4.0 : Introduction

Q.1. Define force and state its units and dimensions.

Ans: i. A force is a push or pull exerted on a body, which produces or tends to produce a change in its velocity.

ii. It is given by F = ma,
where, m = mass of the body
a = acceleration

iii. Unit: newton in S.I. system and dyne in c.G.S. system.

iv. Dimension: $[M^1L^1T^{-2}]$

Q.2. State Newton's first law of motion. How is it related with inertia of a body?

Ans: i. **Newton's first law of motion:** Every body continues to be in a state of rest or of uniform motion along a straight line, unless it is acted upon by an external unbalanced force.

ii. The inability of a body to change by itself its state of rest or of motion is called "inertia" of the body. Inertia of a body is measured by the mass of the body.

iii. Heavier the body, greater is the force required to change its state and hence greater is its inertia.

Q.3. An astronaut accidentally gets separated out of his small spaceship accelerating in inter stellar space at a constant rate of 100 m s^{-2}. What is the acceleration of the astronaut the instant after he is outside the spaceship? (Assume that there are no nearby stars to exert gravitational force on him.)

(NCERT)

Ans: i. Assuming absence of stars in the vicinity, the only gravitational force exerted on astronaut is by the spaceship.

ii. But this force is negligible.

iii. Hence, once astronaut is out of the spaceship net external force acting on him can be taken as zero. From the first law of motion, the acceleration of astronaut is zero.

Q.4. State and explain ewton's second law of motion.

Ans: Statement: The rate of change of linear momentum of a body is directly proportional to the external unbalanced force applied on it and takes place in the direction of force.

Explanation:

i. The linear momentum of a body is the product of mass and its velocity.

ii. If m $\vec{u}$ is the initial linear momentum $\vec{P}_1$ and m $\vec{v}$ is the final linear momentum $\vec{P}_2$ after time t, then using Newton's second law, $\frac{\vec{P}_2 - \vec{P}_1}{t} \propto \vec{F}$

$$\therefore \quad \vec{F} = k\left(\frac{\vec{P}_2 - \vec{P}_1}{t}\right)$$

iii. The constant of proportionality k is 1 when we choose unit of momentum as kg-m/s and unit of force as newton. Similarly in C.G.S. system of units the unit of force is dyne by taking k = I.

$$\therefore \quad \vec{F} = \frac{\vec{P}_2 - \vec{P}_1}{t} = \frac{m\vec{v} - m\vec{u}}{t} = m\left(\frac{\vec{v} - \vec{u}}{t}\right) = m\vec{a}$$

Where, $\vec{a}$ is the acceleration of the body and it is assumed that mass m is constant.

Q.5. Explain why a cricketer moves his hands backwards while holding a catch.

(NCERT)

Ans: i. By drawing hands backward in the act of catching the ball, cricketer allowes longer time for his hands to stop the ball.

ii. By Newton's second law of motion, force applied depends on the rate of change of momentum.

iii. Taking longer time to stop the ball ensures smaller rate of change of momentum.

iv. Due to this the cricketer can stop the ball by applying smaller amount of force and thereby not hurting his hands.

Q.6. State and explain Newton's third law of motion.

Ans: Statement: To every action there is always an equal and opposite reaction.

Explanation:

i. If body A exerts force on body B, then body B also exerts equal and oppositeforce on body A, simultaneously.

ii. Two interacting bodies exert forces which are always equal in magnitude, have the same line of action and are opposite in direction, upon each other. Thus, forces always occur in pairs.

iii. A single isolated force is not possible. Action and reaction forces act always on different bodies. Hence they never cancel each other. Each force produces its own effect.

Q.7. Give the magnitude and direction of the net force acting on:

i. a drop of rain falling down with a constant speed.

ii. a cork of mass 10 g floating on water.

iii. a kite skillfully held stationary in the sky.

iv. a car moving with a constant velocity of 30 kmh^{-1} on a rough road.

v. a high speed electron in space far from all gravitating objects, and free of electric and magnetic fields.

(NCERT)

Ans: i. The drop of rain falls down with a constant speed, hence according to the first law of motion, the net force on the drop ofrain is zero.

ii. Since the 10^9 cork is floating on water, its weight is balanced by the upthrust due to water. Therefore, net force on the cork is zero.

iii. As the kite is skillfully held stationary in the sky, in accordance with first law of motion, the net force on the kite is zero.

iv. As the car is moving with a constant velocity of 30 km/h on a road, the net force on the car is zero.

v. As the high speed electron in space is far from all material objects, and free of electric and magnetic fields, hence it doesn't accelerate and moves with constant velocity. Hence, net force acting on the electron is zero.

Q.8. Explain the concept of impulse. Write down its unit and dimension.

Ans: i. Impulse is defined as the product of the average force and time for which force acts.

ii. Consider an impulsive force $\vec{F}$ acting on a body of mass m for short time t. The velocity of the body changes from $\vec{u}$ to $\vec{v}$.

iii. According to Newton's second law,

$$\therefore \quad \vec{F} = \frac{\vec{P}_2 - \vec{P}_1}{t}$$

$$\therefore \quad \vec{F}t = \vec{P}_2 - \vec{P}_1$$

∴ Impulse of the force or impulse

$$\vec{F}t = \vec{P}_2 - \vec{P}_1 = \vec{j} = m\vec{v} - m\vec{u}$$

iv. Impulse received during an impact is equal to the total change in momentum during the impact.

v. Impulse is a vector quantity. Its direction is same as that of the force.

vi. Unit: S.I. unit of impulse is kg m s^{-1} or Ns. In C.G.S. system, unit of impulse is g em s^{-1} or dyne s.

vii. Dimension: $[M^1L^1T^{-1}]$

Q.9. If a body is not accelerated, does a force act on it or not?

Ans: According to Newton's second law, $\vec{F} = m\vec{a}$. If acceleration a = 0, then the net force acting on the body is zero. Therefore, If a body is not accelerated, F = 0

Q.10. Large force always produces large change in momentum on a body than a small force. Is this correct?

Ans: No. From Newton's second law, we have,

$$\frac{dP}{dt} = F \qquad .... (1)$$

$$dP = Fdt \qquad .... (2)$$

From equation (2), we can infer that a small force acting for a longer time can produce same change in momentum of a body as a large force acting in the same direction for a short time. Hence, the given statement is incorrect.

4.1 : Types of force :

Q.11. What do you mean by real force and pseudo force?

Give examples of each.

Ans: Real force:

i. A force which is produced due to interaction between the objects is called realforce.

ii. It has specific source and origin, external to

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the body experiencing that force.

iii. It can be explained on the basis of fundamental interactions, such as, gravitational, nuclear or electromagnetic.

Examples:

a. The earth revolves around the sun in circular path due to gravitational force of attraction between the sun and the earth.

b. A relative motion between two solid surfaces in contact gives rise to a force of friction.

c. Binding of protons and neutrons in the nucleus of an atom is due to nuclear force.

Pseudo force:

i. A pseudo force or a fictitious force is one which arises due to the acceleration of the observer's frame of reference

OR

A force which does not have a real origin called pseudo force.

ii. When we are travelling by a bus and if its stops suddenly, we feel a push in forward direction.

iii. This force we cannot attribute to any external source. Hence we cannot explain this phenomenon using 'Newton's laws of motion, because according to Newton's law only 'external force' can change the motion ofa body.

iv. However the push that we experience when the bus stops suddenly is due to our own inertia.

Examples:

a. When a vehicle moves along a curved road, a passenger in the vehicle experiences a force in the outward direction i.e. away from the centre of the curved road. This is due to pseudo force.

b. Bus is moving with an acceleration (a) on a straight road in forward direction, a person of mass 'm' experiences a backward pseudo force of magnitude 'ma'.

4.2 : General idea of gravitational, electromagnetic and nuclear force from daily life experiences :

Q.12. Name the different types of fundamental forces in nature.

Ans: Fundamental forces in nature are classified into four types:

i. Gravitational force

ii. Electromagnetic force

iii. Strong nuclear force

iv. Weak nuclear force

Q.13. Define gravitational force. Give its examples.

Ans: Gravitational force:

i. Force of attraction between a body (on account of its mass) and the earth is called gravitational force.

ii. The existence of force of attraction between a body and earth is universal. This force is called gravitational force of attraction.

Examples:

a. The motion of moon, artificial satellites around the earth and motion of planets around the sun is due to gravitational force of attraction.

b. The concept of weight of a body in the earth's gravitational field is due to gravitational force exerted by the earth on a body.

Q.14. State Newton's law of gravitation.

Ans: Newton's law of gravitation:

Every particle of matter attracts every other particle with aforce whose magnitude is

i. directly proportional to the product of their masses and

ii. inversely proportional to the square of the distance between them and independent of the media.

This force is directed along the line joining the centres of the two particles. Let m_1 and m_2 be the masses of two particles separated by a distance r, then according to Newton's law of gravitation,

$$F \propto \frac{m_1 m_2}{r^2}$$

$$\therefore \quad F = G\frac{m_1 m_2}{r^2}$$

where 'G' is constant called the universal gravitational constant.

$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$

Q.15. Write down the main characteristics of gravitational force.

Ans: Characteristics of gravitational force:

i. This force is a central force which is always attractive and never repulsive.

ii. It obeys inverse square law of distances i.e.,

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gravitational force between two bodies varies inversely as the square of the distance between them i.e., $F \propto \frac{1}{r^2}$.

iii. It is independent of the presence of other medium.

iv. It is the weakest of the four basic forces in nature.

v. Its range is unlimited.

vi. It is a conservative force i.e work done against the force is stored in the form of P.E.

Q.16. Define electrostatic force and magnetic force.

Ans: Electrostatic force:

The force between two charges at rest is called electrostaticforce.

Magnetic force:

Force exerted between two poles of a magnet is called magneticforce.

Force between unlike poles is attractive and like poles is repulsive in nature.

Q.17. Write a note on electromagnetic force.

Ans: Electromagnetic force:

i. The forces between electric charges in relative motion are called electromagneticforces.

ii. When a charge is moving, it produces magnetic field and moving charge in magnetic field experiences a magnetic force. This force is an electromagnetic force.

iii. These forces not only depend on charge carried by two bodies but also on the magnitude and direction of their velocities.

iv. These forces hold together the atoms and molecules of a body.

Examples:

a. Force of attraction between electrons and protons in the nucleus.

b. Repulsive force between two electrons or two protons.

c. The electrons inside an atom revolve around the nucleus on account of the force of attraction between positively charged nucleus and negatively charged electrons.

d. Two parallel conductors carrying current in the same direction attract each other and when the current flows in opposite directions they repel each other.

Q.18. State characteristics of electromagnetic force.

Ans: Characteristics of electromagnetic force:

i. These forces can be attractive or repulsive.

ii. It is about 10^{36} times stronger than the gravitational force.

iii. It obeys the inverse square law of distance.

iv. It depends on intervening medium.

v. It is a long-range force.

vi. It is a central force which is conservative.

Q.19. Write a note on nuclear force.

Ans: Nuclear force:

i. The force exerted inside the nucleus of an atom which holds protons and neutrons by making the atom stable is called as nuclearforce.

ii. The phenomenon of radioactivity can be explained on the basis of nuclear forces.

iii. The concept of nuclear force is used in obtaining nuclear energy by the processes of fission and fusion.

iv. There are two types of nuclear forces.

a. Strong nuclear force: The strong interaction, which binds protons and neutrons in the nucleus of an atom is called strong nuclear force.

b. Weak nuclear force: The force of weak interaction which results in the decay of many unstable nuclei is called weak nuclear force.

Q.20. Discuss the characteristics of strong nuclear forces.

Ans: Characteristics of strong nuclear forces:

i. They are basically attractive forces.

ii. They have the shortest range and operate only within a nucleus upto a distance of 10^{-14} m.

iii. It is the strongest force in nature.

iv. They are non-central and nonconservative forces.

v. They are charge independent.

vi. They do not obey the inverse square law of distances i.e., they vary inversely as some higher power of distance between nucleons.

Q.21. Discuss the characteristics of weak nuclear forces.

Ans: Characteristics of weak nuclear forces:

i. It acts between any two elementary particles (pair of subatomic particle).

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ii. It is a stronger force than gravitational force.

iii. It is much weaker than electromagnetic force or strong nuclear force.

iv. It is a very short range force effective only within a nucleon of the order of size of the nucleus (= 10^{-18} m).

Q.22. Define the following terms.

i. Conservative force

ii. Non-conser.vative force

Ans: i. Conservative force:

A force is said to be conservative if the, work done by the force in moving a body between any two points is independent of path followed by the body.

OR

A force is said to be conservative if work done by the force in moving a particle along a closed path is zero.

eg.: Gravitational force

ii. Non-conservative force:

The force is said to be non-conservative if the work done by a force in moving a body over a closed path is non-zero.

eg.: frictional force

4.3 : Law of conservation of Momentum :

Q.23. Define momentum. Discuss its unit and dimension.

Ans: i. The product of mass and velocity of a body is called momentum.

ii. It is a vector quantity.
It is denoted by P. In vector form, it is written as $\vec{P} = m\vec{v}$.

iii. Unit: kg m s^{-1} in SI system and g cm s^{-1} in CGS system.

iv. Dimension: $[M^1L^1T^{-1}]$

v. Momentum is classified into two types.

a. Linear momentum $\vec{p}$:
Momentum possessed by a body by virtue of its linear motion is called linear momentum.

b. Angular momentum $\vec{L}$:
Momentum possessed by a body by virtue of its rotational motion is called angular momentum.

