# DEFINITE INTEGRATION-EXERCISE

**1.** The value of $\int_0^4 \left\{\sqrt{x}\right\} dx$, where { } denotes the fractional part of x is

(1) 16/3 (2) 25/3

(3) 7/3 (4) None of these

**2.** $\lim_{n\to\infty} \frac{1}{n}\left(\sin\frac{\pi}{2n} + \sin\frac{2\pi}{2n} + \sin\frac{3\pi}{2n} + ...... + \sin\frac{n\pi}{2n}\right)$ is equal to

(1) $1/\pi$ (2) $2/\pi$ (3) $-2/\pi$ (4) $\pi/2$

**3.** $\int_0^\infty \frac{dx}{(a^2+x^2)^7}$ is equal to

(1) $\frac{231}{2048}\left(\frac{1}{a^{13}}\right)$ (2) $\frac{231}{2048}\left(\frac{\pi}{a^{13}}\right)$

(3) $\frac{231}{2047}\left(\frac{1}{a^{13}}\right)$ (4) $\frac{232}{2047}\left(\frac{\pi}{a^{13}}\right)$

**4.** $\int_{-1}^{3/2} |x \sin \pi x|\, dx$ equals

(1) $\frac{4}{\pi}$ (2) $\frac{3}{\pi} + \frac{1}{\pi^2}$

(3) $\frac{3}{\pi^2} + \frac{1}{\pi}$ (4) None of these

**5.** $\int_{2-\log 3}^{3+\log 3} \frac{\log(4+x)}{\log(4+x)+\log(9-x)}\, dx =$

(1) $\frac{1}{2} + \log 3$ (2) $\frac{5}{2}$

(3) $1 + 2\log 3$ (4) None of these

**6.** $I_n = \int_0^{\pi/4} \tan^n x\, dx$ then $\lim_{n\to\infty} n\,(I_n + I_{n-2})$ equals

(1) 1/2 (2) 1 (3) $\infty$ (4) 0

**7.** $\int_0^{\pi/4} \cos^{3/2}2\theta \cos\theta\, d\theta$ equals

(1) $\frac{3}{8\sqrt{2}}$ (2) $\frac{3\pi}{16\sqrt{2}}$ (3) $\frac{3\pi}{16}$ (4) 0

**8.** $\int_2^3 \frac{(x+2)^2}{2x^2-10x+53}dx =$

(1) 2 (2) 1 (3) $\frac{1}{2}$ (4) $\frac{5}{2}$

**9.** $\int_{1/2}^{2} \frac{1}{x}\sin\left(x - \frac{1}{x}\right)dx =$

(1) 0 (2) $\frac{3}{4}$ (3) $\frac{5}{4}$ (4) 2

**10.** $\int_1^e ((x+1).e^x \ell nx)dx =$

(1) e (2) $e^e + 1$

(3) $e^e(e-1)$ (4) $e^e(e-1) + e$

**11.** For $n \in N$, the value of $\int_0^{n\pi+V} \sqrt{\frac{1+\cos 2x}{2}}dx$ is where $\frac{\pi}{2} < V < \pi$

(1) $2n + 1 - \cos V$ (2) $2n - \sin V$

(3) $2n + 2 - \sin V$ (4) $2n + 1 - \sin V$

**12.** For $f(x) = x^4 + |x|$, let $I_1 = \int_0^\pi f(\cos x)dx$ and $I_2 = \int_0^{\pi/2} f(\sin x)dx$ then $\frac{I_1}{I_2}$ is equal to :-

(1) 1 (2) $\frac{1}{2}$ (3) 2 (4) 4

**13.** $\int_{-2}^{\pi} \frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}dx$ is equal to :-

(where [·] denotes the greatest integer function)

