at a point on the curve gives the instantaneous velocity at that point.

v. If the slope is positive, the instantaneous velocity is positive, indicating that the particle is moving in the direction of increasing x.

vi. If the slope is negative, the instantaneous velocity is negative, indicating that the particle is moving in the direction of decreasing x.

vii. If the magnitude of the slope is increasing, the velocity is increasing in magnitude.

viii. If the magnitude of the slope is decreasing, the velocity of the particle is decreasing in magnitude.

ix. When the graph has zero slope, the instantaneous velocity is zero.

x. The graph cannot have a sharp turn because such a point will give two different velocities at the same instant.

xi. The position-time graph for the motion of a particle does not represent the path of the motion.

Q.26. The position-time (x-t) graphs for two children A and B returning from their school 0 to their homes P and Q respectively are shown in figure. Choose the correct entries in the brackets below.

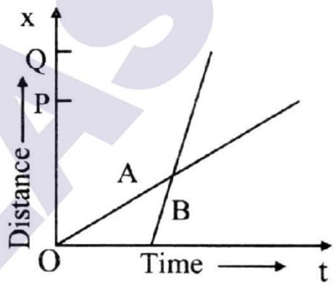
i. (A/B) lives closer to the school than (B/A)

ii. (A/B) starts from the school earlier than (B/A)

iii. (A/B) walks faster than (B/A)

iv. A and B reach home at the (same/different) time

v. (A/B) overtakes (B/A) on the road (once/twice) (NCERT)



Ans: i. A lives closer to the school than B. This is because, OQ > OP, hence B has to cover larger distance than A.

ii. A starts from the school earlier than B. This is because, A starts at t = 0 whereas B starts at some finite time greater than zero.

iii. As slope of B is greater than that of A, hence B walks faster than A.

iv. A and B reach home at different times. This is because the value of 't' corresponding to P and Q for A and B respectively is different.

v. B overtakes A on the road once. This is because A and B meet each other only once on their way back home. As B starts from school later than A and walks faster than A, hence B overtakes A once on his way home.

Q.27. Discuss velocity-time graph of a particle moving with constant velocity.

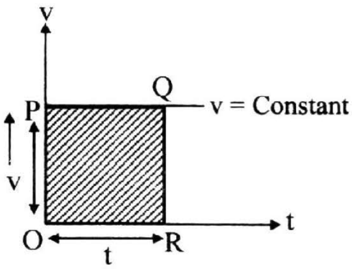
Ans: Velocity-time graph of a particle moving with constant velocity:

i. As the velocity is constant, the graph will be a straight line parallel to time axis .

ii. Acceleration of particle is zero as velocity is constant.

iii. This can also be inferred as zero slope of the line.

iv. Distance covered by a particle moving with uniform velocity is the area under curve of velocity time graph.



v. The distance (s) covered by the particle in time (t) is given by,
Distance = velocity x time

$s = v \times t$

$\therefore \quad s = OP \times OR$

$\therefore \quad s =$ Area of the region OPQR

Q.28. Explain the velocity-time graph of a particle moving with constant acceleration.

Ans: Graph can be explained in following three conditions:

i. Particle is moving with constant acceleration having initial velocity u = 0.

In this case, slope of the graph $= \dfrac{\Delta v}{\Delta t} = +a$

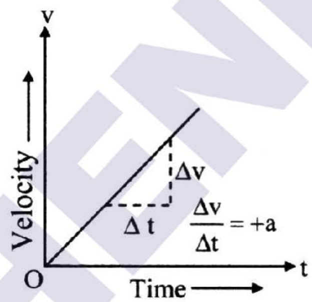


Figure (a)

As acceleration is positive, velocity increases with time [see figure. (a)]

ii. Particle is moving with finite initial velocity $(\vec{u})$:

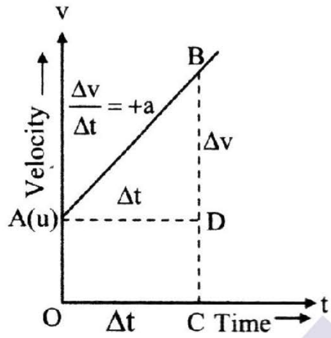


Figure (b)

In this case, distance covered by particle at any instant of time is given by area of closed region OABC as shown in figure.

Area (OABC)

$=$ area (OADC) + area (Δ ABD)

$= (OA \times OC) + (\frac{1}{2} \times AD \times BD)$

$= ut + \frac{1}{2} \times t(v - u)$

Area OABC $= ut + \frac{1}{2} at^2$ ($\because v - u = at$)

$s = ut + \frac{1}{2} at^2$

Slope of velocity-time graph gives acceleration of particle.

In figure (b),

$\text{Slope} = \dfrac{BD}{AD} = \dfrac{v - u}{t} = a$

Slope = acceleration of the body.

Intercept on Y-axis gives initial velocity (u).

iii. Particle is moving with constant negative acceleration:

In this case, velocity decreases with increase in time as shown in figure. (c). Slope of the graph is $\dfrac{\Delta v}{\Delta t} = -a$

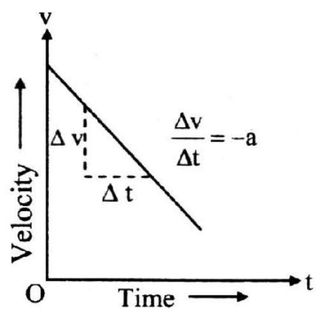


Figure (c)

Q.29. Explain the velocity-time graph of a particle moving with

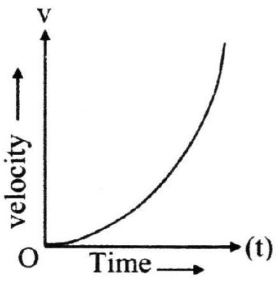
a. increasing acceleration

b. decreasing acceleration

Ans: a. Particle is moving with increasing acceleration:

i. In this case, velocity of particle increases constantly with respect to time.

ii. The graph is a curve and slope of the graph is positive and increases with time.

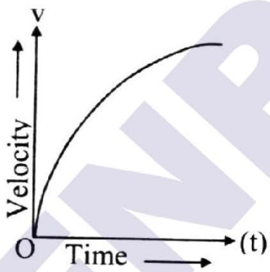


Increasing acceleration

b. Particle is moving with decreasing acceleration.

i. In this case, velocity of particle decreases continuously with time.

ii. Slope of the graph is positive but decreases with time.



Decreasing acceleration

Q.30. Write the characteristics of velocity-time graph.

Ans: i. In case of velocity-time graph, velocity is plotted along Y-axis and time along X-axis.

ii. The area under velocity-time graph between t_1 and t_2 gives the displacement & during the time interval ~t = h - tl and the distance travelled by body.

iii. The slope of the velocity-time graph gives acceleration.

iv. If the slope is positive, the acceleration is in the positive direction of motion.

v. If the slope is negative, the acceleration is in the negative direction of motion.

vi. If acceleration and velocity have opposite signs, then it is retardation and the velocity is decreasing.

vii. The graph is a straight line for constant acceleration or retardation.

viii. For a particle moving with uniform velocity, the graph is a straight line parallel to the time axis.

Q.31. A ball is dropped from a height of 90 m on a floor. At each collision with the floor, the ball loses one tenth of its speed. Plot the speed-time graph of its motion between t = 0 to t = 12 s. (NCERT)

Ans: i. Before collision for the ball,

$u_1 = 0$, s, = 90 m, $a_1 = g = 9.8 \text{ m/s}^2$,

Using,

$v_1^2 = u_1^2 + a_1 s_1$, we get,

$v_1^2 = (0)^2 + 2 \times 9.8 \times 90$

$\therefore \quad v_1 = \sqrt{1764} = 42$ m/s.

ii. Now using,

$v_1 = u_1 + a_1 t_1$ we get,

$42 = 0 + 9.8 \times t_1$

or $\quad t_1 = \dfrac{42}{9.8} \sim 4.3$ s

iii. After collision,

$u_2 = v_1 - 10\%$ of v_1 .... [Given]

$u_2 = v_1 - \dfrac{v_1}{10} = \dfrac{9v_1}{10}$

$u_2 = \dfrac{9}{10} \times 42 = 37.8$m/s

and $v_2 = 0$ .... [at maximum height]

iv. $a_2 = -g = -9.8 \text{ m/s}^2$

Using,

$v_2 = u_2 + a_2 t_2$ we get,

$0 = 37.8 - 9.8 \times t_2$

or $\quad t_2 = \dfrac{37.8}{9.8} \sim 3.9$ s

We know that,

v. time of ascent (t_2) = time of descent (t_3)

$t_3 = 3.9$ s.