Note: Dimensions and units of angular momentum are different than linear momentum.

Q.24. State and derive law of conservation of linear momentum in case of colliding bodies.

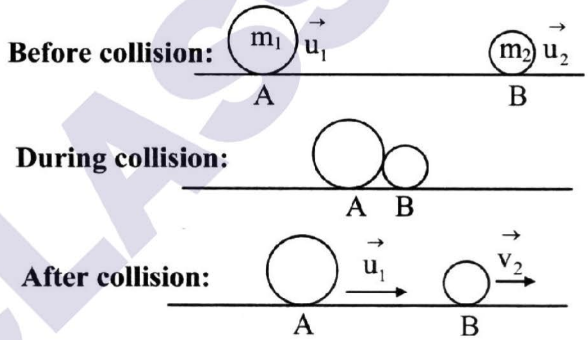
Ans: Statement:

When the resultant external force acting on a system of interacting bodies is zero (i.e., an isolated system), the total linear momentum of the system of interacting bodies remains constant or conserved.

Derivation:

i. Consider two rotating spherical bodies A and B of masses m_1 and m_2 respectively, moving along a straight line with initial velocities $\vec{u}_1$ and $\vec{u}_2$ in the same direction.

ii. If $\vec{u}_1 > \vec{u}_2$, both A and B will collide. Let t be the time duration of collision.



Collision between two bodies

iii. During the collision, both A and B will exert force on each other, in opposite direction. After collision, let their velocity change to $\vec{v}_1$ and $\vec{v}_2$ respectively.

iv. Acceleration of A $= \dfrac{\vec{v}_1 - \vec{u}_1}{t} = \vec{a}$

$\therefore$ Force on A by B, $\vec{F}_{AB} = \dfrac{m_1(\vec{v}_1 - \vec{u}_1)}{t}$

Similarly, acceleration of B $= \dfrac{\vec{v}_1 - \vec{u}_1}{t}$

$\therefore$ Force on B by A $= \vec{F}_{AB} = \dfrac{m_1(\vec{v}_1 - \vec{u}_1)}{t}$

v. By Newton's third law, the forces F_{AB} and F_{BA} are equal and opposite.

$\therefore$ $\vec{F}_{AB} = -\vec{F}_{BA}$

$\therefore$ $\vec{F}_{AB} = \dfrac{m_1(\vec{v}_1 - \vec{u}_1)}{t} = \vec{F}_{AB} = \dfrac{m_1(\vec{v}_1 - \vec{u}_1)}{t}$

$\therefore \quad m_1\vec{v}_1 - m_1\vec{u}_1 = -m_2\vec{v}_2 + m_2\vec{u}_2$

$\therefore \quad m_1\vec{v}_1 - m_1\vec{u}_2 = -m_1\vec{v}_1 + m_2\vec{u}_2$

Thus, total linear momentum before collision is equal to total linear momentum after collision of two spheres, if resultant external force acting on two spheres is zero.

Q.25. Give 2 examples of principle of conservation of linear momentum.

Ans: Examples:

i. **When a person jumps from a boat, the boat is pushed away:** When a person jumps from a small boat on the the shore, he imparts an equal and opposite momentum to the boat. Therefore, the boat is pushed away backward.

ii. **A hammer is rebound when a nail is driven into wall by hitting it:** When a nail is driven into a wall by striking it with a hammer, the hammer is seen to rebound after striking the nail. This is because the hammer imparts a certain amount of momentum to the nail and the nail imparts an equal and opposite amount of momentum to the hammer.

Q.26. Explain recoiling of the gun on the basis of conservation of momentum and obtain an expression of the recoil velocity of gun in terms of muzzle velocity of the bullet.

OR

Explain the principle of conservation of linear momentum, with an example.

Ans: i. When a bullet is fired from a gun, the gun recoils. The bullet and gun move in exactly opposite direction. In this case, linear momentum of the bullet is equal to linear momentum of the gun.

ii. Let m_1 = mass of bullet

m_2 = mass of gun

$\vec{v}_1$ = velocity of the bullet

and $\vec{v}_2$ = velocity of gun

iii. Before firing, the gun and bullet both are at rest, hence total linear momentum is zero. The total linear momentum after firing is given by, $m_1\vec{v}_1 + m_2\vec{v}_2$.

iv. Using conservation of linear momentum,

$\therefore \quad 0 = m_1\vec{v}_1 + m_2\vec{v}_2$

$\therefore \quad m_2\vec{v}_2 = -m_1\vec{v}_1$

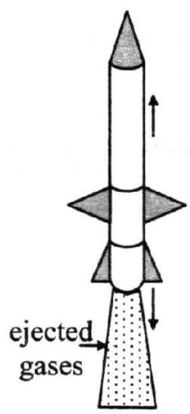
$\therefore \quad \vec{v}_2 = \left(\frac{m_1}{m_2}\right)(-\vec{v}_1)$

The negative sign in the above equation shows that the direction of $\vec{v}_2$ is exactly opposite to $\vec{v}_1$ i.e., gun recoils.

v. As $m_2 >> m_1$, $\vec{v}_2 << \vec{v}_1$ i.e., recoil velocity is very small compared to velocity of the bullet.

Q.27. Explain the principle of conservation of linear momentum in rocket propulsion. (motion)

Ans: i. In rocket, liquid oxygen is bumt in the combustion chamber. The hot gases are ejected through a small opening in the tail, at high pressure and with very high speed. As a result of it, large momentum of the gases is produced in the backward direction. This imparts an equal forward momentum to the rocket as per the law of conservation of linear momentum. Due to this, the rocket moves upwards with high speed.



Motion of Rocket

ii. As the rate of ejection of gas remains uniform, the rate of change of momentum of the rocket is uniform. Hence, force acting on the rocket is constant. Due to escaping gases, mass of the rocket goes on decreasing and velocity and acceleration goes on increasing.

iii. Thus, even though the force remains constant, the velocity and acceleration of rocket increases and attains very high velocity.

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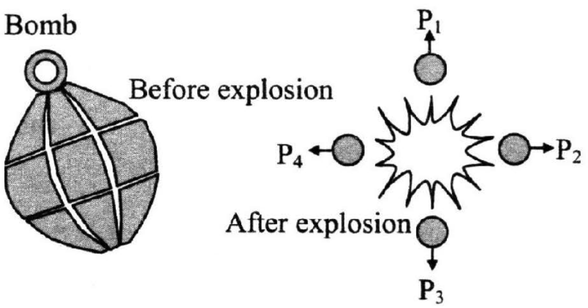
Q.28. Explain, how linear momentum is conserved during the explosion of a bomb.

Ans: i. When bomb is at rest before explosion, the linear momentum of bomb is zero.

ii. When bomb explodes into number of pieces, scattered in different directions, the vector sum of linear momentum of these pieces is also zero. This is as per the law of conservation of linear momentum.

iii. Let the bomb explode into four pieces as shown in the figure. Let m_1, m_2, m_3 and m_4 be the masses of four pieces and $\vec{v}_1, \vec{v}_2, \vec{v}_3$, and $\vec{v}_4$ be their velocities respectively.



iv. Linear momenta of four pieces are given by

$\vec{P}_1 = m_1\vec{v}_1$, $\vec{P}_2 = m_2\vec{v}_2$, $\vec{P}_3 = m_3\vec{v}_3$,

$\vec{P}_4 = m_4\vec{v}_4$

v. As no external force is acting on the system, the total linear momentum is conserved.

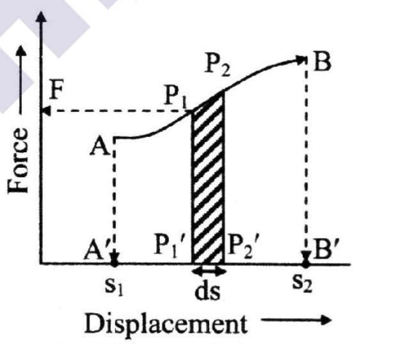
∴ Linear momentum before explosion = linear momentum after explosion.

$0 = m_1\vec{v}_1 - m_1\vec{u}_2 = -m_1\vec{v}_1 + m_2\vec{u}_2$

Thus, linear momentum remains conserved.

Q.29. Explain graphically the work done by a variable force.

Ans:



Graph of Force - Displacement

i. Let the force vary in magnitude between the points A and B.

ii. Let at P_1 the magnitude of force be $F = P_1P_1$.

Due to this force, the body displaces through infinitesimally small displacement ds, in the direction of force.

It moves from P_1 to P_2 ∴ $\overline{ds} = \overline{P_1P_2}$

iii. But direction of force and displacement are same, we have

$\overline{ds} = \overline{P_1P_2}$

iv. Hence, small work done between P_1 to P_2 is dW and is given by

$dW = \vec{F} \cdot \overline{ds} = P_1P_1 \times P_1P_2$.

= Area of the strip PIP2P2'PI'.

v. The total work done can be found out by dividing the portion AB into small strips like $P_1P_2P_2P_1$ and taking sum of all the areas of the strips.

∴ $W = \int_{s_1}^{s_2} \vec{F}.\overline{ds}$ = Area ABB'A'

vi. Graphically work done by the variable force is represented by the area under the portion of force-displacement graph.

Q.30. Explain the concept of work done by a variable force.

OR

Derive an expression for work done in moving a body from one point to another point under the action of variable force. Determine the amount of work done, if force and displacement (a) is along same direction, (b) are perpendicular to each other.

Ans: i. Suppose a body is moving from A to B under the action of variable force as shown in figure.

ii. Let 'O' be the reference point s_1 and s_2 are magnitudes of initial and final displacements of the body at points A and B with respect to 'O' respectively.

iii. Suppose at some instant of time, the body is at P_1 and the force acting at P_1 is $\vec{F}$. Now, due to the application of force, the body moves through infinitesimally small distance $\overline{ds}$ and reaches P_2.

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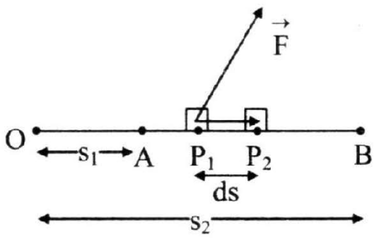

Body under variable force

$\therefore$ $P_1P_2 = \overline{ds}$

iv. Though the force acting on the body is varying, for infinitesimal small displacement $\overline{ds}$ it can be assumed to be constant.

v. Hence, small amount of work done dW by the force $\vec{F}$ between the points P_1 and P_2 is given by,

$dW = \vec{F} \cdot \overline{ds}$ .... (1)

vi. The total work done in moving the body from A to B can be obtained by integrating equation (1) within proper limits of integration.

$$W = \int dW = \int_A^B \vec{F}.\overline{ds}$$

If s_1 and s_2 be the magnitudes of displacements of the body at points A and B w.r.t. point O then,

vii. If θ is the angle between the force $\vec{F}$ acting on the body along the path and displacement ds, then the work done is given as,

$$W = \int_{s_1}^{s_2} F ds \cos\theta \quad .... (2)$$

viii. Equation (2) represents expression for work done in moving a body from one point to another point under the action of variable force.

Case 1:

If $\vec{F}$ and $\overline{ds}$ are along the same direction

then, $W = \int_{s_1}^{s_2} F ds \cos\theta = \int_{s_1}^{s_2} F ds$

$[\because \theta = 0^\circ \therefore \cos 0^\circ = 1]$

Cases 2:

If $\vec{F}$ and $\overline{ds}$ are perpendicular to each other then,

$$W = \int_{s_1}^{s_2} F ds \times 0 = 0$$

Q.31. Work and kinetic energy are equivalent quantities, justify.

Ans: i. When a force does some work on a body, the kinetic energy of the body increases by the same amount.

ii. Conversely, when an opposing force is applied on a body its K.E. decreases. This decrease in kinetic energy of the body is equal to the work done by the body against the opposing force.

iii. Therefore, work and kinetic energy are equivalent quantities.

Q.32. State and explain work-energy theorem.

OR

Show that work done on a body by a force is equal to the change in its kinetic energy

Ans: Statement: Work done by a force in displacing a body (i.e., only speed of system is changed) is equal to the change in the kinetic energy of the body.

Proof:

i. Suppose that the force $\vec{F}$ is applied on a body of mass which is initially at rest. The body now displaces through infinitesimal small displacement ds in the direction of the force, then small work done dW is given by,

$dW = \vec{F}.ds = Fds$ .... (1)

ii. According to Newton's 2nd law of motion

$F = ma$ .... (2)

where 'a' is the acceleration produced in the direction of force.

iii. From equations (1) and (2), we get,

$dW = ma\, ds$

$$dW = m\frac{dv}{dt}ds \qquad (\because a = \frac{dv}{dt})$$

$$= m\frac{dv}{dt}dv$$

$\therefore$ $dW = mv\, dv$ .... (3)

Where $\frac{ds}{dt} = v$ is instantaneous velocity

iv. Total work done to increase velocity from u to v can be found out by integrating equation (3) and is given by,

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$$W = \int_u^v dW$$

$$W = \int_u^v mvdv$$

$$W = m\int_u^v vdv$$

$$= m\left[\frac{v^2}{2}\right]_u^v = \frac{1}{2}m[v^2 - u^2]$$

$$W = \frac{1}{2}mv^2 - \frac{1}{2}\ mu^2 \qquad ...(4)$$

v. From equation (4), work done on a body by a force is equal to change in its kinetic energy.

