1. The equation of tangent at the point (at^2, at^3) on the curve $ay^2 = x^3$ is-

(1) $3tx - 2y = at^3$ (2) $tx - 3y = at^3$

(3) $3tx + 2y = at^3$ (4) None of these

2. The angle between the tangent lines to the graph of the function $f(x) = \int_2^x (2t-5)\,dt$ at the points where the graph cuts the x-axis is

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

3. The angle of intersection between the curves $y^2 = 8x$ and $x^2 = 4y - 12$ at (2, 4) is-

(1) 90° (2) 60° (3) 45° (4) 0°

4. A Spherical balloon is being inflated at the rate of 35cc/min. The rate of increase in the surface area (in cm2/min.) of the balloon when its diameter is 14 cm, is :

(1) $\sqrt{10}$ (2) $10\sqrt{10}$ (3) 100 (4) 10

5. If the surface area of a sphere of radius r is increasing uniformly at the rate $8cm^2/s$, then the rate of change of its volume is :

(1) proportional to r^2 (2) constant

(3) proportional to r (4) proportional to $\sqrt{r}$

6. A curve is represented by the equations, $x = \sec^2 t$ and $y = \cot t$ where t is a parameter. If the tangent at the point P on the curve where $t = \pi/4$ meets the curve again at the point Q then $|PQ|$ is equal to:

(1) $\frac{5\sqrt{3}}{2}$ (2) $\frac{5\sqrt{5}}{2}$ (3) $\frac{2\sqrt{5}}{3}$ (4) $\frac{3\sqrt{5}}{2}$

7. A circle with centre at (15, –3) is tangent to $y = \frac{x^2}{3}$ at a point in the first quadrant. The radius of the circle is equal to

(1) $5\sqrt{6}$ (2) $8\sqrt{3}$ (3) $9\sqrt{2}$ (4) $6\sqrt{5}$

8. At the point $P(a, a^n)$ on the graph of $y = x^n$ $(n \in N)$ in the first quadrant a normal is drawn. The normal intersects the y-axis at the point (0, b).

If $\lim_{a \to 0} b = \frac{1}{2}$, then n equals

(1) 1 (2) 3 (3) 2 (4) 4

9. If the function $f(x) = 2x^2 - kx + 5$ is strictly increasing in [1, 2], then 'k' lies in the interval

(1) $(-\infty, 4)$ (2) $(4, \infty)$

(3) $(-\infty, 8]$ (4) $(8, \infty)$

10. Which one of the following statements does not hold good for the function $f(x) = \cos^{-1}(2x^2 - 1)$?

(1) f is not differentiable at x = 0

(2) f is monotonic

(3) f is even

(4) f has an extremum

11. Complete set of values of K in order that $f(x) = \sin x - \cos x - Kx + b$ decreases for all real values is given by-

(1) $K < 1$ (2) $K \geq 1$

(3) $K \geq \sqrt{2}$ (4) $K < \sqrt{2}$

12. When $0 \leq x \leq 1$, $f(x) = |x| + |x - 1|$ is-

(1) strictly increasing (2) strictly decreasing

(3) constant (4) None of these

13. Let f (x) and g (x) be two continuous functions defined from R → R, such that $f(x_1) > f(x_2)$ and $g(x_1) < g(x_2)$, $\forall\ x_1 > x_2$, then solution set of $f\left(g(\alpha^2 - 2\alpha)\right) > f\left(g(3\alpha - 4)\right)$ is

(1) R (2) ϕ

(3) (1, 4) (4) R – [1, 4]

14. If $2a + 3b + 6c = 0$, then at least one root of the equation $ax^2 + bx + c = 0$ lies in the interval-

(1) (0, 1) (2) (1, 2)

(3) (2, 3) (4) none

15. The function $f : [a, \infty) \to R$ where R denotes the range corresponding to the given domain, with rule $f(x) = 2x^3 - 3x^2 + 6$, will have an inverse provided

(1) $a \geq 1$ (2) $a \geq 0$

(3) $a \leq 0$ (4) $a \leq 1$

16. The function $f(x) = \tan^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is -

(1) strictly increasing in its domain
(2) strictly decreasing in its domain
(3) strictly decreasing in $(-\infty, 0)$ and strictly increasing in $(0, \infty)$
(4) strictly increasing in $(-\infty, 0)$ and strictly decreasing in $(0, \infty)$