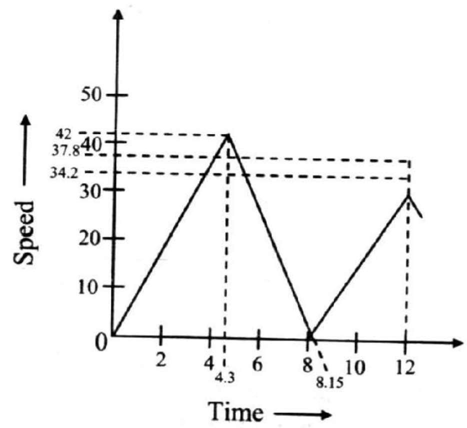
vi. Total time required by the ball

$= t_1 + t_2 + t_3$

$= 4.3 \text{ s} + 3.9 \text{ s} + 3.9 \text{ s}$

$= 12.15 \sim 12$s

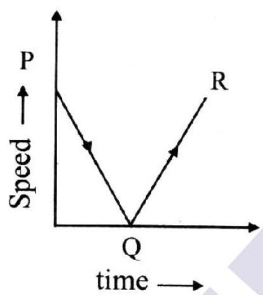
vii.



viii. The speed-time graph of the motion of the ball is plotted above.
Hence from the graph,
the ball will hit the ground with velocity
= 37.8 m/s.

Q.32. A ball thrown vertically upwards from a point P on earth reaches a point Q and returns back to earth striking at a point R. Draw speed - time graph to depict the motion of the ball (Neglect air resistance).

Ans: i.



ii. A ball which is thrown up with a certain initial speed goes up to a certain height where its speed becomes zero.

iii. Now, during its downward motion, the speed goes on increasing from zero and reaches its initial value when it strikes the ground.

iv. The speed - time graph for the motion of a ball is as shown in the figure.

Q.33. What is accelerated motion? Explain with necessary graphs uniformly accelerated linear (one-dimensional) motion. Give two examples of each.

Ans: Accelerated motion:

A body is said to be in accelerated motion when its velocity changes with time [in magnitude or direction or both]

Explanation:

i. The position-time graph for a particle undergoing acceleration during a certain time-interval is a curve having different slopes at different points in that timeinterval.

ii. From the position-time graph, the average velocity of the particle in time interval ,At = $t_2 - t_1$ is the slope of the chord MN:

iii. Instantaneous velocity of the particle at any point R is given by slope of tangent to curve at that point.

iv. The velocity-time graph is a straight line inclined to the time axis and passing through origin.

v. The slope to the straight line gives the average acceleration of the particle.

vi. Examples: A body falling freely in the absence of air resistance, car travelling in a straight line with velocity increasing at a constant rate.

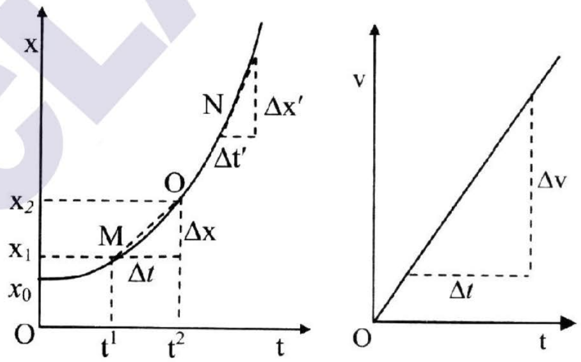


Figure (a)
Position-time graph

Figure (b)
Velocity-time graph

Accelerated motion

Q.34. Read each statement below carefully and state with reasons and examples if it is true and false:

For a particle in one - dimensional motion,

i. with zero speed at any instant may have non-zero acceleration at that instant.

ii. with zero speed has non-zero velocity.

iii. with constant speed must have zero acceleration.

iv. with positive value of acceleration must be speeding up. (NCERT)

Ans: i. Consider a stone thrown vertically upwards. Now, at the highest point, the speed of stone is zero but is acted upon by gravitational acceleration which has a non - zero value. Hence the given statement is justified.

ii. Speed is the distance covered by a body per unit time. Zero speed implies that the body covers no (zero) distance per unit time. In other words, it is at rest, which further implies that body is not displaced at all. Hence it has zero velocity, so the given statement is false.

iii. A body having a constant speed covers equal distances in equal intervals of time. Since the motion is one - dimensional, the velocity of the body is also uniform. This means that the change of displacement per unit time or acceleration is zero. Hence the given statement is justified.

iv. The velocity of a body at any instant t is given by,

v = u + at

For the given statement, there arise two cases depending upon choice of the time instant taken as origin.

Case I: a and u are both positive at the instant of time taken as origin.

For example, in case of a body falling vertically downwards both a and u are positive at the instant of time taken as ongm.

Case II: a is positive but u is negative. For example, consider a body projected vertically upwards. In this case, the body slows down for all the time instants before the one at which velocity becomes zero.

3.3 : Equation of path of a projectile

Q.35. What is projectile? Derive an equation of the path of a projectile.

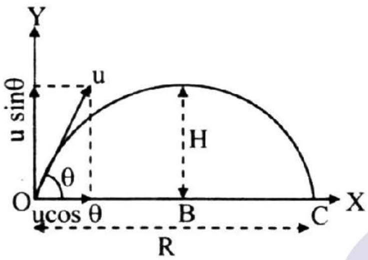
Ans: i. An object thrown in the air with initial velocity in any direction, moving freely under the. action of gravity and making some angle with horizontal is called projectile.

ii. Example: A bullet fired from a gun, football kicked in air, a stone thrown obliquely in air etc.

Equation of path of a projectile:

i. Consider a body projected with velocity u, at an angle e of projection from point 0 in the co-ordinate system of the XY- plane, as shown in figure.

ii. Velocity u can be resolved into two rectangular components:



u_x = u cos e (Horizontal component)
u_y = u sin e (Vertical component)

iii. The horizontal component of velocity ucosfl remains constant throughout the motion because there is no horizontal component of acceleration i.e., ax= O.

iv. Let x = horizontal distance covered by the projectile in time t

x = (horizontal velocity) x (time)

x = (u cose) t

v. Let y be the vertical distance covered by the projectile in time t.

Applying kinematical equation,

$$s = ut + \frac{1}{2}at^2$$

Substituting s = y, u = u sin 9

Vertical component of acceleration, ay= -g

$$y = (u \sin 9)t \ \frac{1}{2}gt^2$$

Substituting t from equation (1) in equation (2), we have

Equation (3) represents the path of the projectile.

vi. If we put tan 9 = a and g/2u² cos²e = β then equation (3) can be written as y = α x – βx^2 where a and β are constants. This is equation of parabola. Hence, path of projectile is a parabola.

Note:

1. Equation $y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$ is valid only when 9 lies between 0 and $\frac{\pi}{2}$.

2. The shape of the trajectory of motion of an object is not determined by acceleration alone but also depends on initial condition of motion.

Q.36. Define the following terms:

- **i. Projectile motion**
- **ii. Point of projection**
- **iii. Velocity of projection**
- **iv. Angle of projection**
- **v. Trajectory of a projectile**

Ans: i. Projectile motion:

Motion of a projectile under the effect of gravity is called projectile motion.

Examples:

Motion of bob from a catapult, motion of canon from a machine gun etc.

ii. Point of projection:

The point from which the body is projected in air is called as point of projection.

iii. Velocity of projection:

The velocity with which an object is projected in air is called the velocity of projection.

iv. Angle of projection:

The angle made by the velocity of projection with the horizontal is called the angle of projection.

v. Trajectory of projectile:

The curved path followed by the projectile in air is called trajectory. It is parabolic in nature.

Q.37. State the main assumptions which are used at the time of studying projectile motion.

Ans: i. The effect of air resistance is negligible. This is .true when projectile is moving with small speed.

ii. Acceleration due to gravity is constant at every point of the trajectory and is directed vertically downwards. This is true when height reached by projectile is very small as compared to radius of the earth.

iii. Effect of rotation of earth on the projectile is negligible. This is true when horizontal distance travelled by projectile is small as compared to radius of the earth.

iv. The only force acting on the projectile is gravitational force, acting in vertically downward direction.

3.4 : Time of fight (T)

Q.38. Define time of flight of a projectile. Obtain an expression for it.