(1) $\pi + \sin2\cos2$ (2) $\pi - 2 + \sin2\cos2$

(3) $\pi - 2 - \sin2\cos2$ (4) None

**14.** $\int_{-\pi/4}^{\pi/4} \frac{e^x \sec^2 x}{e^{2x}-1}dx =$

(1) 0 (2) 1 (3) 2 (4) e

**15.** $\int_{-1}^{0} \frac{4x^2+4x+3}{1+e^{2x+1}}dx =$

(1) $\frac{7}{3}$ (2) 0 (3) $\frac{7}{6}$ (4) $\frac{7}{12}$

**16.** $\int_{0}^{11\pi/2} (\sin^4 x + \cos^4 x)dx =$

(1) $\frac{33\pi}{4}$ (2) $\frac{33\pi}{8}$

(3) $\frac{33\pi}{16}$ (4) None

**17.** If $x^2 f(x) + f\left(\frac{1}{x}\right) = 2$ for all x except at x = 0, then

$\int_{1/3}^{3} f(x)dx =$

(1) $\frac{4}{3}$ (2) $\frac{8}{3}$ (3) $\frac{1}{3}$ (4) None

**18.** If $I_n = \int_0^1 x^n e^{-x} dx$, $n \in N$ then $I_7 - 7I_6 =$

(1) $\frac{1}{e}$ (2) $\frac{2}{e}$ (3) $1 - \frac{1}{e}$ (4) $-\frac{1}{e}$

**19.** Given $\int_0^{\pi/2} \frac{dx}{1+\sin x+\cos x} = \ell n2$, then the value of definite integral $\int_0^{\pi/2} \frac{\sin x dx}{1+\sin x+\cos x} dx =$

(1) $\frac{1}{2}\ell n2$ (2) $\frac{\pi}{2} - \ell n2$

(3) $\frac{\pi}{4} - \frac{1}{2}\ell n2$ (4) $\frac{\pi}{2} + \ell n2$

**20.** The true solution set of the inequility

$\sqrt{5x-6-x^2} + \frac{\pi}{2}\int_0^x dz > x\int_0^{\pi} \sin^2 x dx$ is :-

(1) R (2) (1, 6) (3) (–6, 1) (4) (2, 3)

**21.** $\lim_{n\to\infty} \left\{\tan\frac{\pi}{2n}.\tan\frac{2\pi}{2n}......\tan\frac{\pi}{2}\right\}^{1/n} =$

(1) 1 (2) $\frac{\pi}{2}$ (3) e (4) None

**22.** If $\int_a^b \frac{x^n}{x^n+(16-x)^n}dx = 6$ then $(n \in R)$

(1) a = 4, b = 12 (2) a = – 4, b = 20
(3) a = 2, b = 14 (4) None

**23.** If $x = \int_0^y \frac{dt}{\sqrt{1+t^2}}$ the value of $\frac{d^2y}{dx^2}$ is equal to

(1) $\frac{y}{\sqrt{1+y^2}}$ (2) y (3) $\frac{2y}{\sqrt{1+y^2}}$ (4) 4y

**24.** Let f(x) = $\int_2^x \frac{dt}{\sqrt{1+t^4}}$ and g be the inverse of f. then the value of g'(0) is

(1) 1 (2) 17 (3) $\sqrt{17}$ (4) None

**25.** $\int_0^{2\pi/3} [\sin 3x]\, dx = ?$, where $[\bullet] \to$ G.I.F.

(1) $-\frac{\pi}{3}$ (2) $-\frac{\pi}{6}$ (3) $-\frac{\pi}{2}$ (4) $-\frac{\pi}{12}$

**26.** If $\int_0^4 [3x]\, dx = K\int_0^6 \{4x\}\, dx$ then K is equal to, where $[\bullet]$ and {0} denotes G.I.F. and fractional part functions

(1) $\frac{22}{3}$ (2) $\frac{3}{22}$ (3) $\frac{11}{3}$ (4) $\frac{3}{11}$

**27.** If f(x) = $\int_0^x \sin^4 t\, dt$ then f(x + π) is equal to :-

(1) f(π) (2) f(x)
(3) f(x) + f(π) (4) f(x) . f(π)