Q.33. When ball rolls down an inclined plane, its momentum increases. Is the momentum conserved?

Ans: i. The law of conservation of momentum states that total momentum of an isolated system is conserved.

ii. This implies that the law of conservation of momentum is applicable to an isolated system i.e., system subjected only to internal forces.

iii. In the statement of the question, the ball is considered as a system.

iv. Now, a ball rolling down an inclined plane is subjected to external forces such as gravitational and frictional force.

v. Thus, it cannot be considered as an isolated system and hence momentum conservation of the ball cannot be expected.

Q.34. The sign of work done by a force on a body is important to understand. State carefully if the following quantities are positive or negative:

i. Work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket.

ii. Work done by gravitational force in the above case.

iii. Work done by friction on a body sliding down an inclined plane.

iv. Work done by an applied force on a body moving on a rough horizontal plane with uniform velocity.

v. Work done by the resistive force of air on a vibrating pendulum in bringing it to rest. (NCERT)

Ans: i. As the applied force and displacement of the bucket are in the same direction, i.e., upward direction, $\theta = 0$ and

work done by man, W = Fs cos θ

W = Fs cos 0^o = Fs $[\because \cos 0^o = 1]$

= Positive work

ii. As gravitational force acts in the downward direction and displacement of bucket is in the upward direction,

$\theta = 180°$.

Work done by gravitational force,

W = Fs cos θ

W = Fs cos 180° $[\because \cos 180° = -1]$

= –Fs

= Negative work

iii. Work done by friction on a body sliding down an inclined plane.

Friction always opposes the motion of a body, hence acts in a direction opposite to the - motion of body. Therefore, $\theta = 180°$.

Work done,

W = Fs cos θ

= Fs cos 180° $[\because 180° = -1]$

= –Fs

= Negative work

iv. As the applied force and motion of body are in the same direction, $\theta = 0°$.

Work done,

W = Fs cos θ

= Fscos θ $[\because \cos\theta = 1]$

= Fs

= Positive work

v. As the resistive force of air opposes the motion of bob of pendulum, $\theta = 180°$.

Work done,

W = Fs cos θ

= Fs cos 180°

= –Fs $[\because \cos\theta = 1]$

= Negative work

4.4 : Elastic and inelastic collisions in one and two dimensions :

Q.35. Explain elastic collision with examples.

Ans: i. Collision between two bodies in which total momentum and kinetic energy along with total energy of colliding bodies remain conserved is called as elastic collision.

ii. In an elastic collision,

$$m_1\vec{v}_1 - m_1\vec{u}_2 = -m_1\vec{v}_1 + m_2\vec{u}_2$$

It means that total momentum is conserved.

iii. In an elastic collision,

$$\frac{1}{2}\,m_1u_1^2 + \frac{1}{2}\,m_1u_2^2 = \frac{1}{2}\,m_1v_1^2 + \frac{1}{2}\,m_2v_2^2$$

It means that kinetic energy is conserved. eg.: At normal temperature, Collisions between gas molecules.

Q.36. Explain inelastic collision with two examples.

Ans: i. Collision between two bodies in which only linear momentum and total energy are conserved but kinetic energy is not conserved is called inelastic collision.

ii. In an inelastic collision,

$$m_1\vec{v}_1 - m_1\vec{u}_2 = -m_1\vec{v}_1 + m_2\vec{u}_2\,.$$

It means that linear momentum is conserved.

iii. In an inelastic collision,

$$\frac{1}{2}\,m_1u_1^2 + \frac{1}{2}\,m_1u_2^2 \neq \frac{1}{2}\,m_1v_1^2 + \frac{1}{2}\,m_2v_2^2$$

It means that kinetic energy in inelastic collision is not conserved.

Examples:

a. Collision between a bat and a ball, Collision between two vehicles.

b. When a ball is allowed to fall on a hard floor from a certain height, the collision is said to be inelastic. In this case, ball does not rebound to the original height due to loss of kinetic energy in the collision.

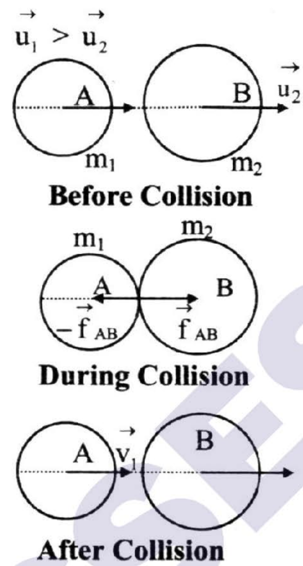
Q.37. Define perfectly inelastic collision.

Ans: A type of collision in which two bodies stick together after collision and move with common velocity or may stop moving is called perfectly inelastic collision.

eg.: Meteorite collides head-on on the earth, lump of mud thrown on a wall sticks to the wall due to the loss of kinetic energy.

Q.38. In case of an elastic head on collision between two bodies, derive an expression for the velocities of the bodies in terms of their masses and velocities before collision.

Ans: Head on elastic collision of two spheres:



Collision between two bodies

i. Consider two rotating smooth bodies A and B of masses m_1 and m_2 respectively moving along the same straight line.

ii. Let $\vec{u}_1$ = initial velocity of the sphere A , before collision.

$\vec{u}_2$ = initial velocity of the sphere B before collision.

$\vec{v}_1$ = velocity of the sphere A after collision.

$\vec{v}_2$ = velocity of the sphere B after collision.

iii. After the elastic collision, the spheres separate and move along the same straight line without rotation.

iv. According to the law of conservation of momentum,

$$m_1\vec{v}_1 - m_1\vec{u}_2 = -m_1\vec{v}_1 + m_2\vec{u}_2 \quad ....(1)$$

According to the law of conservation of energy (as kinetic energy is conserved during elastic collision),

$$\frac{1}{2}\,m_1u_1^2 + \frac{1}{2}\,m_1u_2^2 \neq \frac{1}{2}\,m_1v_1^2 + \frac{1}{2}\,m_2v_2^2$$

v. Since kinetic energy is a scalar quantity, the terms involved in the above equations are scalars.

vi. The equation (1) can be written in scalar form as,

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$m_1(u_1 - v_1) = m_2(v_2 - u_2)$ .....(3)

vii. Also the equation (2) can be written as,

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$$\frac{1}{2}m_1(u_1^2 - v_1^2) = \frac{1}{2}m_2(v_2^2 - u_2^2)$$

$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$$

$$m_1(u_1 - v_1)(u_1 - v_1) = m_2(v_2 - u_2)(v_2 - u_2) \quad ....(4)$$

viii. Now dividing equation (4) by (3) we get,

$$(u_1 + v_1) = (u_2 + v_2)$$

$$u_1 + v_1 = u_2 + v_2$$

$$v_2 = u_1 - u_2 + v_1 \quad ....(5)$$

ix. From equation (3),

$$m_1(u_1 - v_1) = m_2(v_2 - u_2)$$

$$v_2 = u_2 + \frac{m_1}{m_2}(u_1 - v_1) \quad ....(6)$$

x. Comparing equation (5) and (6),

$$u_1 - u_2 + v_1 = u_2 + \frac{m_1}{m_2}(u_1 - v_1)$$

$$v_1 + \frac{m_1 v_1}{m_2} = 2u_2 + \frac{m_1 u_1}{m_2} - u_1$$

$$v_1\left[1 + \frac{m_1}{m_2}\right] = u_1\left[\frac{m_1}{m_2} - 1\right] + 2u_2$$

$$v_1\left[\frac{m_1 + m_2}{m_2}\right] = u_1\left[\frac{m_1 - m_2}{m_2}\right] + 2u_2$$

$$v_1 = u_1\left[\frac{m_1 - m_2}{m_1 + m_2}\right] + u_2\left[\frac{2m_2}{m_1 + m_2}\right] \quad ...(7)$$

Substituting the value of VI in equation (5), we have

$$v_2 = u_1 - u_2 + u_1\left(\frac{m_1 - m_2}{m_1 + m_2}\right) + u_2\left(\frac{2m_2}{m_1 + m_2}\right)$$

$$= u_1\left(\frac{m_1 - m_2}{m_1 + m_2} + 1\right) + u_2\left(\frac{2m_2}{m_1 + m_2} - 1\right)$$

$$v_2 = u_1\left[\frac{2m_2}{m_1 + m_2}\right] + u_2\left[\frac{m_2 - m_2}{m_1 + m_2}\right] \quad ...(8)$$

Equations, (7) and (8), represent the final velocity of two spheres after collision.

Special cases:

Case 1:

If $m_1 = m_2$, then $v_1 = u_2$ and $v_2 = u_1$

Thus, A and B will exchange their velocities after head on elastic collision.

Case 2:

If second particle is at rest i.e., $u_2 = 0$ and $m_1 = m_2$ then,

$$v_1 = u_1\left(\frac{m_1 - m_2}{m_1 + m_2}\right) = 0,$$

$$v_2 = u_1\left(\frac{2m_2}{m_1 + m_2}\right) = u_1$$

Thus, A comes to rest and B will move continuously with velocity of A.

Case 3:

If m_1 is mass of a light body and m_1 is mass of heavy body i.e., $m_1 << m_2$, $u_2 = 0$, thus, m_1 can be neglected.

Hence $v_1 = -u_1$ and $v_2 = 0$.

Hence, B continues to be at rest while A reverses its velocity.

Q.39. Define coefficient of restitution between two colliding bodies and state its value for perfectly elastic and perfectly inelastic collision.

Ans: i. Coefficient of restitution is defined as the ratio of relative velocity of separation after collision to the relative velocity of approach before collision between two colliding bodies.

ii. We have,

$$v_2 - v_1 \propto u_1 - u_2$$

$$\therefore \quad v_2 - v_1 = e(u_1 - u_2)$$

$$\therefore \quad e = \frac{v_2 - v_1}{u_1 - u_2}$$

where e = constant called coefficient of restitution.

iii. Value of e lies between 0 and 1 i.e., $0 \le e \le 1$

iv. For perfectly elastic collision,

$$v_2 - v_1 = u_1 - u_2$$

$$\therefore \quad e = 1$$

v. For an inelastic collision. $0 < e < 1$.

vi. For a perfectly inelastic collision,

$$v_2 - v_1 = 0 \qquad \therefore \quad e = 0$$

Note:

1. Coefficient of restitution IS a dimensionless quantity and depends upon the materials of the bodies. It does not depend upon its mass.
2. The value of coefficient of restitution (e) shows the nature of collision between the bodies.
3. For two smooth glass balls, e = 0.94 and for lead balls, e = 0.2.

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Q.40. Two particles undergo one-dimensional, head-on, perfectly inelastic collision. Derive an expression for their final velocity in terms of their masses and initial velocities.

Ans: Expression for final velocity in inelastic head on collision.

i. Let two particles A and B of masses m_1 and m_2 move with initial velocity $\vec{u}_1$ and $\vec{u}_2$ respectively such that particle A collides head-on with particle B i.e.,
$u_1 > u_2$.

ii. If the collision is perfectly inelastic, the particles stick together and move with a common velocity $\vec{v}$ after the collision along the same straight line.

iii. By the law of conservation of momentum,
$m_1u_1 + m_2 u_2 = (m_1 + m_2) v$

$$\therefore \quad v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}$$

This is the required common velocity after collision.

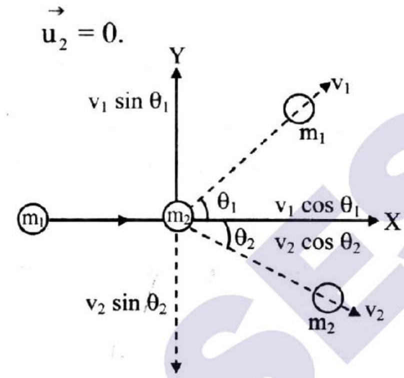
Q.41. Distinguish between elastic and inelastic collision.

Ans:

No.	Elastic Collision	Inelastic Collision
i.	In an elastic collision, both momentum and kinetic energy are conserved.	In an inelastic collision, momentum is conserved but kinetic energy is not conserved.
ii.	The total kinetic energy after collision is equal to the total kinetic energy before collision.	The total kinetic energy after the collision is less than the total kinetic energy before collision.
iii.	Coefficient of restitution (e) is equal to one.	Coefficient of restitution (e) is less than one. For a perfectly inelastic collision coefficient of restitution is equal to zero.
iv.	Bodies do not stick together in elastic collision.	Bodies stick together in a perfectly inelastic collision.
v.	Sound, heat and light are not produced.	Sound or light or heat or all these may be produced.

Q.42. Explain elastic collision in two dimensions.