Ans: Time of flight:

Time taken by the projectile to cover the entire trajectory is called time of flight (T).

Expression for time of flight:

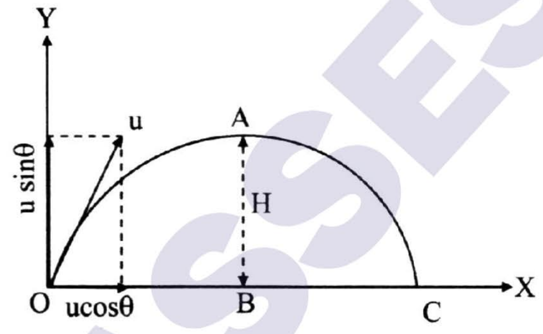
i. Let

u = Velocity of projectile

u_x = Horizontal component of velocity

u_y = Vertical component of velocity

t_A = time of ascent = time taken to reach maximum height



ii. Using kinematical equation v = u + at for vertical direction.

$v_y = u_y + a_y t$

At maximum height substituting

$v_y = 0$, $t = t_A$, $ay = -g$, $u_y = u \sin \theta$

$0 = u \sin \theta - g t_A$

$u \sin \theta = g t_A$

$$t_A = \frac{u \sin \theta}{g} \quad \text{....(1)}$$

This is time of ascent of projectile.

iii. At any instant of time (t), the position of the projectile (y) along Y-axis is given by,

$$y = (u \sin \theta)t - \frac{1}{2} g t^2 \quad \text{....(2)}$$

iv. The projectile returns to the ground level (point C) after time 'T'. At this point, the net vertical displacement of the projectile is zero.

i.e. y = 0 at t = T.

From equation (2),

$$0 = (u \sin \theta)T - \frac{1}{2} gT^2$$

$$(u \sin \theta)T - \frac{1}{2} gT^2$$

$$T = 2u \sin \theta \quad \text{....(3)}$$

Equation (3) represents time of flight of projectile.

From equation (1) and (2),

T= 2tA

'T' is the time in which the projectile returns to original plane of projection i.e. at point C.

Q.39. For the projectile motion, show that time of ascent is equal to time of descent.

Ans: i. Let t_1 = time taken by projectile to reach maximum height, from 0 to A.

t_2 = time taken by projectile to descent from A to C as shown in figure

we have to prove: $t_1 = t_2$

Using equation, $v = u + at$, $v = v_y' u = uy$ and $a = -ay$.

$v_y = u_y - a_y t$

At maximum height,

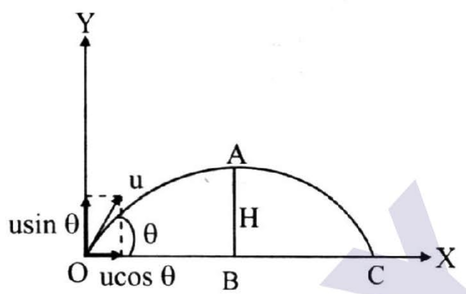
$v_y = 0, a_y = (-g)$

$u_y = u \sin \theta$

$0 = u \sin \theta - gt^1$

$gt_1 = u \sin \theta$

$$t_1 = \frac{u \sin\theta}{g} \quad ...(1)$$



iii. But time of flight of a projectile is given as,

$$T = \frac{2u\sin\theta}{g}$$

$$t_1 + t_2 = \frac{2u\sin\theta}{g}$$

$$t_2 = \frac{2u\sin\theta}{g} - t_1$$

Substitute t_1 from (1), we get

$$t_2 = \frac{2u\sin\theta}{g} - \frac{u\sin\theta}{g}$$

$$t_2 = \frac{u\sin\theta}{g}$$

iv. Comparing equation (1) and (2) we see that

$$t_1 = t_2 = \frac{u\sin\theta}{g}$$

Hence, for a projectile motion,

time of ascent = time of descent

3.5 : Horizontal range (R)

Q.40. What is the horizontal range of projectile? Derive an expression for it.

Ans: Horizontal range of projectile:

The horizontal distance between the point of projection and the point on the same horizontal plane, at which the projectile returns after moving along its trajectory is called the horizontal range (R) of the projectile.

Expression for horizontal range:

i. Suppose a body is projected obliquely with velocity u making an angle θ with positive direction of X-axis.

From equation of trajectory of the projectile,

$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

ii. At point C, y = 0 and x = R

$$0 = R \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

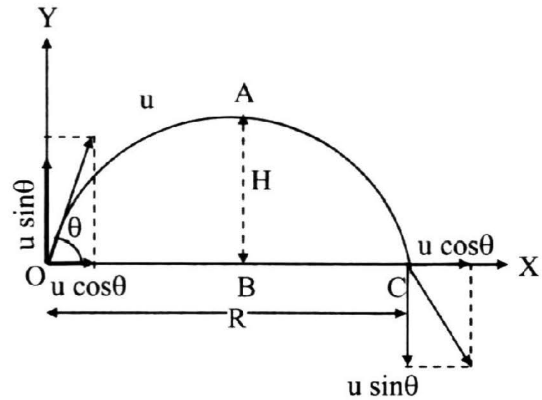
iii. $$R \tan \theta = \frac{gx^2}{2u^2 \cos^2 \theta}$$

$$\tan \theta = \frac{gR}{2u^2 \cos^2 \theta}$$

$$gR = 2u^2 \cos/ \theta \times \tan \theta$$

$$gR = 2u^2 \cos 2\theta \sin\theta \times \frac{\sin\theta}{\cos\theta}$$

iv.



v. $$R = \frac{u^2 (2\sin\theta.\cos\theta)}{g}$$

$$R = \frac{u^2 \sin\theta 2\theta}{g} \quad [\because \sin 2\theta = 2\sin\theta.\cos\theta]$$

This is required expression for horizontal range of the projectile.

vi. **Special cases:**

Case 1:

When $\theta = 0°$,

$$R = \frac{u^2 \sin\theta^0}{g} = 0 \quad [\because \sin 0° = 0]$$

Case 2:

When $\theta = 45°$,

$$R = \frac{u^2 \sin(2\times 45°)}{g} = \frac{u^2 \sin 90°}{g}$$

$$R = \frac{u^2}{g} \qquad [\because \sin 90° = 1]$$

This value is maximum for a projectile

$$R_{max} = \frac{u^2}{g}$$

Case 3:

When $\theta = 90°$,

$$R = \frac{u^2 \sin 2\times 90°}{g} = \frac{u^2 \times 0}{g} = 0$$

$[\because \sin 90° = 1]$

3.6 : Maximum height of projectile

Q.41.A. Define maximum height of a projectile.

B. Derive an expression for maximum height attained by a projectile (when projected obliquely).

Ans: **A. Maximum height of a projectile:**

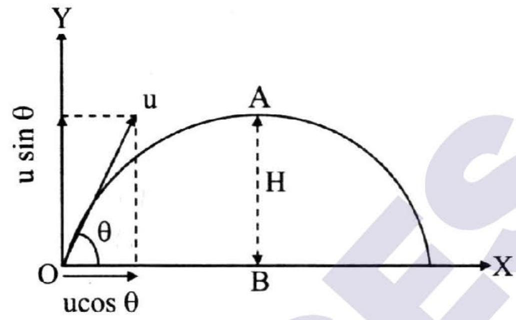
The maximum vertical distance travelled by the projectile from the ground level during its motion is the maximum height of a projectile.

B. Expression for maximum height:

i. Let H be the maximum height reached by the projectile. At maximum height,
$v = 0$, $u = u_y = u \sin\theta$,
$a = a_y = -g$, $s = H$

ii. From kinematic equation
$v^2 = u^2 + 2as$
$0 = (u \sin\theta)^2 - 2gH$
$u^2 \sin^2\theta = 2gH$

$$H = \frac{u^2 \sin^2\theta}{2g}$$



This equation represents maximum height of projectile. From the formula it is observed that maximum height of projectile depends on:

i. initial velocity of projection (u)

ii. angle of projection (θ)

Q.42. When the horizontal range of an obliquely projected body is maximum, show that it is four times greater than the maximum height reached.