**28.** $\int_0^{\pi} [\tan x]\, dx$ : where $[\bullet]$ is GIF

(1) $-\frac{\pi}{2}$ (2) $-\frac{2}{\pi}$ (3) $\frac{2}{\pi}$ (4) None

**29.** $\lim_{n\to\infty}\left(\frac{n}{n^2} + \frac{n}{n^2+1} + \frac{n}{n^2+2^2} + .... + \frac{n}{2n^2-2n+1}\right)$ is equal to :-

(1) 1 (2) tan1 (3) $\frac{\pi}{4}$ (4) None

**30.** Let f(x) = $\sqrt{1+x\sqrt{1+(x+1)\sqrt{1+(x+2)(x+4)}}}\, dx$

then $\int_0^{100} f(x)dx$ is

(1) 5010 (2) 5050
(3) 5100 (4) 5049

**31.** If f(x) is a nonzero differentiable function such that $\int_0^x f(t)\,dt = (f(x))^2$; $\forall x \in R$ then f(2) equals

(1) 3 (2) 2 (3) 1 (4) –1

**32.** If $\int_a^b (2+x-x^2)\,dx$ is maximum then (a + b) is equal to

(1) 3 (2) 2 (3) 1 (4) –1

**33.** Let g(x) = $\int_0^x f(t)\,dt$ where $\frac{1}{2} \le f(t) \le 1$, $t\in[0, 1]$ and $0 \le f(t) \le \frac{1}{2}$ for $t \in (1, 2)$ for :-

(1) $-\frac{3}{2} \le g(2) \le \frac{1}{2}$ (2) $\frac{1}{2} \le g(2) \le \frac{3}{2}$

(3) $\frac{3}{2} < g(2) \le \frac{5}{2}$ (4) None

**34.** Given that,

$$\int_0^\infty \frac{x^2dx}{(x^2+a^2)(x^2+b^2)(x^2+c^2)} = \frac{\pi}{2(a+b)(b+c)(c+a)}$$

then find the value of $\int_0^\infty \frac{dx}{(x^2+4)(x^2+9)}$ is :-

(1) $\frac{\pi}{32}$ (2) $\frac{\pi}{30}$

(3) $\frac{\pi}{60}$ (4) $\frac{\pi}{15}$

**35.** Let y = $f$(x) be a differentiable curve satisfying $\int_2^x f(t)dt = \frac{x^2}{2} + \int_x^2 t^2 f(t)dt$, then $\int_{-\pi/4}^{\pi/4} \frac{f(x)+x^9-x^3+x+1}{\cos^2 x}dx$ equals :-

(1) 0 (2) 1 (3) 2 (4) None

**36.** If $f$(x) be a real valued function : $f(x) + f(x + 4) = f(x + 2) + f(x + 6)$, g(x) = $\int_x^{x+8} f(t)\,dt$, then g'(x) is equal to :-

(1) $f$(x) (2) $f$(x + 8)

(3) 8 (4) 0

**37.** If $\int_{-1}^{4} f(x)dx = 4$ and $\int_2^4 [3-f(x)]dx = 7$ then the value of $\int_2^{-1} f(x)dx$ is :-

(1) 2 (2) –3 (3) –5 (4) None

**38.** The value of $\int_{-2}^{2} \min\left(x-[x], -\left(x-[x]\right)\right)dx$ is ([.] denotes the greatest integer function) :-

(1) 0 (2) 1 (3) 2 (4) None

**39.** If $\int_0^{10} f(x)dx = 5$ then $\sum_{K=1}^{10}\int_0^1 f(K-1+x)dx$ is :-

(1) 50 (2) 10 (3) 5 (4) None

## ANSWER KEY

Exercise-I

| Que. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Ans. | 3 | 2 | 2 | 2 | 1 | 2 | 2 | 3 | 1 | 4 |
| Que. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| Ans. | 3 | 3 | 2 | 1 | 3 | 2 | 2 | 4 | 3 | 4 |
| Que. | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| Ans. | 1 | 3 | 2 | 3 | 1 | 1 | 3 | 4 | 3 | 3 |
| Que. | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | |
| Ans. | 3 | 3 | 2 | 3 | 3 | 4 | 3 | 2 | 3 | |