Ans: i. Suppose a particle of mass m_1 moving with initial velocity u_1 collides with another particle of mass m_2 at rest, $\vec{u}_2 = 0$



ii. After collision, the particle of mass m_1 moves with velocity $\vec{v}_1$ at an angle θ_1 with respect to the x-axis while the particle of mass m_2 moves with velocity $\vec{v}_2$ at an angle θ_2 with respect to the x-axis.

iii. Total linear momentum before collision

$= m_1\vec{u}_1 + m_2\vec{u}_2$

$= m_1\vec{u}_1 + 0 \qquad [\because \vec{u}_2 = 0] \qquad ....(1)$

Total linear momentum after collision

$= m_1 \vec{v}_1 + m_2 \vec{v}_2 \qquad .....(2)$

iv. According to the law of conservation of linear momentum, we have,

$m_1\vec{u}_1 = m_1 \vec{v}_1 + m_2 \vec{v}_2 \qquad ....(3)$

v. The x-component of equation (3) is given by,

$m_1\vec{u}_1 = m_1 \vec{v}_1 \cos \theta_1 + m_2 \vec{v}_2 \cos \theta_2 ..(4)$

y-component of equation (3) is given by,

$0 = m_1v_1 \sin \theta_1 - m_1 v_2 \sin \theta_2 \qquad ....(5)$

vi. In elastic collision, kinetic energy is also conserved.
Hence, we have,

$$\frac{1}{2} m_1u_1^2 + \frac{1}{2} m_1u_2^2 \neq \frac{1}{2} m_1v_1^2 + \frac{1}{2} m_2v_2^2$$

vii. From above equations, it is clear that motion of the spheres after collision depends on the angle between the direction of initial velocity and the line of force during the collision.

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4.5 : Inertial and non–inertial frames :

Q.43. Define the term frame of reference. Give example and its importance.

Ans: i. A system of co–ordinate axes with reference to which the position or motion of an object is described is called aframe of reference.

ii. A state of a body can be clearly described with the help of such a coordinate system. The frame of reference is thus, a frame of co–ordinates.

iii. The frame of reference is a convenient way to specify the position or motion of a body.

iv. For example: Let us consider a ball thrown by a person standing on the ground. The person observes the ball falling vertically downwards after sometime. But to a person sitting in a moving car, the ball is seen to be falling along a parabolic path.

v. Thus, the same event of falling of a ball is described differently by the persons in different frames of reference. To avoid this confusion, it is necessary to specify the frame, with reference to which an event is described.

Q.44. Explain the terms inertial and non–inertial frame of reference.

Ans: Inertial frame of reference:

i. Inertial frame of reference is a coordinate system in which Newton's laws of motion holds good.

ii. A frame of reference moving with a constant velocity with respect to an inertial frame is also an interial frame of reference. eg.: A space ship drifting with constant speed and having no spin motion.

Non–inertial frame of reference:

A frame of reference in which Newton's laws of motion do not hold good is called noninertial frame of reference.

eg.: If bus moving with constant velocity is suddenly brought to rest by applying brake then the bus will be in a non–inertial frame of reference.

Q.45. Distinguish between inertial frame of reference and non–inertial frame of reference.

Ans:

No.	Inertial frame of reference	Non-inertial frame of reference
i.	The frame of reference moves with a constant velocity (including zero)	Velocity of the frame of reference is variable.
ii.	Newton's laws are obeyed.	Newton's laws are not obeyed.
iii.	In this frame, force acting on a body is a real force.	The acceleration of the frame gives rise to a pseudo force.

Q.46. Explain the concept of pseudo force in non–inertial frame of reference.

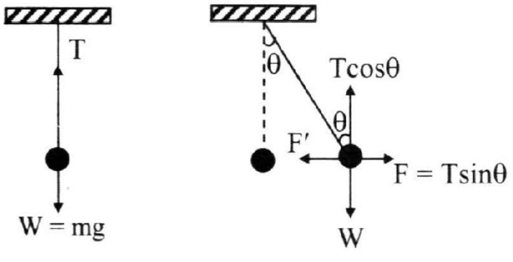
Ans: Concept of pseudo force in non–inertial frame of reference:

i. The force originating due to the acceleration of the frame of reference and not from any interaction with another body is called pseudo force.

ii. We know that a train at rest or moving with uniform velocity is an example of inertial reference frame, while accelerating train is an example of noninertial reference frame.

iii. When a pendulum is suspended from the roof of a train, it remains in equilibrium along the vertical as long as train is at rest or moving with uniform velocity.

iv. In this case, two forces acting on pendulum are, gravity in downward direction and tension in string in upward direction. These forces balance each other as shown in figure (a).



(a) Pendulum when train is at rest **(b) Pendulum in an accelerating train**

v. As the train accelerates in forward direction, pendulum is observed to incline in backward direction as shown in figure (b).

vi. If Newton's laws are applied to this situation, the weight of the bob is balanced by vertical component T cosd and the bob is acted upon by external, unbalanced force T sin θ ....(1)

vii. Yet, the pendulum is in equilibrium (inclined position) ... (2)

viii. Statements (1) and (2) are contradictory to each other and arise due to application of Newton's law to a noninertial frame of reference.

ix. Newton's laws can explain the state of equilibrium of the bob provided a force, F' = ma is introduced in opposite direction to the component of tension as shown in figure (b). This force is the pseudo force.

Q.47. Explain existence of pseudo force in
a. a lift
b. a car
c. a moving bus

Ans: a. Pseudo force in a lift:

i. If a person is standing In a lift moving with uniform velocity then he is in inertial frame of reference.

ii. If the lift is ascending with an acceleration i.e., accelerated frame of reference, the person feels a force in the downward direction called as pseudo force.

iii. If the lift is descending with an acceleration the pseudo force acts on person in the upward direction.

b. Pseudo force in a car:

i. A person sitting in a closed car, feels that he is at rest, if car is moving with uniform velocity. i.e., car is an inertial frame of reference.

ii. But if car is accelerated or decelerated, person experiences backward or forward push.

iii. This is because the person is now in non–inertial frame of reference.

iv. The backward and forward motion of a person is due appearance of psuedo force acting on him.

c. Pseudo force in a moving bus:

i. A passenger sitting in a bus moving along a circular path experiences a force towards the left or right.

ii. In this case bus performs circular motion, which is accelerated motion, hence, passenger is in accelerated frame (non inertial frame) of reference.

iii. A (pseudo) force acts on the passenger in the direction opposite to that of the acceleration.

iv. This force is along the radius and away from the centre of circular path.

4.6 : Moment of Force :

Q.48. Define the following terms.
i. Rigid body
ii. Axis of rotation

Ans: i. Rigid body:

A body is said to be rigid if the relative distance between any two particles of the body does not change under the application offorce of any magnitude.
Its shape and size does not change due to applied force.
In practice, there is no perfectly rigid body.

ii. Axis of rotation:

When a rigid body produces purely rotational motion, all the particles of the body move in circles. The centres of all circles lie along a straight line, called as axis of rotation.
When we apply force on a door to open or close then it gets rotated about hinges. This is axis of rotation of the door.

Q.49. Define moment arm.

Ans: The perpendicular distance between axis of rotation and line of action of force is called moment arm.

Q.50. Define the term moment of force about a point. State its S.I. units and dimensions. Write the vector form.

Ans: i. The ability of a force to produce rotational motion is measured by its turning effect called "moment of force or torque ".

ii. The magnitude of the moment of force is equal to the product of the magnitude of force and moment arm.

∴ Moment of force = force × moment arm

iii. Unit: Nm in SI system and dyne em in CGS system

iv. Dimension = $[M^2L^1T^{-2}]$

v. In vector form, moment of force can be expressed as $\vec{\tau} = \vec{r} \times \vec{F}$

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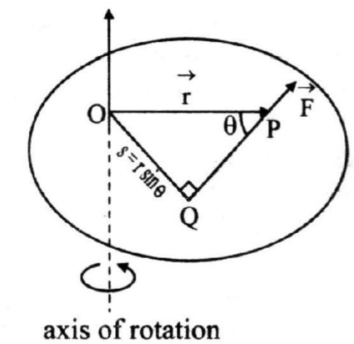
where $\vec{r}$ is the position vector of the point of application of force with respect to the axis.

Q.51. Prove that in rotational motion of a rigid body, $\vec{\tau}=\vec{r}\times\vec{F}$. Also discuss its direction of rotation.

Ans:



axis of rotation

i. Consider a rigid body rotating about an axis passing through a point 'O' and perpendicular to the plane of rotation.

ii. Let $\vec{F}$ be the force applied at point 'P' and 's' be the perpendicular distance between the axis of rotation and the line of action of the force.

iii. By definition, the magnitude of the moment of force (r) is given by,

$\tau = F.s$ ....(1)

iv. Let $\vec{r}$ be the position vector of the point of application of the force and 'θ' be the smaller angle between the direction of the force $\vec{F}$ and $\vec{r}$.

v. From ΔOPQ ,

$\sin\theta = \frac{s}{r}$

$\therefore$ $s = r\sin\theta$

vi. From equation (1),

$\tau = F.s = F.r\sin\theta = rF\sin\theta$

But, $rF\sin\theta = \vec{r}\times\vec{F}$

$\vec{\tau}=\vec{r}\times\vec{F}$ .... (2)

This shows that the moment of force (torque) is a vector directed perpendicular to the plane containing $\left(\vec{r}\times\vec{F}\right)$.

vii. The direction of $\vec{r}$ can be given by the right handed screw rule or right hand rule. The direction of τ is perpendicular to the plane of figure and in the outward direction.

4.7 : Couple and properties of couple :

Q.52. What do you mean by couple? State and explain its properties.

Ans: Two equal and parallel forces acting in opposite directions at two different points of a given body form a couple.

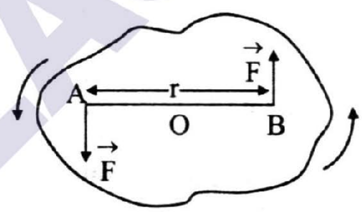
Example:

a. The forces applied to turn the steering wheel of a vehicle.

b. Forces are applied to open or close water tap and also for tightening or loosing the screw.

Explanation:

i. Let two forces of equal magnitude and opposite direction (F) act on a body separated by a perpendicular distance 'r' as shown in figure.



ii. The moment of couple or torque is measured in terms of product of magnitude of one of the forces and the perpendicular distance between them.

iii. Moment of couple = torque = one force × perpendicular distance between two forces.

$\tau = F.r$

iv. Moment of couple or torque is a vector quantity.

v. If $\vec{r}$ is the vector drawn from point of application of one force to that of other, then moment of couple or torque is expressed as a vector product of $\vec{r}$ and $\vec{F}$.

$\therefore$ $\vec{\tau}=\vec{r}\times\vec{F}$

Properties of a couple:

i. A couple produces purely rotational motion because there is no unbalanced force to produce translational motion in body.

ii. A couple can be balanced by equal and opposite couple. If anticlockwise couple acting on a body produce turning effect, its effect can be neutralised by applying clockwise couple of equal magnitude.

iii. A single force cannot neutralise the effect of a couple.

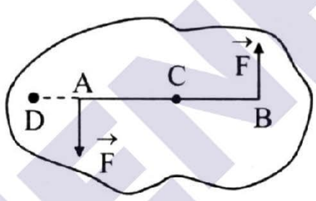
iv. The direction of moment of couple is the vector which is perpendicular to the plane containing two forces and is given by right handed screw rule or the right hand rule.

v. The moment of a couple or torque about any point in its plane is constant in magnitude and direction.

vi. The magnitude of moment of couple (torque) due to a couple about any point is always constant and equal to the product of magnitude of one of the forces and perpendicular distance between them.

Q.53. Show that moment of couple about any point in its plane is always constant and equal to the product of magnitude of one of the forces and perpendicular distance between them. OR Show that moment of a couple does not depend on the point about which you take the moments. (NCERT)

Ans: i. Let two forces each of magnitude 'F' act in opposite direction at the points A and B of a rigid body as shown in figure. AB is the perpendicular distance between them. Let 'C' be the point on line AB and point 'D' along line BA produced.



ii. Moment of couple about any point is the sum of moments of the forces forming the couple about that point.

iii. Using sign conventions, clockwise moment will be taken as negative and anticlockwise moment will be taken as positive.

∴ Moment of couple about the point A,

$= (F \times 0) + F(AB) = F \times AB$ ....(1)

iv. Moment of couple about the point C,

$= (F \times AC) + F(BC)$

$= F \times (AC + BC) = F \times AB$ ....(2)

v. Moment of couple about point D,

$= -F \times (AD) + F \times (BD)$

$= F(BD - AD)$

$= F \times AB$ ....(3)

vi. From equations (1), (2) and (3) moment of couple about any point in its plane is always constant and is equal to the product of magnitude of one of the forces and perpendicular distance between them.

vii. The moment of couple does not depend upon the position of the point about which moment is found.

4.8 : Centre of mass :

Q.54. Define centre of mass.

Ans: Centre of mass of a body is defined as that point at which the whole mass of the body is supposed to be concentrated, in order to study motion of the body in accordance with Newton's laws of motion.

Q.55. Explain the concept of centre of mass using suitable example.