Ans: i. At $R = R_{max}$ ($\because 2\theta = 90°$)

From $R = \frac{u^2 \sin 2\theta}{g}$, we can say that R is maximum when $\sin 2\theta = 1$

ii. Hence, for a given value of u, the horizontal range is maximum when $\theta = 45°$. i.e.

$$R_{max} = \frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin 90°}{g} = \frac{u^2}{g}$$

iii. Now, the maximum height reached is

$$H_{max} = \frac{u^2 \sin^2\theta}{2g}$$

$$= \frac{u^2 \sin^2 45°}{2g} = \frac{u^2\left(\frac{1}{\sqrt{2}}\right)^2}{2g}$$

$$\therefore \quad H_{max} = \frac{u^2}{4g}$$

iv. Now,

$$\frac{R_{max}}{H_{max}} = \frac{u^2/g}{u^2/4g} = 4$$

$R_{max} = 4H_{max}$

Q.43. Discuss the condition for the maximum range of a projectile. Show that for a given velocity of projection, there are two angles of projection which give the same range of projectile.

Ans: Condition for maximum range:

i. Horizontal range of projectile is given by

$$R = \frac{u^2 \sin 2\theta}{g} \quad ...(i)$$

The range will be maximum when $\sin 2\theta = 1$

$$2\theta = 90° \text{ or } \theta = \frac{90}{2} = 45°$$

Thus, horizontal range is maximum at angle of projection 45°

From eq. (i) we get,

$$R_{max} = \frac{u^2 . \sin^2 90}{g} = \frac{u^2(1)}{g} = \frac{u^2}{g'} =$$

ii. The horizontal range of an obliquely projected body with a velocity u, at the angle of projection θ is given by $R = \frac{u^2 \sin 2\theta}{g}$

If u remains the same, but the angle of projection is made $(90 - \theta)$

$$R = \frac{u^2 \sin^2(90° - \theta)}{g}$$

$$= \frac{u^2 \sin^2(180° - 2\theta)}{g}$$

Since, $\sin(180° - 28) = \sin 2\theta$

$$R = \frac{u^2 \sin 2\theta}{g}$$

The value of R remains the same in the two cases.

Q.44. Galileo, in his book 'Two new sciences', stated that "for elevations which exceed or fall short of 45° by equal amounts, the tangents are equal." Prove this statement. (NCERT)

Ans: i. Consider a body projected with an initial velocity u making an angle θ with the horizontal.

ii. Then, its range is given by,

$$R = \frac{u^2 \sin 2\theta}{g}$$

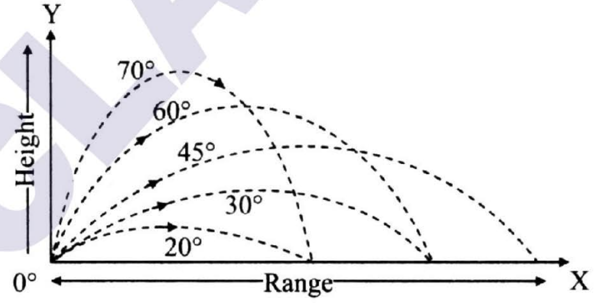
iii. Now, for the angles, $\theta = (45 + \alpha)°$ and $\theta = (45 - \alpha)°$, the corresponding values of 2θ are $(90 + 2\alpha)°$ and $(90 - 2\alpha)°$ respectively.

iv. We know that, $\sin(90 + 2\alpha)°$ and $\sin(90 - 2\alpha)°$ are the same which is equal to $\cos 2\alpha$.

v. Hence the ranges are equal for elevations which exceed or fall short of 45° by equal amounts α.

Q.45. Draw trajectory curves which represent same velocity of projectile and different angles of projection. Discuss the conclusion drawn from it.

Ans:



Trajectories of projectile projected with same velocity and different angles.

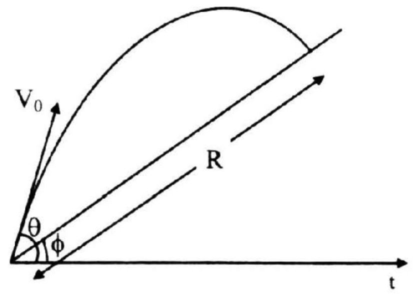
Conclusion:

i. At an angle of projection 45°, range of projectile is maximum.

ii. If the projectile is projected at angle less than 45°, range of projectile and maximum height reached would be less than that at angle of projection (8) = 45°

iii. If it is projected at an angle greater than 45° then horizontal range is small and maximum height reached is greater than that at angle of projection $(\theta) = 45°$

Q.46. A particle is projected with speed Vo at an angle θ to the horizontal on an inclined surface making an angle ϕ ($\phi < \theta$) to the horizontal. Derive an expression for its range along the inclined surface.



Ans: i. The equation of trajectory of projectile is given by,

$$y = (\tan\theta)x - \left(\frac{g}{2u^2\cos^2\theta}\right) \quad ... (1)$$

ii. In this case to find R substitute,

$y = R\sin\phi$ ....(2)

and $x = R\cos\phi$ ....(3)

Solved Examples :

Type I : Problems based on kinematical equations

Example 1

A ball is dropped from the top of a building 122.5 m high. How long will it take to reach the ground? What will be its velocity when it strikes the ground?

Solution:

Given: $h = 122.5$ m, $u = 0$

To find: i. Time taken to reach the ground (t)
ii. Velocity of ball when it strikes ground (v)

Formulae: i. $h = ut + \frac{1}{2}gt^2$
ii. $v = u + gt$

Calculation:
From formula (i),

$$122.5 = 0 + \frac{1}{2} \times 9.8\, t^2$$

$$t^2 = \frac{122.5}{4.9} = 25$$

$\therefore$ $t = \sqrt{25}$

$\therefore$ **t = 5 second**

From formula (ii),

$v = u + gt$

$v = 0 + 9.8 \times 5$

$\therefore$ **v = 49 m/s**

Ans: i. Time taken to reach the ground is **5 s**.
ii. Velocity of the ball when it strikes the ground is **49 m/s**.

Example 2

A body starts from rest and moves with a uniform acceleration of 5 m/s², What will be the velocity when it has covered a distance of 250 m? How much time will it require to cover this distance?

Solution:

Given: $u = 0$, $a = 5$ m/s², $s = 250$ m

To find: i. Velocity after covering 250 m (v)
ii. Time required to cover 250 m (t)

Formulae: i. $v^2 = u^2 + 2as$
ii. $v = u + at$

Calculation:
From formula (i),

$v^2 = 0 + 2 \times 5 \times 250$

$\therefore$ $v = \sqrt{2 \times 5 \times 250}$

$\therefore$ **v = 50 m/s**

From formula (ii),

$$t = \frac{v-u}{a} = \frac{50-0}{5} = \frac{50}{5}$$

$\therefore$ **t = 10 s**

Ans: i. Velocity of the body after covering 250 m is **50 m/s**
ii. Time required to cover 250 m is **10 s**.

Example 3

A car moves at a constant speed of 60 kmlhr for 1 km and 40 km/hr for next 1 km. What is the average speed of the car?

Solution:

Given: $v_1 = 60$ km/hr, $x_1 = 1$ km,
$v_2 = 40$ km/hr, $x_2 = 1$ km

To find: Average speed of car (v_{avg})

Formula: $$v_{avg} = \frac{\text{total path length}}{\text{total time interval}}$$

Calculation:
From given data,

$$t_1 = \frac{x_1}{v_1} = \frac{1}{60} \text{ hour}$$

$$t_2 = \frac{x_2}{v_2} = \frac{1}{40} \text{ hour}$$

From formula,

$$\text{Average speed of car} = \frac{x_1 + x_2}{t_1 + t_2}$$

$$= \frac{1+1}{\frac{1}{60}+\frac{1}{40}} = \frac{2}{\frac{40+60}{60\times 40}}$$

$$= \frac{2\times 60\times 40}{40+60}$$

$$= \frac{2\times 60\times 40}{100}$$

∴ Average speed of car = **48 km/hr**

Ans: The average speed of the car is **48 km/hr.**

Example 4

A train travels at a speed 50 km/hr for 0.5 hr, at 30 km/hr for next 0.26 hr and then 70 km/hr for next 0.76 hr. What is the average speed of the train?

Solution:

Given : $v_1 = 50$ km/hr, $t_1 = 0.5$ hr
$v_2 = 30$ km/hr, $t_2 = 0.26$ hr
$v_3 = 70$ km/hr, $t_3 = 0.76$ hr

To find : Average speed of train (v_{avg})

Formula : $$v_{avg} = \frac{\text{total path length}}{\text{total time interval}}$$

Calculation :

$x_1 = v_1 \times t_1 = 50 \times 0.5 = 25$ km
$x_2 = v_2 \times t_2 = 30 \times 0.26 = 7.8$ km
$x_3 = v_3 \times t_3 = 70 \times 0.76 = 53.2$ km

From formula,

$$v_{avg} = \frac{x_1 + x_2 + x_3}{t_1 + t_2 + t_3}$$

$$\therefore \quad v_{avg} = \frac{0.5 + 0.26 + 0.76}{0.5+0.26+0.76} = \frac{86}{1.52}$$

∴ v_{avg} = **56.58 km/hr**

Ans: Average speed of the train is **56.58 km/hr**.

Example 5

Figure shows variation of speed of the car with time.

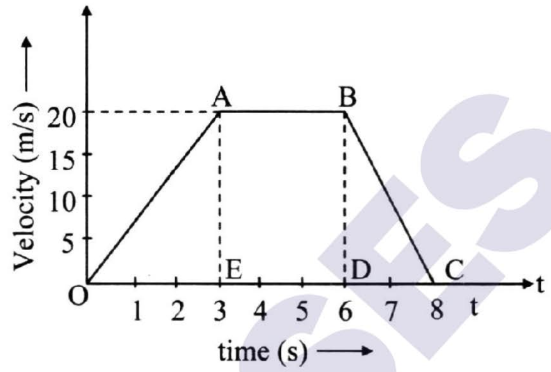
i. What is initial speed of the car?

ii. What is the maximum speed attained by the car?

iii. Which part of the graph shows zero acceleration?

iv. Which part of the graph shows varying retardation?

v. Find the distance travelled by the car in first 6 second.



Solution:

i. Initial speed is at origin i.e. 0 m/s

ii. V_{max} = speed at A = 20 m/s

iii. The part of the graph which shows zero acceleration is between t = 3 sand t = 6 s i.e., AB. This is because, during AB there is no change in velocity.

iv. Varying retardation is not shown in the graph the graph shows uniform retardation from t = 6 s to t = 8 s i.e., BC.

v. Distance travelled by car in first 6 s
= Area of OABDO
= A(Δ OAE) + A(rect. ABDE)

$$= \frac{1}{2} \times 3 \times 20 + 3 \times 20$$

= 30 + 60
= **90 m**

Example 6

A car is moving on a straight road with uniform acceleration. The speed of car varies with time as follows:

Speed (m/s)	5	10	15	20	25	30
Time (s)	0	10	20	30	40	50

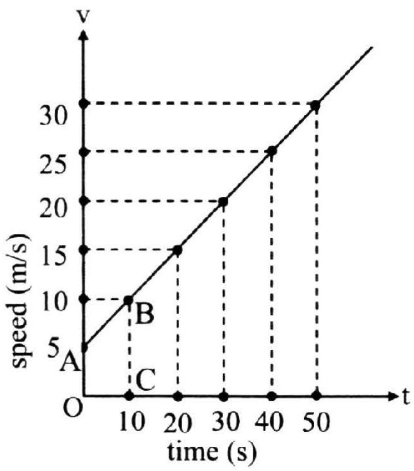
Draw the speed-time graph by choosing convenient scale.

Calculate:

i. the acceleration of the car.

ii. distance travelled by the car in 10 second.

Solution:



i. Acceleration, $a = \frac{dv}{dt}$

$$= \frac{30-5}{50-0} = \frac{25}{50}$$

$\therefore$ $a = 0.5\ m/s^2$

ii. Distance travelled in 10 s

= Area of trapezium OABC

$$= \frac{1}{2} \times (5 + 10) \times 10$$

$= 15 \times 5$

$\therefore$ s = **75 m**

Ans: i. The acceleration of the car is $0.5\ m/s^2$.

ii. Distance travelled by the car in 10 s is **75 m**.

Example 7

A car moving along a straight highway with speed of 126 km h^{-1} is brought to a stop within a distance of 200 m. What is the retardation of the car (assumed uniform) and how long does it take for the car to stop? (NCERT)

Solution:

Given: $u = 126\ kmh^{-1} = 126 \times \frac{5}{18}$

$= 35\ ms^{-1}$

$s = 200\ m, v = 0$

To find: i. Retardation of the car (a)

ii. Time taken by car (t)

Formulae: i. $v^2 - u^2 = 2as$

ii. $v = u + at$

Calculation:

From formula,

$0 - (35)^2 = 2a \times 200$

$$a = \frac{-(35)^2}{400} = -\mathbf{3.06\ ms^{-2}}$$

Also,

$$\therefore\ t = \frac{v-u}{a} = \frac{0-35}{-3.06} = \mathbf{11.4\ s}$$

Ans: i. Retardation of the car is $3.06\ ms^{-2}$ (in magnitude).

ii. Time taken by the car to stop is **11.4 s**.

Example 8

The speed of a car is reduced from 90 km/hr to 36 km/hr in 5 s. What is distance travelled by the car during this time interval?

Solution:

Given: $u = 90\ km/hr = 90 \times \frac{5}{18} = 25\ m/s$

$v = 36\ km/hr = 36 \times \frac{5}{18} = 10\ m/s$

$t = 5\ s$

To find: Distance travelled (s)

Formula: $s = ut + \frac{1}{2}at^2$

Calculation:

Since, $a = \frac{v-u}{t}$

$$a = \frac{10-25}{5} = \frac{-15}{5} = -3\ m/s^2$$

From formula,

$$s = 25 \times 5 + \frac{1}{2} \times (-3) \times 25$$

$= 125 - 37.5$

s = **87.5 m**

Ans: Distance travelled by the car in 5 s is **87.5 m**.

Example 9

Two trains A and B of length 400 m each are moving on two parallel tracks with a uniform speed of 72 km h^{-1} in the same direction, with A ahead of B. The driver of B decides to overtake A and accelerates by 1 ms-2. If after 50 s, the guard of B just brushes past the driver of A, what was the original distance between them? (NCERT)

Solution:

Given: Train A: $u_A = 72\ km\ h^{-1}$

$= 72 \times \frac{5}{18}\ ms^{-1}$

$= 20\ ms^{-1}$

$t = 50$ s, $a = 0$

Train B : $u_B = 72\ km\ h^{-1}$

$= 20\ ms^{-1}$

$t = 50$ s, $a = 1 m/s^2$

To find : Original distance between trains $(s_B - s_A)$

Formula : $s = ut + \frac{1}{2}at^2$

Calculation :

For train A,

From formula,

$s_A = 20 \times 50 + \frac{1}{2} \times 0 \times 50^2 = 1000$ m.

For train B,

From formula,

$s_B = 20 \times 50 + \frac{1}{2} \times 1 \times 50^2 = 2250$ m

Considering the guard of train B in the last compartment of train B,

Original distance between two trains + length of train A + length of train B = $s_B - s_A$

∴ Original distance between two trains + 400 + 400 = 2250 – 1000

∴ Original distance between two trains = 1250 – 800 = **450 m**

Ans: The original distance between the trains was **450 m**.

Type II : Problems based on relative velocity

Example 10

A car moving at a speed 10 m/s on a straight road is ahead of car B moving in the same direction at 6 m/s. Find the velocity of A relative to Band vice-versa.

Solution:

Given : $v_A = 10 m/s$, $v_B = 6$ m/s,

To find : i. Velocity of A relative to B $(v_A - v_B)$

ii. Velocity of B relative to A $(v_B - v_A)$

Formulae: i. $v_{AB} = v_A - v_B$

ii. $v_{BA} = v_B - v_A$

Calculation :

From formula (i),

$V_{AB} = 10 - 6 = 4$ m/s

From formula (ii),

$V_{SA} = 6 - 10 = -4 m/s$

–ve sign indicates that driver of car A sees the car B lagging behind at the rate of 4 mls.

∴ $V_{AB} = 4$ m/s, $V_{SA} = -4$ m/s

Ans: i. Velocity of A relative to B is 4 m/s.

ii. Velocity of B relative to A is –4 m/s.

Example 11

Two trains 120 m and 80 m in length are running in opposite directions with velocities 42 km/h and 30 km/h respectively. In what time will they completely cross each other?