Ans: i. To explain the concept of centre of mass consider a sphere rolling along a smooth horizontal surface.

ii. The sphere has both translational and rotational motion. Different particles of sphere move in translational as well as rotational motion.

iii. There is only one point i.e., centre of the sphere, which has only translational motion along straight line. The entire mass of the sphere can be considered to be situated at its centre.

iv. Any external force acting on the sphere will produce an acceleration of this point as per Newton's law. Thus, the centre of the sphere is its centre of mass.

v. Thus, using concept of centre of mass, we assume that:

a. A body is replaced by a "single particle" having mass equal to that of the body situated at the centre of mass of the body.

b. The motion of the centre of mass represents the translational motion of the body.

vi. The centre of mass of some bodies of uniform density and of regular (symmetric) shape are as follow:

a. Disc → centre

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b. Rod $\rightarrow$ mid point of the rod

c. Rectangular Lamina $\rightarrow$ point of intersection of its diagonals.

d. Triangular plate $\rightarrow$ Intersection of medians

Q.56. Give the location of the centre of mass of a (i) sphere, (ii) cylinder. (iii) ring, and (iv) cube, each of uniform mass density. Does the centre of mass of a body necessarily lie inside the body? (NCERT)

Ans:

No.	Object (with uniform mass density)	Location of centre of mass
i.	Sphere (hollow/ solid)	centre of the sphere
ii.	cylinder	centre of its axis of symmetry
iii.	ring	centre of ring
iv.	cube	geometrical centre of the cube

The centre of mass of a body does not necessarily lie inside the body. In case of a ring, hollow sphere or hollow cylinder etc, it lies at the centre, thus, outside the body.

Q.57. A child sits stationary at one end of a long trolley moving uniformly with a speed V on a smooth horizontal floor. If the child gets up and runs about on the trolley in any manner, what is the speed of the CM of the (trolley + child) system? (NCERT)

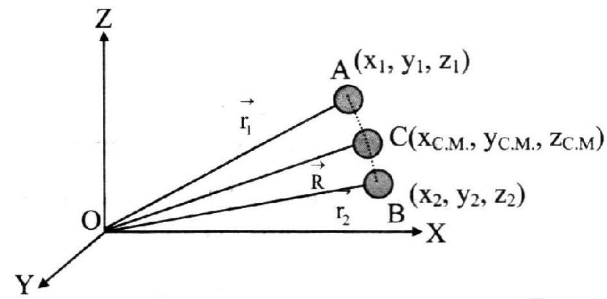
Ans: i. Centre of mass of a body is affected by external force only. Changes in internal forces of the body do not affect it.

ii. In the given situation trolley and the child forms the system.

iii. Hence irrespective of the movements of the child inside the trolley in any manner, speed of the C.M. of the system remains the same.

Q.58. Derive an expression for the position of centre of mass of a system of two particles. How can the treatment be extended for n masses?

Ans: Expression for position of centre of mass:



i. Consider two particles with masses m_1 and m_2 at point A and B respectively and separated by a short distance as shown in figure.

ii. Let $\vec{r}_1$, and $\vec{r}_2$ be the position vectors of two particles drawn from origin O.

iii. Let $\vec{R}$ be the position vector of centre of mass of two particles situated at point C.

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

iv. For 'n' particles of masses m., m2 ... m, having position vectors $\vec{r}_1, \vec{r}_2, ... \vec{r}_n$ respectively, the position vector of their centre of mass is given by,

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + ...m_n\vec{r}_n}{(m_1 + m_2 + m_3 + ...m_n)}$$

$$= \frac{\sum_{i=1}^{n} m_i\vec{r}_i}{\sum_{i=1}^{n} m_i} = \frac{\sum_{i=1}^{n} m_i\vec{r}_i}{M}$$

Where $M = m_1 + m_2 + ... + m_n$ = Total mass of the system.

v. Let $(x_1, y_1\ z_1), (x_2, y_2, z_2) ... (x_n, y_n, z_n)$ be the co–ordinates of the 'n' particles and $(x_{C.M.} y_{CM}., z_{C.M})$ be the co–ordinates of the centre of mass of the system then,

$$x_{C.M} = \frac{\sum_{i=1}^{n} m_i x_i}{M},$$

$$y_{C.M.} = \frac{\sum_{i=1}^{n} m_i y_i}{M}$$

$$z_{CM.} = \frac{\sum_{i=1}^{n} m_i z_i}{M}$$

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4.9 : Centre of gravity :

Q.59. Define centre of gravity of a body. Under what conditions the centre of gravity and centre of mass coincide?

Ans: i. Centre of gravity is defined as a fixed point through which the whole weight of the body always acts vertically downwards, whatever may be the position of the body.

ii. The centre of mass coincides with centre of gravity when the body is in a uniform gravitational field.

iii. If size of body is small as compared to the size of the earth, the acceleration due to gravity g_1, g_2 .... g_n is equal to acceleration due gravity 'g'. In this case, centre of mass of the body coincides with the centre of gravity.

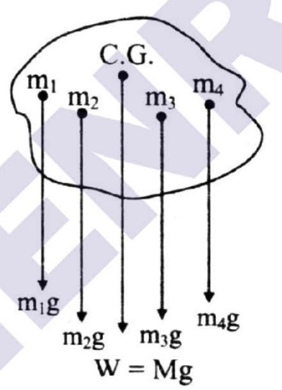
iv. For bodies of small size, centre of mass and centre or gravity coincides.

Q.60. Explain the concept of centre of gravity.

Ans: i. Consider a body of mass 'M' composed of large number of particles of masses m_1, m_3 .... m_n.

ii. Each particle is separately acted upon by gravitational force of attraction directed towards the centre of the earth.

iii. Since all these points are at different distances from the centre of the earth they experiences different gravitational force.



The resultant of these like parallel forces gives the total weight 'W' ofthe body.

$W = m_1g + m_2g + ... + m_ng = Mg$.

iv. Where g is the acceleration due to gravity at a point where a point object with mass M will produce the same gravitational force as experienced by this extended object. This point is the centre of gravity (C.G) of the extended object.

Q.61. Explain the method to determine centre of gravity of irregular bodies.

Ans: Method to determine centre of gravity:

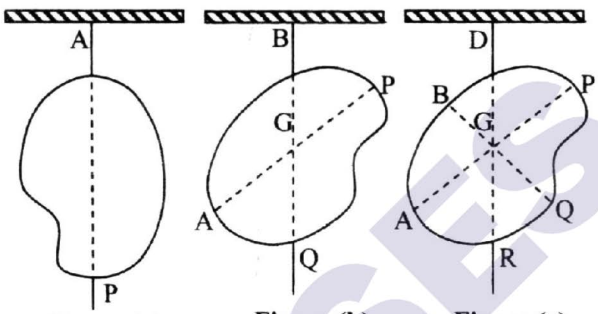


Figure (a) Figure (b) Figure (c)

i. First suspend the body from a point A on its surface. When the body is at rest, the centre of gravity must lie vertically below the point of support, somewhere on the line AP as shown in figure (a).

ii. Suspend the body from another point B on its surface as shown in figure (b). Again the centre of gravity must lie somewhere on the line BQ.

iii. Suspend the body from another point D on its surface as shown in figure (c). Again the centre of gravity must lie some where on line DR.

iv. The common point to the lines AP, BQ and DR is G. Therefore, G is the centre of gravity of the body i.e., the intersection of the vertical lines through the point of suspension is the centre of gravity ofthe body.

Q.62. Explain with an example, for very large bodies, the centre of gravity does not coincide with the centre of mass.

Ans: i. The magnitude of $\vec{g}$ changes with distance from the centre of the earth. Hence, in case of very large bodies, such as a mountain range, acceleration due to gravity is not the same for all particles.

ii. In fact, C.G. is greater for particles near the centre of the earth.

iii. In such cases, the centre of gravity and the centre of mass do not coincide.

iv. For example, suppose that a long uniform rod AB is extending vertically upwards for several hundred kilometres from the earth's

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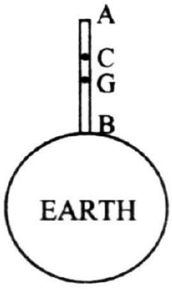
surface. The centre of mass of the rod will be at its geometrical centre G. A



v. However, the average magnitude of $\vec{g}$ for the upper half of the rod AC will be less than the average magnitude of $\vec{g}$ for lower half of the rod CB.

vi. In other words, the lower half of the rod will be heavier than the upper half. Hence, the centre of gravity G of the rod will lie in the lower half portion below the centre of mass C.

Q.63. In the ship or boat, heavy load is kept at the bottom. Explain why.

Ans: i. A ship or a boat floats on water. The equilibrium position is obtained when the centre of gravity G of the floating body and the centre of buoyancy B lie in the same vertical line. (figure a).

ii. In the tilted position, let the vertical line through the shifted centre of buoyancy cut the centre line of the ship at a point P.

iii. For the equilibrium to be stable, P must lie above G. (figure b).

iv. If P lies below G, the equilibrium is unstable, (figure c).

v. The centre of gravity G of the floating body is thus kept as low as possible by keeping the heavy load at the bottom of a ship or boat to increase its stability and protect it from tumbling.

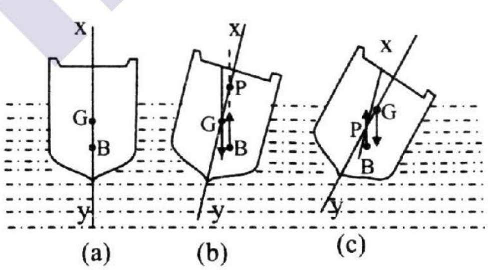


Figure (a) and (b) stable equilibrium (c) unstable equilibrium of a floating body

Q.64. State and explain the conditions of equilibrium of a body under the action of a number of coplanar forces.

Ans: i. A rigid body is said to be in equilibrium position, if under the application of forces, it remains in the state of rest or in uniform motion.

ii. Rigid bodies have two types of motion, translational and rotational. Hence, there are the two conditions of equilibrium of a rigid body.

a. Translational equilibrium:

i. A rigid body is said to be in translational equilibrium if the resultant force acting on the body is zero.

ii. For the translational equilibrium of a rigid body, the vector sum of all the forces acting on a rigid body is equal to zero. This is the first condition of equilibrium.

iii. Let F_1 F_2, F_3 ...., F_n be the forces acting on a rigid body then according to the condition of translational equilibrium,

$F_1 + F_2 + ... + F_n = 0$

$\Sigma \ \vec{F} = 0$

iv. If the forces are not in the same plane and are acting in X–Y–Z plane then each force can be resolved into three mutually perpendicular components parallel to X, Y and Z axes. If the vector sum of forces is zero, the vector sum of their components along each axis must be equal to zero.

$\therefore \quad \Sigma\vec{F}_x = 0, \Sigma\vec{F}_y = 0$ and $\Sigma\vec{F}_z = 0$

b. Rotational equilibrium:

i. A body is said to be in rotational equilibrium, if the resultant torque acting on the body is zero.

ii. If $\vec{\tau}_1$, $\vec{\tau}_2$, $\vec{\tau}_3$ are the torque acting on body, then for rotational equilibrium,

$\vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 + ... = 0$

i.e. $\Sigma\vec{\tau} = 0$

i.e. $\Sigma\vec{r} \times \vec{F} = 0$

iii. For coplanar forces, the condition for rotational equilibrium is that, the vector sum of moments of all forces (torque) about any point must be zero.

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iv. Condition of complete equilibrium of rigid body under the action of coplanar forces are,

$$\Sigma \vec{F}_x = 0$$

$$\Sigma \vec{F}_y = 0$$

$$\Sigma \vec{r} \times \vec{F} = 0$$

$$\Sigma \vec{\tau} = 0$$

Q.65. State the conditions of equilibrium of a body under the action of a number of coplanar forces.

Ans: i. A rigid body is said to be in equilibrium position, if under the application of forces, it remains in the state of rest or in uniform motion.

ii. Rigid bodies have two types of motion, translational and rotational. Hence, there are the two conditions of equilibrium of a rigid body.

iii. A rigid body is said to be in translational equilibrium if the resultant force acting on the body is zero.

iv. A body is said to be in rotational equilibrium, if the resultant torque acting on the body is zero.

Formulae :

1. **Gravitational force between two bodies:**

$$F = \frac{Gm_1m_2}{r^2}$$

2. **Force due to inertia:** $\vec{F} = m\vec{a}$

3. **Force in terms of momentum:** $\vec{F} = \frac{\vec{P}_2 - \vec{P}_1}{t}$

4. **Impulse:** $\vec{J} = \vec{F}t = m(\vec{v} - \vec{u})$

5. **Linear momentum:** $\vec{P} = m\vec{v}$

6. **Work done by variable force:**

$$W = \int_b^a \vec{F}.\vec{ds} = \int_b^a F ds \cos\theta$$

7. **Laws of conservation of linear momentum:**

i. $\sum_{i=1}^{n} mv = \text{constant}$

ii. $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

8. **Recoil velocity:** $v_2 = -\frac{m_1v_1}{m_2}$

9. **Velocity of connected body:**

$$v = \frac{m_1v_1 + m_2v_2}{(m_1 + m_2)}$$

10. **Coefficient of restitution:** $e = \frac{v_2 - v_1}{u_1 - u_2}$

11. **Velocities of colliding bodies after elastic collision:**

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right).u_1 + \left(\frac{2m_2}{m_1 + m_2}\right).u_2$$

$$v_2 = \left(\frac{2m_2}{m_1 + m_2}\right).u_1 + \left(\frac{m_1 - m_2}{m_1 + m_2}\right).u_2$$

12. **Magnitude of moment of force (Torque):**

$\tau = F r \sin\theta$

13. **Position vector of centre of mass:**

i. For two particle system,

$$R_{C.M.} = \frac{m_1r_1 + m_2r_2}{m_1 + m_2}$$

ii. For n particles system,

$$R_{C.M.} = \frac{1}{n}\sum_{i=1}^{n} m_i r_i$$

14. **Cartesian co–ordinate of centre of mass of two particles system:**

$$x = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}, \qquad y = \frac{m_1y_1 + m_2y_2}{m_1 + m_2}$$

$$z = \frac{m_1z_1 + m_2z_2}{m_1 + m_2}$$

15. **Equilibrium of rigid body:**

i. for translational equilibrium

$\sum F = 0$ i.e.