Solution:

Given : $l_1 = 120$ m, $l_2 = 80$ m,

$v_A = 42$ km/h $= 42 \times \frac{5}{18}$

$= \frac{35}{3}$ m/s

$v_B = -30$ km/h $= -30 \times \frac{5}{18}$

$= \frac{-25}{3}$ m/s

To find : Time taken by trains to cross each other (t)

Formula : $\text{Time} = \frac{\text{Distance}}{\text{speed}}$

Calculation :

Total distance to be travelled

= sum of lengths of two trains

= 120 + 80 = 200m

Relative velocity of A with respect to B is v_{AB}

$v_{AB} = v_A - v_B$

$= \frac{35}{3} - \left(\frac{-25}{3}\right) = \frac{60}{3}$

∴ $v_{AB} = 20$ m/s

From formula,

∴ Time taken to cross each other (t)

$= \frac{\text{Distance}}{\text{speed}}$

$= \frac{200}{20} = 10$ s

Ans: Time taken by the two trains to cross each other is **10 s**.

Example 12

A jet aeroplane travelling at the speed of 500 km/hr ejects its products of combustion at speed of 1500 km/hr relative to jet plane. What is the relative velocity of the latter with respect to an observer on the ground?

Solution:

Let us cpnsider the positive direction of motion towards the observer on the ground. Suppose $\vec{v}_a$ and $\vec{v}_{cj}$ be the velocities of the aeroplane and relative velocity of combustion products w.r.t. aeroplane respectively.

$\therefore \quad \vec{v}_{cj}$ = 1,500 km/hr (towards the observer on the ground) and $\vec{v}_a$ = 500 km/hr (away from the observer on the ground)

$\therefore \quad -\vec{v}_a$ = – 500 km/hr (towards the observer on the ground)

Let $\vec{v}_c$ be the velocity of the combustion products towards the observer on ground then,

$\therefore \quad \vec{v}_c = \vec{v}_{cj} + \vec{v}_a = 1500 + (-500) = 1000$ km/hr

$\therefore \quad \vec{v}_c = 1000$ km/hr

Ans: The relative velocity of the combustion products W.r.t. the observer is 1000 km/hr.

Example 13

Two towns A and B are connected by a regular bus service with a bus leaving in either direction every T min. A man cycling with a speed of 20 km/h in the direction A to B notices that a bus goes past him every 18 min. in the direction of his motion, and every 6 min. in the opposite direction. What is the period T of the bus service and with what speed (assumed constant) do the buses ply on the road? (NCERT)

Solution:

Let v km h^{-1} be the constant speed with which the buses ply between the towns A and B. The relative velocity of the bus (for the motion A to B) with respect to the cyclist (i.e. in the direction in which the cyclist is going = (v – 20) km h^{-1}.

The relative velocity of the bus from B to A with respect to the cyclist = (v + 20) km h^{-1}.

The distance travelled by the bus in time T (minutes) = v T

Given that $\dfrac{vT}{v-20} = 18$ or v T = 18 v – 18 × 20 ... (i)

and $\dfrac{vT}{v+20} = 6$ or v T = 6v + 20 × 6 ... (ii)

Equating (i) and (ii) we get,

18v – 18 × 20 = 6v + 20 × 6 or

12 v = 20 × 6 + 18 × 20 = 480 or v = 40 km h^{-1}

Substituting this value of v in (i) we get

40 T = 18 × 40 – 18 × 20 = 18 × 20 or

T = 18 × 20/40 = **9 min**

Ans: Period of the bus service is **9 min** and buses ply on the road with a speed of **40 km h^{-1}**.

Type In : Problems based on projectile motion

Example 14

A body is projected with a velocity of 30 ms^{-1} at an angle of 30° with the vertical. Find

i. the maximum height

ii. time of flight and

iii. the horizontal range

Solution:

Given : $u = 30\ ms^{-1}$, $\theta = 90° - 30° = 60°$

To find : i. The maximum height reached (H)

ii. Time of flight (T)

iii. The horizontal range (R)

Formula : i. $H = \dfrac{u^2 \sin^2\theta}{2g}$

ii. $T = \dfrac{u^2 \sin^2\theta}{g}$

iii. $R = \dfrac{u^2 \sin 2\theta}{g}$

Calculation :

From formula (i),

$$H = \frac{30^2 \times \sin^2 60^\circ}{2 \times 9.8} = \frac{30 \times 30}{2 \times 9.8} \times \left(\frac{\sqrt{3}}{2}\right)^2$$

$\therefore$ H = **34.44 m**

From formula (ii),

$$T = \frac{2 \times 30 \sin 60^\circ}{9.8}$$

$\therefore$ T = **5.3 s**

From formula (iii),

$$R = \frac{30^2 \times \sin 120^\circ}{9.8} = \frac{30^2 \sin 60^\circ}{9.8}$$

$\therefore$ R = **79.53 m**

Ans: i. The maximum height reached by the body is **34.44 m**.

ii. The time of flight of the body is **5.3 s**.

iii. The horizontal range of the body is **19.53 m.**

Example 15

A ball is dropped from the top of a building 122.5 m high. How long will it take to reach the ground? What will be its velocity when it strikes the ground?

Solution:

Given: $h = 122.5$ m, $u = 0$

To find:
i. Time taken to reach the ground (t)
ii. Velocity of ball when it strikes ground (v)

Formulae:
i. $h = ut + \frac{1}{2}gt^2$
ii. $v = u + gt$

Calculation:
From formula (i),

$$122.5 = 0 + \frac{1}{2} \times 9.8\, t^2$$

$$t^2 = \frac{122.5}{4.9} = 25$$

$\therefore$ $t = \sqrt{25}$

$\therefore$ $t =$ **5 second**

From formula (ii),

$v = u + gt$

$v = 0 + 9.8 \times 5$

$\therefore$ $v =$ **49 m/s**

Ans: i. Time taken to reach the ground is **5 s**.

ii. Velocity of the ball when it strikes the ground is **49 m/s**.

Example 16

A body starts from rest and moves with a uniform acceleration of 5 m/s², What will be the velocity when it has covered a distance of 250 m? How much time will it require to cover this distance?

Solution:

Given: $u = 0$, $a = 5$ m/s², $s = 250$ m

To find:
i. Velocity after covering 250 m (v)
ii. Time required to cover 250 m (t)

Formulae:
i. $v^2 = u^2 + 2as$
ii. $v = u + at$

Calculation:
From formula (i),

$v^2 = 0 + 2 \times 5 \times 250$

$\therefore$ $v = \sqrt{2 \times 5 \times 250}$

$\therefore$ $v =$ **50 m/s**

From formula (ii),

$$t = \frac{v-u}{a} = \frac{50-0}{5} = \frac{50}{5}$$

$\therefore$ $t =$ **10 s**

Ans: i. Velocity of the body after covering 250 m is **50 m/s**

ii. Time required to cover 250 m is **10 s**.

Example 17

A car moves at a constant speed of 60 kmlhr for 1 km and 40 km/hr for next 1 km. What is the average speed of the car?

Solution:

Given: $v_1 = 60$ km/hr, $x_1 = 1$ km,
$v_2 = 40$ km/hr, $x_2 = 1$ km

To find: Average speed of car (v_{avg})

Formula: $v_{avg} = \dfrac{\text{total path length}}{\text{total time interval}}$

Calculation:
From given data,

$$t_1 = \frac{x_1}{v_1} = \frac{1}{60} \text{ hour}$$

$$t_2 = \frac{x_2}{v_2} = \frac{1}{40} \text{ hour}$$

From formula,

$$\text{Average speed of car} = \frac{x_1 + x_2}{t_1 + t_2}$$

$$= \frac{1+1}{\frac{1}{60} + \frac{1}{40}} = \frac{2}{\frac{40+60}{60 \times 40}}$$

$$= \frac{2 \times 60 \times 40}{40 + 60}$$

$$= \frac{2 \times 60 \times 40}{100}$$

$\therefore$ Average speed of car = **48 km/hr**

Ans: The average speed of the car is **48 km/hr.**

Example 18

A train travels at a speed 50 km/hr for 0.5 hr, at 30 km/hr for next 0.26 hr and then 70

km/hr for next 0.76 hr. What is the average speed of the train?

Solution:

Given : $v_1 = 50$ km/hr, $t_1 = 0.5$ hr
$v_2 = 30$ km/hr, $t_2 = 0.26$ hr
$v_3 = 70$ km/hr, $t_3 = 0.76$ hr

To find : Average speed of train (v_{avg})

Formula : $$v_{avg} = \frac{\text{total path length}}{\text{total time interval}}$$

Calculation :

$x_1 = v_1 \times t_1 = 50 \times 0.5 = 25$ km
$x_2 = v_2 \times t_2 = 30 \times 0.26 = 7.8$ km
$x_3 = v_3 \times t_3 = 70 \times 0.76 = 53.2$ km

From formula,

$$v_{avg} = \frac{x_1 + x_2 + x_3}{t_1 + t_2 + t_3}$$

$$\therefore \quad v_{avg} = \frac{0.5 + 0.26 + 0.76}{0.5+0.26+0.76} = \frac{86}{1.52}$$

$$\therefore \quad v_{avg} = \mathbf{56.58\ km/hr}$$

Ans: Average speed of the train is **56.58 km/hr**.

Example 19

Figure shows variation of speed of the car with time.

i. What is initial speed of the car?
ii. What is the maximum speed attained by the car?
iii. Which part of the graph shows zero acceleration?
iv. Which part of the graph shows varying retardation?
v. Find the distance travelled by the car in first 6 second.

Solution:

i. Initial speed is at origin i.e. 0 m/s
ii. V_{max} = speed at A = 20 m/s
iii. The part of the graph which shows zero acceleration is between t = 3 sand t = 6 s i.e., AB. This is because, during AB there is no change in velocity.
iv. Varying retardation is not shown in the graph the graph shows uniform retardation from t = 6 s to t = 8 s i.e., BC.
v. Distance travelled by car in first 6 s
= Area of OABDO
= A(Δ OAE) + A(rect. ABDE)

$$= \frac{1}{2} \times 3 \times 20 + 3 \times 20$$

$= 30 + 60$
$= \mathbf{90\ m}$

Example 20

A man throws a ball to maximum horizontal distance of 80 m. Calculate the maximum height reached.

Solution:

Given : $R_{max} = 80$ m

To find : Maximum height reached (H_{max})

Formula : $R_{max} = 4H_{max}$

Calculation :

From formula,

$$\therefore \quad H_{max} = \frac{R_{max}}{4}$$

$$= \frac{80}{4} = \mathbf{20\ m}$$

Ans: The maximum height reached by the ball is **20 m**.

Example 21

A body is thrown with a velocity of 40 m/s in a direction making an angle of 30° with the horizontal Calculate

a. Horizontal range
b. Maximum height and
c. Time taken to reach the maximum height.

Solution:

Given : u = 40 m/s, 9 = 30°

To find :
i. Horizontal range (R)
ii. Maximum height (H_{max})
iii. Time to reach max. height (t_A)

Formulae :

i. $$R = \frac{u^2 \sin 2\theta}{g}$$ ii. $$H_{max} = \frac{u^2 \sin^2 \theta}{2g}$$

iii. $$t_A = \frac{u \sin \theta}{g}$$

Calculation :

a. From the formula (i),

$$R = \frac{(40)^2 \times \sin 60^\circ}{9.8}$$

$$= \frac{40 \times 40 \times \sqrt{3}}{2 \times 9.8} = \mathbf{141.4\ m}$$

b. From the formula (ii),

$$H_{max} = \frac{40 \times 40 \times \sin^2 30^o}{2 \times 9.8} = \frac{40 \times 40 \times 1}{2 \times 9.8 \times 4}$$

∴ H_{max} = **20.41 m**

c. From the formula (iii),

$$t_A = \frac{40 \times \sin 30^o}{9.8} = \frac{40 \times 1}{9.8 \times 2}$$

∴ t_A = **2.041 s**

Ans: i. Horizontal range of the body is **141.4 m**.

ii. Maximum height reached by the body is **20.41 m**.

iii. Time taken by the body to reach the maximum height is **2.041 s**.

Additional Theory Questions :

1. Define:

i. Displacement
ii. Path length (distance)
iii. Uniform motion
iv. Speed
v. Velocity
vi. Average velocity
vii. Instantaneous velocity
viii. Uniform velocity
ix. Average speed
x. Instantaneous speed
xi. Accelaration
xii. Average acceleration
xiii. Instantaneous acceleration
xiv. Uniform acceleration
xv. Retardation.
xvi. Acceleration due to gravity
xvii. Projectile
xviii. Horizontal range.

Ans: i. Refer Q.2.
ii. Refer Q.2.
iii. Refer Q.8.
iv. Refer Q.7. A.(i)
v. Refer Q. 7. B. (i)
vi. Refer Q.9.a.(i)
vii. Refer Q.9.b.(i)
viii. Refer Q.9. c. (i)
ix. Refer Q.9.d. (i)
x. Refer Q.9.e.(i}
xi. Refer Q.10.a.(i)
xii. Refer Q.1 O.b.(i)
xiii. Refer Q.10.c.(i)
xiv. Refer Q.10.d.(i)
xv. Refer Q.10.e.(i)
xvi. Refer Q.14. (i)
xvii. Refer Q.35.(i)
xviii. Refer Q.40.A.

2. Distinguish between speed and velocity.
Ans: Refer Q.7

3. Describe the term relative velocity with the help of an example.
Ans: Refer Q.16.

4. Describe variable velocity. State four characteristics of motion with variable velocity.
Ans: Refer Q.19.

5. Describe the position-time graph of a particle moving with variable velocity.
Ans: Refer Q.24

6. Describe the velocity-time graph of a particle moving with constant velocity.
Ans: Refer Q.27

7. Discuss the velocity-time graph of a particle moving with constant acceleration for the following cases: The particle
i. is having initial velocity zero.
ii. is moving with a finite initial velocity.
iii. is moving with a constant negative acceleration.
Ans: Refer Q.28

8. Describe with necessary graphs uniformly accelerated motion in one-dimension. Give two examples.
Ans: Refer Q.33.

9. Define projectile. Give two examples.
Ans: Refer Q.35.A.

Practice Problems :

Type I: Problems based on kinematical equations

1. The initial velocity of a car is 5 m/s. It accelerates uniformly at 0.5 m /s^2 for 30 s and then retards uniformly at 2 m/s^2. Find the distance covered by the car before it comes to rest. For how much time is the car in motion?

2. A body starts from rest and moves with uniform acceleration of 0.25 m /S^2. Find the distance travelled by it in 10 s and its velocity. Find also the distance coverer! by it in the 10th second of its motion.

3. A body is projected vertically upwards with a velocity of 98 m/s. How high will it rise and how much time will it take to return to its point of projection ? [g = 9.8 m/s^2]

4. A body let fall from the top of a vertical cliff covers 83.3 m in the last second oi' its motion. Find the height of the cliff. [g = 9.8 m /s^2]

5. A body is thrown horizontally from the top of a tower and strikes the ground after two seconds at an angle of 45° with the horizontal. Find the height of the tower and the speed with which the body was thrown. [Take g = 9.8 ms^{-2}.]

6. Two bodies are projected vertically upwards from the ground with velocities of 49 m/s and 21 m/s respectively. Find the difference between the maximum heights reached by them.

Type II: Problems based on relative velocity

7. Train A moves with a uniform velocity of 60 kmh $^{-1}$. Another train B moves in the same direction with a uniform velocity of 80 kmh^{-1}. (i) What is the relative velocity of A with respect of B? (ii) What is the relative velocity of B with respect to A?

8. Two trains A and B are moving on parallel tracks with velocities of 60 kmh^{-1} and 90 kmh^{-1} respectively but in opposite directions. Find (i) the relative velocity of train A w.r.t. train Band (ii) the relative velocity of train A w.r.t. ground.

9. Ajet airplane travelling at the speed of 450 kmh^{-1} ejects the burnt gases at the speed of 1200 kmh^{-1} relative to the jet airplane. Find the speed of the burnt gases w.r.t. a stationary observer on earth.

10. Two trains 120 m and 100 m in length are running in opposite directions on parallel tracks with velocities 40 kmh^{-1} and 32 kmh^{-1}. In what time will they completely cross each other?

Type III: Problems based on projectile motion

11. A projectile is projected at an angle of 60°. If its time of flight is 16 sec, calculate its horizontal range.

12. Two particles are projected horizontally from the top of a tower 122.5 m high. Their velocities of projection are 20 m/s and 50 m/s respectively. Compare their times of flight.

13. A boy playing on the roof of a 10m high building throws a ball with a speed of 10 ms^{-1} at an angle of 30° with horizontal. How far from the throwing point will the ball be at the height of 10m from the ground?

[g = 10 m/s^{-1}]

14. A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high above the ground can the cricketer throw the same ball?

15. A shell is fired at an angle of 30° to the horizontal with a velocity of 196 ms^{-1}. Find the total time of flight, horizontal range and maximum height attained.

16. A stone is thrown up with a velocity of 39.2 m/s at 30° to the horizontal. Find at what values of time will it be at a height of 14.7 m?