$F_1 + F_2 + F_3 + ... = 0$

ii. For rotational equilibrium

$\sum \tau = 0$ i.e.,

$\tau_1 + \tau_2 + \tau_3 + ... = 0$

Solved Examples :

Type I : Problems based on force and impulse

Example 1

If a constant force of 800 N produces an acceleration of 5 m/s² in a body, what is its mass? If the body starts from rest, how much

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distance will it travel in 10 s?

Solution:

Given : $F = 800$ N, $a = 5$ m/s^2, $u = 0$ $t = 10$ s

To find : mass (m), distance travelled (s)

Formulae : i. $F = ma$

ii. $s = ut + \frac{1}{2}at^2$

Calculation : From formula (i),

$\therefore \quad m = \frac{F}{a} = \frac{800}{5} = \mathbf{160\ kg}$

From formula (ii),

$s = \frac{1}{2} \times 5 \times (10)^2 \qquad [\because u = 0]$

$\therefore \quad s = \mathbf{250\ m}$

Ans: Mass of the body is **160 kg** and the distance travelled by the body is **250 m**.

Example 2

Two billiard balls each of mass 0.05 kg moving in opposite directions with speed 6 m s^{-1} collide and rebound with the same speed. What is the impulse imparted to each ball due to the other?

(NCERT)

Solution:

Given : $m = 0.05$ kg, $u = 6$ m/s, $v = –6$ m/s

To find : Impluse (J)

Formula : $J = m(v – u)$

Calculation : From formula,

$J = 0.05\ (–6 –6)$

$= \mathbf{–0.6\ kg\ m\ s^{-1}}$

Ans: Impulse received by each ball is **–6.6 kg m s^{-1}**.

Example 3

A bullet of mass 0.1 kg moving horizontally with a velocity of 20 m/s strikes a target and brought to rest in 0.1 s. Find the impulse and average force of impact.

Solution:

Given : $m = 0.1$ kg, $u = 20$ m/s, $t = 0.1$ s

To find : Impluse (J), Average force (F)

Formulae :i. $J = mv – mu$

ii. $F = m\frac{(v-u)}{t}$

Calculation : From formula (i),

$J = m\ (v – u) = 0.1\ (0 – 20) = \mathbf{–2\ s}$

From formula (ii),

$F = \frac{(v-u)}{t}$

$= \frac{2}{0.1} = \mathbf{20\ N}$

Ans: Magnitude of impulse is **2 Ns**, average force of impact is **20 N**.

Example 4

A batsman deflects a ball by an angle of 45° without changing its initial speed which is equal to 54 km/h. What is the impulse imparted to the ball?

(Mass of the ball is 0.15 kg.) (NCERT)

Solution:

Let the point B represents the position of bat. The ball strikes the bat with velocity v along the path AB and gets deflected with same velocity along BC, such that ∠ABC = 45°

Initial momentum of the ball = $mv \cos\left(\frac{\theta}{2}\right)$ along NB

Final momentum of the ball = $mv \cos\left(\frac{\theta}{2}\right)$ along BN

Hence, Impulse = change in momentum

$= mv \cos\left(\frac{\theta}{2}\right) - [\,- mv \cos\left(\frac{\theta}{2}\right)\,]$

$= 2mv \cos\left(\frac{\theta}{2}\right) \qquad (\because v = 54\ km/hr = 15\ m/s)$

$= 2 \times 0.15 \times 15 \times \cos(22.5°)$

$= 4.157$ kg m s^{-1}

Thus, impulse imparted to the ball is **4.157 kg m s^{-1}**.

Example 5

A cricket ball of mass 150 g moving with a velocity of 12 m/s is turned back with a velocity of 20 m/s on hitting the bat. The force of the ball lasts for 0.01 s, Find the average force exerted on the ball by the bat.

Solution:

Given : $m = 0.150$ kg, $v = 20$ m/s, $u = –12$ m/s and $t = 0.01$ s

To find : Average force (F)

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Formula : $F = m\frac{(v-u)}{t}$

Calculation : From formula,

$$F = \frac{0.150[20-(-12)]}{0.01} = \mathbf{480\ N}$$

Ans: The average force exerted on the ball by the bat is **480 N**.

Example 6

A 20 g bullet leaves a machine gun with a velocity of 200 m/s. If the mass of the gun is 20 kg, find its recoil velocity. If the gun fires 20 bullets per second, what force is to be applied to the gun to prevent recoil?

Solution:

Given : $m_1 = 20\ g = 0.02\ kg$, $m_2 = 20\ kg$, $v_1 = 200\ m/s$, $t = \frac{1}{20}\ s$,

To find : Recoil velocity (v_2), applied force (F)

Formulae : i. $v_2 = -\frac{m_1 v_1}{m_2}$ ii. $F = ma$

Calculation : From formula (i),

$$\therefore\ v_2 = -\frac{0.02}{20} \times 200 = \mathbf{-0.2\ m/s}$$

Negative sign shows that the machine gun moves in a direction opposite to that of the bullet.

$$a = \frac{v-u}{t} = \frac{0.2-0}{\frac{1}{20}} = 4\ ms^{-2}$$

From formula (ii),

$\therefore$ $F = m_2 \times a = 20 \times 4 = \mathbf{80\ N}$

Ans: The recoil velocity of gun is **0.2 m/s** and the required force to prevent recoil is **80 N**.

Example 7

A constant force acting on a body of mass 3 kg changes its speed from 2 m s⁻¹ to 3.5 m/s in 25 s, The direction of motion of the body remains unchanged. What is the magnitude and direction of the force? (NCERT)

Solution:

Given : $u = 2\ m\ s^{-1}$, $m = 3\ kg$, $v = 3.5\ m\ s^{-1}$, $t = 25\ s$

To find : Force (F)

Formula : $F = ma$

Calculation : Since, $v = u + at$

$\therefore$ $3.5 = 2 + a \times 25$

$$a = \frac{3.5-2}{25} = \mathbf{0.06\ m\ s^{-2}}$$

From formula,

$F = 3 \times 0.06 = \mathbf{0.18\ N}$

Since, the applied force increases the speed of the body, it acts in the direction of the motion.

Ans: The applied force is **0.18 N** along the direction of motion.

Example 8

A constant retarding force of 50 N is applied to a body of mass 20 kg moving initially with a speed of 15 m s⁻¹. How long does the body take to stop? (NCERT)

Solution:

Given : $m = 20\ kg$, $u = 15\ m\ s^{-1}$ $v = 0$, $F = -50\ N$ (retarding force)

To find : Time (t)

Formula : $v = u + at$

Calculation : Since, $F = ma$

$$\therefore\ a = \frac{F}{m} = \frac{-50}{20} = -2.5\ m\ s^{-2}$$

From formula,

$0 = 15 + (-2.5) \times t$

$\therefore$ $t = 6\ s$

Ans: Time taken to stop the body is **6 s**.

Example 9

A body constrained to move along the z–axis of a coordinate system is subject to a constant force F given by F = $(-\hat{i}+2\hat{j}+3\hat{k})$ N where $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along the x, y and z axis of the system respectively. What is the work done by this force in moving the body a distance of 4 m along the z–axis? (NCERT)

Solution:

Given : $F = (-\hat{i}+2\hat{j}+3\hat{k})$ N, $\hat{s} = 4\hat{k}$

To find : work done (W)

Formula : $W = \vec{F}.\vec{s}$

Calculation : From formula,

$$W = (-\hat{i}+2\hat{j}+3\hat{k}).\left(4\hat{k}\right)$$

$$= 12\hat{k}.\hat{k} = 12\ J$$

Ans: The work done by the force is in moving the body **12 J**.

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Type II : Problems based on work and energy

Example 10

A liquid drop of 1.00 g falls from height of cliff 1.00 km. It hits the ground with a speed of 50 m s⁻¹. What is the work done by the unknown force?

Solution:

Given : $m = 1.0\ g = 1.0 \times 10^{-3}\ kg$, $h = 1\ km = 10^3\ m$, $v = 50\ ms^{-1}$

To find : Work done (W_f)

Formula : $W_f = \Delta K.E - W_g$

Calculation :

i. The change in kinetic energy of the drop

$$\Delta K.E = (K.E.)_{final} - (K.E.)_{initial}$$

$$\Delta K.E = \frac{1}{2} mv^2 - 0$$

$$= \frac{1}{2} \times 1.0 \times 10^{-3} \times (50)^2$$

$$\Delta K.E = 1.25\ J$$

ii. Work done by the gravitational force is W_g

$W_g = mgh = 1.0 \times 10^{-3} \times 9.8 \times 10^3 = 9.8\ J$

$\therefore$ $W_g = 9.8\ J$

From formula,

$$W_f = \Delta K.E - W_g$$
$$= 1.25 - 9.8$$
$$W_f = \mathbf{-8.55\ J}$$

Ans: Work done by the unknown force is **– 8.55 J**.

Example 11

A body of mass 0.5 kg travels in a straight line with velocity $v = ax^{3/2}$, where $a = 5\ m^{-1/2}\ s^{-2}$. What is the work done by the net force during its displacement from x = 0 to x = 2 m? (NCERT)

Solution:

Given : $M = 0.5\ kg$, $v = ax^{3/2}$, where $a = 5\ m^{-1/2}\ s^{-1}$

Let v_1 and v_2 be the velocities of the body, when x = 0 and x = 2 m respectively. Then,

$v_1 = 5 \times 0^{3/2} = 0$, $v_2 = 5 \times 2^{3/2} = 10\sqrt{2}$ m

To find : Work done (W)

Formula : Work done = Increase in kinetic energy

$$W = \frac{1}{2} M (v_2^2 - v_1^2)$$

Calculation :

From formula,

$$W = \frac{1}{2} \times 0.5 \times \left[(10\sqrt{2})^2 - 0^2\right]$$

$\therefore$ $W = \mathbf{50\ J}$

Ans: Work done by the net force on the body is **50 J**.

Example 12

A particle of mass 12 kg is acted upon by a force $f = (100 - 2x^2)$where f is in newton and 'x' is in metre. Calculate the work done by this force in moving the particle x = 0 to x = –10m. What will be the speed at x = 10 m if it starts from rest?

Solution:

Given : $F = 100 - 2x^2$

at A, x = 0 and at B, x = –10 m

To find : Work done (W), speed (v)

Formulae : i. $W = \int_A^B \vec{F}.ds$ ii. $v = \sqrt{\frac{2K.E}{m}}$

Calculation : From formula (i),

$$W = \int_A^B \vec{F}.\overline{ds} = \int_{x=0}^{x=-10} F dx$$

$$\therefore W = \int_{x=0}^{x=-10} (100 - 2x^2)dx$$

$$= \int_{x=0}^{x=-10} 100\ dx - \int_{x=0}^{x=-10} 2x^2\ dx$$

$$= [100x]_0^{-10} - \left[\frac{2x^3}{3}\right]_0^{-10}$$

$$= -1000 - \frac{(2)(-1000)}{3}$$

$$= -1000 + \frac{2000}{3}$$

$$W = \frac{-1000}{3}$$ Neglecting the negative sign,

$\therefore$ $W = \mathbf{333.3\ J}$

The work done by the variable force is converted into K.E. with speed v.

$\therefore$ W = K.E. = 333 J

From formula (ii),

$$v = \sqrt{\frac{2 \times 333.3}{12}}$$

$\therefore$ v = **7.45 m/s**

Ans: i. Work done by the force on the particle is **333.3 J**.

ii. The speed of the particle at x = 10 will be **7.45 m/s.**

Example 13

A position dependent force f = 7 – 2x + 3x² newton acts on a small body of mass 2 kg and displaces from x = 0 to x = 5, calculate the work done.

Solution:

Given : $F = 7 - 2x + 3x^2$, x = 0 at A and x = 5 at B.

To find : Work done (W)

Formula : $W = \int_A^B \vec{F}.\overline{ds}$

Calculation : From formula,

$$W = \int_{x=0}^{x=5} (7 - 2x + 3x^2)dx$$

$$= \int_{x=0}^{x=5} 7\,dx - \int_{x=0}^{x=5} 2x\,dx + \int_{x=0}^{x=5} 3x^2\,dx$$

$$= [7x]_0^5 - \left[\frac{2x^2}{2}\right]_0^5 + \left[\frac{3x^3}{3}\right]_0^5$$

$$= 35 - 25 + 125$$

$\therefore$ W = **135 J**

Ans: The work done is **135 J**.

Example 32

Three masses 3 kg, 4 kg and 5 kg are located at the corners of an equilateral triangle of side 1 m. Locate the centre of mass of the system.

Solution:

Suppose the equilateral triangle lies in the XY – plane with mass 3 kg at the origin. Let (x, y) be the co–ordinates of C.M.