Type IV: Miscellaneous

17. Find the angle of projection so that a body when projected has the horizontal range equal to the maximum height attained.

18. A car travels from A to B with a uniform velocity of 30 km/hr and returns back with a uniform velocity of 40 km/hr. Find the average velocity and the average speed of the car if the distance AB is 120 km.

19. A stone is falling freely from the top of a tower 400 m high. At the same time another stone is projected upwards from the bottom of the tower with a speed of 100 m/s. Find when and where the two stones will meet?

[g = 9.8 ms^{-1}]

Multipal Choice Questions

1. Which of the following changes, when a particle is moving with uniform velocity?

a) speed b) velocity

c) acceleration d) position vector

2. If distance covered by a particle is zero, what can you say about its displacement?

a) It mayor may not be zero

b) It cannot be zero

c) It is negative

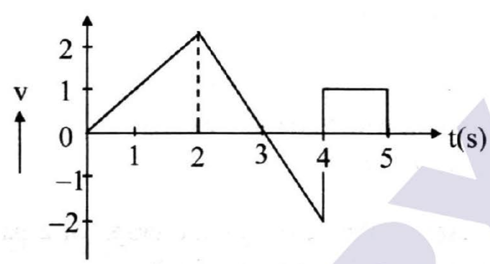
d) It must be zero

3. A car travels half the distance with constant velocity 50 km/h and another half with a constant velocity of 40 km/h, along a straight line. The average velocity of the car in kmlh is

a) 45 b) 44.4

c) 0 d) $\sqrt{(50\times 40)}$

4. The velocity v versus t graph of a body in a straight line is as shown in figure. The displacement of the body in five seconds is



a) 2m b) 3 m

c) 4m d) 5 m

5. A ball thrown up is caught by the thrower 6 s after start. The height to which the ball has risen is [$g = 10\ m/s^{-1}$]

a) 0 m b) 30 m

c) 45 m d) 90 m

6. A bullet fired into a block loses half its velocity after moving 0.9 m. Before it comes to rest, distance travelled would be

a) 1.2 m b) 0.8 m

c) 2.0 m d) 1.6 m

7. The range of projectile is 1.5 km when it is projected at an angle of 15° with horizontal. What will be its range when it is projected at an angle of 45° with the horizontal?

a) 0.75 km b) 1.5 km

c) 3 km d) 6 km

8. A stone thrown upward with a speed u from the top of the tower reaches the ground with a velocity 3u. The height of the tower is

a) $3\ u^2/g$ b) $4\ u^2/g$

c) $6\ u^2/g$ d) $9\ u^2/g$

9. With what speed must a ball be thrown down for it to bounce 10m higher than its original level? Neglect any loss of energy in striking the ground.

a) 14 m/sec b) 20 m/sec

c) 5 m/sec d) none

10. The velocity-time relation of a particle starting from rest is given by v = kt where $k = 2\ m/s^2$. The distance travelled in 3 sec is

a) 9 m b) 16 m

c) 27 m d) 36 m

11. A particle at rest is dropped under gravity from a height h ($g = 9.8\ m/sec^2$) and if it travels a distance $\frac{9}{25}$ h in the last second, then the height h is

a) 100 m b) 122.5 m

c) 145 m d) 168.5 m

12. Which of the following remains constant for a projectile fired from the earth?

a) Momentum

b) Kinetic energy

c) Vertical component of velocity

d) Horizontal component of velocity

13. In case of a projectile, what is the angle between the instantaneous velocity and acceleration at the highest point?

a) 45° b) 180°

c) 90° d) 0°

14. What determines the nature of the path followed by a particle?

a) Velocity b) Speed

c) Acceleration d) straight line

15. A player kicks up a ball at an angle 8 with the horizontal. The horizontal range is maximum when 8 is equal to

a) 30° b) 45°

c) 60° d) 90°

16. The greatest height to which a man can throw a stone is h. The greatest distance to which he can throw it will be

a) h /2 b) 2 h
c) h d) 3h

17. Two balls are projected at an angle 8 and (90° – 8) to the horizontal with the same speed. The ratio of their maximum vertical heights is
a) 1 : 1 b) tan 8 : 1
c) 1 : tan 8 d) $\tan^2 8$: 1

18. When air resistance is taken into account while dealing with the motion of the projectile, which of the following properties of the projectile show an increase?
a) range
b) maximum height
c) speed at which it strikes the ground
d) the angle at which the projectile strikes the ground

19. The maximum height attained by projectile is found to be equal to 0.433 of horizontal range. The angle of projection of this projectile is
a) 30° b) 45°
c) 60° d) 75°

20. A projectile is thrown with an initial velocity of 50 m/s. The maximum horizontal distance which this projectile can travel is
a) 64 m b) 128 m
c) 5 m d) 255 m

21. A large number of bullets are fired in all directions with the same speed v. What is the maximum area on the ground on which these bullets will spread?
a) $\pi \frac{v^2}{g}$ b) $\pi \frac{v^4}{g^2}$
c) $\pi^2 \frac{v^2}{g^2}$ d) $\pi^2 \frac{v^2}{g^2}$

22. A body is thrown vertically upwards, maximum height is reached, then it will have
a) zero velocity and zero acceleration.
b) zero velocity and finite acceleration.
c) finite velocity and zero acceleration.
d) finite velocity and finite acceleration.

23. The velocity-time graphs of two objects make angles 30° and 60° with time axis, then the ratio of their acceleration is
a) $\frac{1}{6}$ b) $\frac{1}{4}$
c) $\frac{1}{3}$ d) $\frac{1}{2}$

24. The relative velocity of two objects moving with same speed and in the same direction is
a) negative b) zero
c) positive d) infinite

25. A jet airplane travelling at the speed of 500 kmh^{-1} ejects the burnt gases at the speed of 1400 kmh^{-1} relative to the jet airplane. The speed of burnt gases relative to stationary observer on the earth is
a) 2.8 kmh^{-1} b) 190 kmh^{-1}
c) 700 kmh^{-1} d) 900 kmh^{-1}

26. Which of the following is NOT a projectile?
a) A bullet fired from gun.
b) A shell fired from cannon.
c) A hammer thrown by athlete.
d) An aeroplane in flight.

27. A projectile projected with certain angle reaches ground with
a) double angle
b) same angle
c) greater than 90°
d) angle between 90° and 180°

28. A projectile projected with certain velocity reaches ground with (magnitude)
a) zero velocity
b) smaller velocity
c) same velocity
d) greater velocity

29. For uniformly accelerated motion, the slope of the v – t graph is
a) negative b) zero
c) positive d) infinite

30. If the particle is at rest, then the x – t graph can be only
a) parallel to position - axis
b) parallel to time - axis
c) inclined with acute angle
d) inclined with obtuse angle

31. The x – t graph for an object having uniformly accelerated motion is
a) straight line b) circle
c) parabola d) hyperbola

32. The slope of x – t graph at any point gives
a) instantaneous velocity
b) instantaneous acceleration

c) force at that instant
d) momentum at that instant

33. The area under v – t graph gives
a) uniform acceleration
b) velocity
c) displacement
d) force

34. Two balls A and B of same masses are thrown from the top of the building. A thrown upward with velocity v and B thrown downward with velocity v, then
a) Velocity of A is more than B at the ground
b) Velocity of B is more than A at the ground
c) Both A and B strike the ground with same velocity
d) None of these

Answer Keys

1. d)	2. d)	3. b)	4. b)	5. c)	6. a)	7. c)	8. b)	9. a)	10. a)
11. b)	12. d)	13. c)	14. a)	15. b)	16. b)	17. d)	18. d)	19. c)	20. d)
21. b)	22. b)	23. c)	24. b)	25. d)	26. d)	27. b)	28. c)	29. c)	30. b)
31. c)	32. a)	33. c)	34. c)						

Answers to Practice Problems :

1. 475 m, 40 s
2. 12.5 m, 2.5 mis, 2.375 m
3. 490 m, 20 s
4. 396.9 m
5. 19.6 m, 19.6 ms^{-1}
6. 100 m
7. –20 kmh^{-1} , 20 kmh^{-1}
8. i. 150 kmh^{-1}
   ii. –60 kmh^{-1}
9. 750 kmh^{-1}
10. 11s
11. 724.25 m
12. 1 : 1
13. 8.66m
14. 50m
15. 20 s, 3395 m, 490 m
16. 1 sand 3 s
17. 75°58'
18. Zero, 34.3 kmlhr
19. 4 seconds, 321.6 m from ground