From the figure,

$$AB\ \sqrt{OB^2 - OA^2} = \sqrt{1^2 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2}\text{m}$$

$x_1 = 0$, $x_2 = 1$ m, $x_3 = OA = 0.5$ m

$m_1 = 3$ kg, $m_2 = 4$ kg, m, = 5 kg

$$\therefore \quad x = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$= \frac{3\times0 + 4\times1 + 5\times0.5}{3+4+5}$$

$$= \frac{6.5}{12} = \mathbf{0.54\ m}$$

Again, $y_1 = 0$, $y_2 = 0$, $y_3 = OB = \frac{\sqrt{3}}{2}$

$$\therefore \quad y = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

$$= \frac{3\times0 + 4\times0 + 5\times(\sqrt{3}/2)}{3+4+5}$$

$$= \frac{5\times\sqrt{3}}{2\times12} = \mathbf{0.36\ m}$$

Ans: The center of mass of the system of three particles lies at **(0.54m, 0.36m)** with respect to the particle of mass 3kg as the origin.

Example 33

A ball whose mass is 100 g is dropped from a height of 2 m to the floor. It rebounds vertically upwards after colliding with the floor to a height 1.5 m. Find

i. the momentum of the ball before and after colliding with the floor,

ii. The average force exerted by the floor on the ball, Assume that collision lasts for 10^{-8} s.

Solution:

Given : m = 100g = 0.1 kg, h = 2m, $h_1 = 1.5$ m, $t = 10^{-8}$ s

To find : i. Momentum before (P_1) and after (P_2) collision

ii. Average force (F)

Calculation :

Formulae : i. mgb = mv^2

ii. P = mv

iii. $|F| = \frac{|P_2 - P_1|}{t}$

i. While falling K.E. of ball increases and P.E. decreases.

Before collision, decrease in P.E. = Increase in K.E.

$$\therefore \quad m \times 9.8 \times 2 = \frac{1}{2}\ m \times v^2$$

$$\therefore \quad v = \sqrt{2\times9.8\times2}$$

$$= 2 \times 3.13 = 6.26\ \text{m/s}$$

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Momentum before colliding with floor

$P_1 = mv = 0.1 \times 6.26$

$P_1 = +0.626$ kg m/s

During rebounds

Decrease in K.E. = Increase in P.E.

$\frac{1}{2} m v_1^2 = mgh_1$

$v_1^2 = 2gh_1$

$\therefore \quad v_1 = \sqrt{2gh_1} = \sqrt{2 \times 9.8 \times 1.5} = 5.42$ m/s

Since direction of v_1 is opposite to v

$\therefore \quad v_1 = -5.42$ m/s

Momentum after collision, $P_2 = mv_1$

$\therefore \quad P_2 = 0.1 \times (-5.42)$

$\therefore \quad P_2 = \mathbf{-0.542}$ **kg m/s**

ii. Magnitude of average force $|F| = \frac{|P_2 - P_1|}{t}$

$= \frac{|-0.542 - 0.626|}{10^{-8}} = \frac{|-1.168|}{10^{-8}}$

$F = \mathbf{1.168 \times 10^8}$ **N**

Ans: i. Momentum before and after collision is **0.626 kg m/s** and **–0.542 kg m/s** respectively.

ii. Average force exerted by the floor on the ball is **1.168 × 10⁸ N**.

Example 34

A bullet of mass 10 gram strikes a target with a velocity of 100 m s⁻¹ and comes to rest after piercing 5 cm into it. Calculate the average force of resistance offered by the target.

Solution:

Given : $s = 5$ cm $= 0.05$ m, $v = 0$,
$u = 100$ ms^{-1},
$M = 10$g $= 0.01$ kg

To find : Average force of resistance (F)

Calculation :

Let F be the average force of resistance offered by the target.

From work-energy theorem,

$-Fs = \frac{1}{2}Mv^2 - \frac{1}{2}Mu^2$

$\therefore \quad -F \times 0.05 = \frac{1}{2} \times 0.01 \times [(0)^2 - (100)^2]$

$= \frac{1}{2}(0.01)(100)^2$

$\therefore \quad F \times 0.05 = \frac{1}{2}(0.01)(100)^2$

$\therefore \quad F = \mathbf{1000}$ **N**

Ans: The average force of resistance offered by the target is **1000 N**.

Example 35

Two masses m_1 and m_2 are connected to the two ends of massless string passing over a frictionless massless pulley and m_1 is placed on an inclined plane. Assume the friction between the mass m_2 and the plane is zero. Find the acceleration of the masses and the. tension in the string if m_1 = 20.5 kg, m_2 = 10.5 kg and θ = 45°.

Solution:

Given : $\theta = 45°$, $m_1 = 20.5$ kg, $m_2 = 10.5$ kg

To find : acceleration (a), tension (T)

Formulae : For mass m_1,

$m_1 g - T = m_1 a$ .... (i)

Where 'a' is an acceleration acting in downward direction as shown in figure. (assuming system is moving from inclined plane from m_1 to m_2 over the pulley)

For mass m_2,

$T - m_2 g \sin\theta = m_2 a$ ...(ii)

Calculation :

Adding equations (i) and (ii) we have,

$m_1 g - T - m_2 g \sin\theta + T = m_1 a + m_2 a$

$\therefore \quad m_1 g - m_2 g \sin\theta = (m_1 + m_2)a$

$\therefore \quad g(m_1 - m_2 \sin\theta) = (m_1 + m_2)a$

$$\therefore \quad a = \frac{g(m_1 - m_2 \sin\theta)}{m_1 + m_2}$$

$$= \frac{9.8(20.5 - 10.5\sin 45°)}{20.5 + 10.5}$$

$$= \frac{9.8\left(20.5 - 10.5 \times \frac{1}{\sqrt{2}}\right)}{31}$$

$a = \mathbf{4.13}$ **m/s²**

Positive sign shows that due to acceleration m_1 block is moving in downward direction on inclined plane.

Substitute value of 'a' in equation (i)

$T - 20.5 \times 9.8 = (20.5)(-4.13)$

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$T = 20.5 \times 9.8 - 20.5 \times 4.13$

$T = \mathbf{116.2\ N}$

Ans: The acceleration of the masses is **4.13 m/s²** and the tension in the string is **116.2 N**.

Example 36

A bullet of mass 0.012 kg and horizontal speed 70 m s⁻¹ strikes a block of wood of mass 0.4 kg and instantly comes to rest with respect to the block. The block is suspended from the ceiling by means of thin wires. Calculate the height to which the block rises and also estimate the amount of heat produced in the block.

Solution:

Given : $m = 0.012$ kg, $v = 70\ ms^{-1}$, $M = 0.4$ kg

To find : i. Height (h)

ii. Heat produced (E)

Formula : i. $E = \frac{1}{2} mv^2$

ii. $v_1 = \frac{mv + Mu}{(m + M)}$

iii. $v^2 = u^2 + 2as$

Calculation :

i. K.E. is possessed by the bullet. It is converted completely into heat energy E. The block is at rest on the ground so it possesses zero energy.

$\therefore E = \frac{1}{2} mv^2 = \frac{1}{2} \times 0.012 \times (70)^2$

$E = \mathbf{29.4\ J}$

ii. The bullet strikes the block of mass M and suddenly stops. The block is initially at rest

$\therefore u = 0.$

v_1 is final velocity of block.

According to law of conservation of linear momentum, The bullet is embedded in the block

Final linear momentum = Initial linear momentum

$\therefore (m - M)\, v_1 = mv + Mu$

$\therefore v_1 = \frac{mv + Mu}{(m + M)}$

$= \frac{0.012 \times 70 + 0.4 \times 0}{(0.012 + 0.4)}$

$= \frac{0.012 \times 70}{0.412}$

$\therefore v_1 = \mathbf{2.04\ m/s}$

iii. The block possesses initial velocity of v_1 and due to which it rises to height h till its final velocity becomes zero.

$\therefore u = v_1 = 2.04$ m/s, $v = 0$, $a = -9.8$ m/s², $s = h$

Using formula (iii),

$v^2 = v_1^2 + 2(-9.8)h$

$\therefore 0 = (2.04)^2 - 2 \times 9.8\ h$

$\therefore 2 \times 9.8\ h = (2.04)^2$

$\therefore h = \frac{(2.04)^2}{2 \times 9.8} = 0.2123$ m

$\therefore h = \mathbf{21.23\ cm}$

Ans: i. The height to which the block rises is nearly **21.23 cm**.

ii. The amount of heat produced in the block is **29.4 J**.

Additional Theory Questions :

Q.1. Distinguish between real force and pseudo force.

Ans: Refer to Q.11.

Q.2. Explain the conditions of translational equilibrium of a body under the action of a number of coplanar forces

Ans: Refer Q.64. (a).

Q.3. Explain the conditions of rotational equilibrium of a body under the action of a number of coplanar forces.

Ans: Refer Q.64. (b).

Practice Problems :

Type I: Problems based on force and impulse

1. A constant force acts on a body of mass 100 g for 4 s. The body starting from rest covers a distance of 0.64 m during this time. Find the force.

2. An engine of mass 60 metric tons has a velocity of 72 km/h. What force will stop it within a distance of 100 m? What force will stop it in 20 s?

3. What constant force is needed to increase the momentum of a body from 2300 kg mls to 3000 kg mls in 50 s?

4. A bullet leaves the rifle of mass one kg and the

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rifle recoils thereby with a velocity of 30 cm/s. If the mass of bullet is 3 g find the velocity of the bullet.

5. A machine gun fires 180 bullets per minute on a steel plate. Each bullet has a mass of 20 g and velocity 1 km/s. The bullets return with half their velocity after hitting the plate. Find the force necessary to hold the plate so as not to move it.

6. While launching a rocket of mass 2×10^4 kg, a force of 5×10^5 N is applied for 20 s. Calculate the velocity attained by the rocket at the end of 20 s.

7. A bullet weighing 10 g is fired with a velocity of 800 m s^{-1}. After passing through mud wall 1 m thick, its velocity decreases to 100 m s^{-1}. Find the average force of resistance offered by the mud wall?

Type II: Problems based on work and energy

8. A bullet of mass 25 g was moving at the rate of 500 m s^{-1}. After passing through a solid substance, it continued to move at the rate of 100 m s^{-1}. How much work the bullet had to do in passing through the substance?

Type III : Problems based on head-on collision

9. A body of mass 1 kg and velocity 10 m/s collides with another body of mass 3 kg and velocity 2 mls corning from the exactly opposite direction. If on collision the bodies merge into one, find the magnitude and direction of their common velocity.

10. A body of mass 100 kg moving with a velocity of 50 mls strikes another body of mass 200 kg which is at rest. If the two bodies after collision stick together, then calculate the velocity of the composite body.

11. A sphere of mass 2 kg travelling with a velocity of 22 mls makes a head – on collision with a sphere of mass 4 kg travelling with a velocity of 10 m/s in the same direction. If the coefficient of restitution is 0.8. find the velocities of the two spheres after collision.

Type IV : Problems based on moment of force (Torque)

12. A uniform rod balances about a point 2 m from one end when a weight of 0.18 kg is suspended from that end. It also balances about a point 3 m from the same end when a weight of 0.08 kg is suspended from the same end. Find the length and weight of the rod.

13. A uniform rod of length 6 m and weight 200 gwt. is suspended horizontally by two cords of equal lengths tied to it at distances 1.6 n and 2.4 m respectively from the mid–point. Find the tension in each cord.

14. A uniform rod AB is resting on two supports which are at distances 3 m and 4 m from the ends A and B respectively. Masses of 10 kg and 8 kg are suspended from the ends A and B respectively. The weight of the rod is 45 kg and its length is 12m . Find the reaction at the supports.

15. A uniform ladder 5 m Long and weighing 20 kg leans against a vertical frictionless wall with its lower end 4 m away from the foot of the wall. Find the magnitudes of the forces exerted on the ladder at each of the ends.

Type V : Problems based on Centre of mass and Centre of gravity

16. Three–equal masses each of lkg are placed on vertices of an equilateral triangle whose each side is 1 m long. Find the position of the centre of mass.

17. Locate the centre of mass of a system of two uniform spherical masses of 5 kg and 35 kg, kept with their centres 70 cm apart.

Type VI : Miscellaneous

18. A ball of mass 0.2 kg moving with' a–velocity of 2 m/s on a plane surface, collides with another ball of the same mass and initially at rest. If the coefficient of restitution between the two balls is 0.6, find the velocities of the rru two balls after collision

19. A constant retarding force of 50 N is applied to a body of mass 20 kg moving initially with a speed of 15 m s^{-1}. How long does the body take to stop?

20. A 30 g bullet leaves a rifle with a velocity of 300 m s^{-1} and the rifle recoils with a velocity of 0.60 m s^{-1}. Find the mass of the rifle.

21. An elevator weighs 4000 kg. When the upward tension in the supporting cable is 48000 N, what is the upward acceleration? Starting from rest, how far does it rise in 3 s?

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22. Point masses of 1 kg; 2 kg, :3 kg and 4 kg are placed at the corners A, B, C and D respectively of a square whose each side is 1 metre long. Find the position of the centre of mass of the system.

23. A constant force acts on a body for 3 seconds and then ceases to act. During next 5 seconds the body describes 75 m . If the mass of the body is 8 kg, find the magnitude of the force.

24. Four particles of masses 2 kg, 3 kg, 4 kg and 5 kg are placed at the corners of a square of length 2 m of each side. Find the co–ordinates and position vector of its centre of mass.

25. Two masses 7 kg and 12 kg are connected at the two ends of a light inextensible string that passes over a frictionless pulley. Find the acceleration of the masses and the tension in the string, when the masses are released.

26. A block of.mass 0.8 kg, is dragged along a level surface at constant velooity by a 'banging block of mass 0.2 kg. Calculate the tension in the string and the acceleration of the system.

27. A man weighs 70 kg, He stands on a weighing machine in a lift, which is moving

a. upwards with uniform speed of 10 m s^{-1}

b. downwards with a uniform acceleration of 5 m s^{-2} C. upwards with uniform acceleration of 5 m s^{-2}.

What is the reading on the scale in each case?

28. A bolt of mass 0.3 kg falls from the ceiling of an elevator moving down with a uniform speed of 7 m s^{-1}. It hits the floor of the elevator which is 3 m below and' does not rebound. How much is the heat energy produced?

29. A force $\vec{F} = (2\hat{i} + 3\hat{j})$ N acts on a particle, whose position with respect .to the origin 'O' of an inertial reference frame is given by,

$\vec{r} = (3\hat{i} + 2\hat{j})$ m. Determine the torque acting on the particle.

30. Two weights 4 kg and 6 kg are suspended from a uniform rod AB of length 10m and weighing 2 kg. The distances of the weights from one end are 2.5 m and 7.5 m. Find the point at which the rod balances.

Multipal Choice Questions

1. A body of mass 2 kg moving on a horizontal surface with initial velocity of 4 m s^{-1} comes to rest after two seconds. If one wants to keep this body moving on the same surface with a velocity of 4 m s^{-1}, the force required is

a) 2N b) 4 N
c) 0 d) 8N

2. What force will change the velocity of a body of mass 1 kg from 20 m s^{-1} to 30 m s^{-1} in two seconds?

a) 1 N b) 5N
c) 10 N d) 25N

3. A machine gun has a mass 5 kg. It fires 50 gram bullets at the rate of 30 bullets per minute at a speed of 400 ms^{-1}. What force is required to keep the gun in position?

a) 10 N b) 5 N
c) 15 N d) 30 N

4. A fprce of 5 newton acts on a body of weight 9.80 newton. What is the acceleration produced in metre per sec2?

a) 0.51 b) 1.96
c) 5.00 d) 49.00

5. A body of mass m strikes a wall with. velocity v and rebounds with the same speed. Its change in momentum is

a) 2 mv b) mv / 2
c) – mv d) Zero

6. A ball of mass 0.2 kg is dropped from a certain height above the ground. It bounces pack after an elastic collision with the floor. If the speed with which the ball strikes the ground is 10 mis, then the impulse imparted by the ball on the floor is

a) 120 kg m/s b) 8 kg m/s
c) 4 kg m/s d) 2 kg m/s

7. A force of 6 N acts on a body, of mass 1 kg initially at rest and during this time, the body attains a velocity of 30 m/s. The time for which the force acts on a body is

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a) 10 second b) 8 second
c) 7 second d) 5 second

8. A bullet of mass 10 g is fired from a gun of mass 1 kg with recoil velocity of gun = 5 m/s. The muzzle velocity will be
a) 30 km/min b) 60 km/min
c) 30 m/s d) 500 m/s

9. The velocity of rocket with respect to ground is v_1 and velocity of gases ejecting from rocket with respect to ground is v_2. Then velocity of gases with respect to rocket is given by
a) v_2 b) $v_1 + v_2$
c) $v_1 \times v_2$ d) v_1

10. A rocket of initial mass 6000 kg ejects gases at a constant rate of 16 kg/s with constant relative speed of 11 kmls. What is the acceleration of rocket one minute after the blast?
a) 25 m/s² b) 50 m/s²
c) 10 m/s² d) 35 m/s²

11. Two bodies A and B of masses 1 kg and 2 kg moving towards each other with velocities 4 m/s and 1 m/s suffers a head on collision and stick together. The combined mass will
a) move in direction of motion of lighter mass.
b) move in direction of motion of heavier mass.
c) not move.
d) move in direction perpendicular to the line of motion of two bodies.

12. A machine gun fires a bullet of mass 40 gram with velocity 1200 m/s. The man holding it can exert a maximum force of 144 N on the gun. How many bullets can he fire per second at the most?
a) only one
b) 3
c) any number of bullets
d) 144 × 48

13. Which of the following has maximum momentum?
a) A 100 kg vehicle moving at 0.02 m s^{-1}.
b) A 4 g weight moving at 1000 cm s^{-1}.
c) A 200 g weight moving with kinetic energy of 10^{-6} J.
d) A 200 g weight after falling through one kilometre.

14. A bullet hits and gets embedded in a solid block resting on a horizontal frictionless table. What is conserved?
a) Momentum alone.
b) KE alone.
c) Momentum and K.E. both.
d) P.E. alone.

15. The force exerted by the floor of an elevator on the foot of a person standing there, is more than his weight, if the elevator is
a) going down and slowing down.
b) going up and speeding up.
c) going up and slowing down.
d) either a) and b).

16. If E, G and N represents the magnitudes of electromagnetic, gravitational and nuclear forces between two electrons at a given separation, then,
a) N = E = G b) E < N < G
c) N > G < E d) E > G > N

17. A body subjected to three concurrent forces is found to be in equilibrium. The resultant of any two forces
a) is equal to third force.
b) is opposite to third force.
c) is collinear with the third force.
d) all of the above statements are true.

18. For an inelastic collision, the value of e is
a) greater than 1 b) less than 1
c) equal to 1 d) none of these

19. For a perfect elastic collision
a) e > 1 b) e < 1
c) e = 1 d) none of these

20. A perfect inelastic body collides head on with a wall with velocity v. The change 10 momentum is
a) mv b) 2mv
c) zero d) none of these.

21. Two masses m, and m, moving with velocities v_a and v_b in opposite direction collide elastically and after the collision m_a and m_h move with velocities v_b and v_a respectively.
Then the ratio m.m, is
a) $\frac{v_a - v_b}{v_a + v_b}$ b) $\frac{m_a - m_b}{m_a}$
c) 1 d) $\frac{1}{2}$

22. The frictional force acts ______
a) in direction of motion
b) against the direction of motion

c) perpendicular to the direction of motion
d) at any angle to the direction of motion

23. A marble of mass X collides with a block of mass Z, with a velocity Y, and sticks to it. The final velocity of the system is

a) $\frac{Y}{X+Y}Y$ b) $\frac{X}{X+Z}Y$

c) $\frac{X+Y}{Z}$ d) $\frac{X+Z}{X}$

24. Two balls lying on the same plane collide. Which of the following will be always conserved?
a) heat
b) velocity
c) kinetic energy
d) linear momentum.

25. A body is moving with uniform velocity of 50 km h^{-1}, the force required to keep the body in motion in S.I unit is
a) zero b) 10
c) 25 d) 50

26. A coolie holding a suitcase on his head of 20 kg and travels on a platform, then work done in joule by the coolie is
a) 198 b) 98
c) 49 d) zero

27. Out of the following forces, which force is non-conservative?
a) gravitational b) electrostatic
c) frictional d) magnetic

28. The work done m conservative force is _____
a) negative b) zero
c) positive d) infmite

29. The angle between the line of action of force and displacement when no work done (in degree) is
a) zero b) 45
c) 90 d) 120

30. If the momentum of a body is doubled, its K.E. increases by
a) 50% b) 300%
c) 100% d) 400%

31. In perfectly inelastic collision, which is conserved?
a) P.E. only
b) KE. only
c) momentum only
d) KE. and momentum

32. A man pushes a wall and fails to displace it, hedoes _____
a) negative work
b) no work at all
c) positive work but not maximum work
d) maximum positive work

33. KE. ofa body is ______
a) negative
b) positive
c) can be positive, zero
d) infinite

34. In case of elastic collision, which is true?
a) Momentum and KE. is conserved.
b) Momentum conserved and KE. not conserved.
c) Momentum not conserved and KE. conserved.
d) Momentum and KE. both not conserved.

35. Pseudo force is true only in
a) frame of reference which is at rest.
b) inertial frame of reference.
c) frame of reference moving with constant velocity.
d) non–inertial frame of reference.

36. A man weighing 80 kg carries a stone of 20 kg to the top of the building 30 m high. The work done by him is ($g = 9.8\ m/s^{-1}$)
a) 80 J b) 100 J
c) 980 J d) 29,400 J

37. A weight lifter is holding a weight of 100 kg on his shoulders for 45 s, the amount of work done by him in joules is
a) 4500 b) 100
c) 45 d) zero

38. A work performed on an object does NOT depend upon
a) force applied.
b) angle at which force is inclined.
c) initial velocity of the object.
d) displacement.

39. If m is the mass of a body and E its KE., then its linear momentum is
a) $m\sqrt{E}$ b) $2\sqrt{m}\,E$
c) $\sqrt{m}\,E$ d) $\sqrt{2mE}$

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Answer Keys

1. d)	2. b)	3. a)	4. c)	5. a)	6. d)	7. d)	8. d)	9. b)	10. d)
11. a)	12. b)	13. d)	14. a)	15. d)	16. d)	17. d)	18. b)	19. c)	20. a)
21. c)	22. b)	23. b)	24. d)	25. a)	26. d)	27. c)	28. b)	29. c)	30. b)
31. c)	32. b)	33. c)	34. a)	35. d)	36. d)	37. d)	38. c)	39. d)	

Answers to Practice Problems :

1. 8×10^{-3} N
2. 12×10^4 N; 6×10^4 N
3. 14 N
4. 100 m/s
5. 90 N
6. 500 m s^{-1}
7. 3150 N
8. 3000J
9. 1 m/s in the direction offormer
10. 16.67 m/sec
11. 7.6 m/s, 17.2 m/s
12. 10 m, 0.12 kg
13. 120 gwt , 80 gwt
14. 27.6 kgwt, 35.4 kgwt
15. 13.3 kg wt ,24 kg wt
16. x = 0.5m, y = 0.29m
17. 0.6125 m from centre of 5 kg mass
18. 0.4 m/s, 1.6 m/s
19. 6 s
20. 15 kg
21. 2.2 m s^{-2}, 9.9 m
22. 0.7 m, 0.5 m.
23. 40N
24. $(1, \frac{9}{7})$, $1\,i + \frac{9}{7}\,j$ m
25. 2.58 m s^{-2}, 86.66 N
26. 1.568 N, 1.96 m s^{-2}
27. 686 N , 336 N , 1036 N
28. 8.82 J
29. 5 Nm, Positive z–axis
30. 5.4 m from A

Hints to Multiple Choice Questions :

3. From the principle of momentum conservation,

$m_g v_g = m_b v_b$ (considering magnitudes)

$\therefore \quad v = \frac{0.05 \times 400}{5} = 4$ m/s

$(\because m_b = 50 \text{ g} = 0.05 \text{ kg})$

The gun fires 30 bullets in 1 minute i.e., in 60 s. This means 1 bullet is fired every 2 s.
From Newton's second law,

$$F = \frac{P_2 - P_1}{t}$$

Where P_2, P_1 is final and intial momentum of gun respectively.

$$\therefore \quad F = \frac{m_g v_g - 0}{2} = \frac{5 \times 4}{2} = 10 \text{ N}$$

10. Let 'm_0' be the initial mass of rocket. Its ejection speed of gases $\left(\frac{dm}{dt}\right) = 16$ kg/s

Hence after t = 1 min = 60 s, its mass will be

$$m = m_0 - \left(\frac{dm}{dt}\right)t$$

$$= 6000 - (16) \times 60$$

$\therefore \quad m = 5040$ kg

At this time instant thrust on the rocket is,

$$F = u\frac{dm}{dt}$$

where u is constant relative speed.

$$ma = u\frac{dm}{dt}$$

$$a = \frac{(udm/dt)}{m} = \frac{11 \times 10^3 \times 16}{5040}$$

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$= 34.92 \text{ m/s}^2$

12. To hold the gun stable, rate of change of momentum of the gun should not exceed maximum exerted force

Hence $F = \frac{\Delta P}{t}$

For t = 1 s, $F = \Delta P \Rightarrow 144$ N.s

From the principle of conservation of momentum.

Momentum of gun = momentum of bullet

$= 40 \times 10^{-3} \times 12 \times 10^2$

$\Delta P = 48$ kg m/s.

So, number of bullets that can be fired per second,

$$\frac{\Delta P}{\Delta P'} = \frac{144}{48} = 3$$

13. Momentum of vehicle = 100 × 0.02 = 2 kg m/s .... (i)

Momentum of weight $= 4 \times 10^{-3} \times 10^3 \times 10^{-2}$

$= 4 \times 10^{-2}$ kg m/s .... (ii)

For 200 g weight, K.E. $= \frac{1}{2} mv^2 = 10^{-6}$ J

$$\therefore \quad v = \left(\frac{2\times10^{-6}}{0.2}\right)^{1/2} = 10^{-5/2}$$

Hence, its momentum $= 0.2 \times 10^{-5/2}$ kg m/s .. (iii)

For a weight falling from h = 1 km = 10^3 m

$(P.E)_{max} = (K.E)_{max}$

$$mgh = \frac{1}{2} mv^2$$

$$\therefore \quad v = \sqrt{2gh} = \sqrt{2\times9.8\times10^3} = 140 \text{ m/s}$$

Hence, its momentum = 0.2 × 140 = 28 kg m/s .... (iv)

Comparing the values, momentum of a 200 g weight after falling through 1 km has maximum value.

21. Relative velocity of approach relative velocity of separation

Therefore $\frac{v_2 - v_1}{u_1 - u_2} = e$

For perfectly elastic collisions, e = 1.

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