17. Given $f'(1) = 1$ and $\frac{d}{dx}(f(2x)) = f'(x)\ \forall x > 0$.

If $f'(x)$ is differentiable then there exists a number $c \in (2, 4)$ such that $f''(c)$ equals

(1) $-1/4$ (2) $-1/8$ (3) $1/4$ (4) $1/8$

18. The equation $\sin x + x\cos x = 0$ has at least one root in

(1) $\left(-\frac{\pi}{2}, 0\right)$ (2) $(0, \pi)$

(3) $\left(\pi, \frac{3\pi}{2}\right)$ (4) $\left(0, \frac{\pi}{2}\right)$

***19.** If $f(x) = |x| + |x-1| + |x-2|$, then-

(1) f(x) has minima at x = 1
(2) f(x) has maxima at x = 0
(3) f(x) has neither maxima nor minima at x = 3
(4) None of these

20. The maximum area of a right angled triangle with hypotenuse h is :-

(1) $\frac{h^2}{\sqrt{2}}$ (2) $\frac{h^2}{2}$ (3) $\frac{h^2}{4}$ (4) $\frac{h^2}{2\sqrt{2}}$

21. The cost of running a bus from A to B, is Rs. $\left(av + \frac{b}{v}\right)$, where v km/h is the average speed of the bus. When the bus travels at 30 km/h, the cost comes out to be Rs. 75 while at 40 km/h, it is Rs. 65. Then the most economical speed (in km/h) of the bus is :

(1) 40 (2) 60
(3) 45 (4) 50

22. Difference between the greatest and the least values of the function $f(x) = x(\ln x - 2)$ on $[1, e^2]$ is

(1) 2 (2) e (3) e^2 (4) 1

23. The sum of lengths of the hypotenuse and another side of a right angled triangle is given. The area of the triangle will be maximum if the angle between them is :

(1) $\pi/6$ (2) $\pi/4$ (3) $\pi/3$ (4) $5\pi/12$

24. If $f(x) = x^3 + ax^2 + bx + c$ is minimum at x = 3 and maximum at x = –1, then-

(1) a = –3, b = –9, c = 0
(2) a = 3, b = 9, c = 0
(3) a = –3, b = –9, $c \in R$
(4) none of these

25. If $f(x) = \int_x^{x^2} (t-1)\,dt$, $1 \le x \le 2$, then global maximum value of f(x) is

(1) 1 (2) 2 (3) 4 (4) 5

26. Range of the function $f(x) = \frac{\ell n x}{\sqrt{x}}$ is

(1) $(-\infty, e)$ (2) $(-\infty, e^2)$

(3) $\left(-\infty, \frac{2}{e}\right]$ (4) $\left(-\infty, \frac{1}{e}\right)$

27. Two sides of a triangle are to have lengths 'a' cm & 'b' cm. If the triangle is to have the maximum area, then the length of the median from the vertex containing the sides 'a' and 'b' is

(1) $\frac{1}{2}\sqrt{a^2+b^2}$ (2) $\frac{2a+b}{3}$

(3) $\sqrt{\frac{a^2+b^2}{2}}$ (4) $\frac{a+2b}{3}$

28. A rectangle has one side on the positive y-axis and one side on the positive x - axis. The upper right hand vertex of the rectangle lies on the curve $y = \frac{\ell n x}{x^2}$. The maximum area of the rectangle is

(1) e^{-1} (2) $e^{-1/2}$ (3) 1 (4) $e^{1/2}$

29. P is a point on positive x-axis, Q is a point on the positive y-axis and 'O' is the origin. If the line passing through P and Q is tangent to the curve $y = 3 - x^2$ then the minimum area of the triangle OPQ, is

(1) 2 (2) 4 (3) 8 (4) 9

30. The least area of a circle circumscribing any right triangle of area S is :

(1) πS (2) $2\pi S$

(3) $\sqrt{2}\pi S$ (4) $4\pi S$

31. Number of critical points of the function,

$$f(x) = \frac{2}{3}\sqrt{x^3} - \frac{x}{2} + \int_1^x \left(\frac{1}{2} + \frac{1}{2}\cos 2t - \sqrt{t}\right)dt$$

which lie in the interval $[0, 2\pi]$ is :

(1) 2 (2) 6 (3) 4 (4) 8

32. The bottom of the legs of a three legged table are the vertices of an isosceles triangle with sides 5, 5 and 6. The legs are to be braced at the bottom by three wires in the shape of a Y. The minimum length of the wire needed for this purpose, is

(1) $4 + 3\sqrt{3}$ (2) 10

(3) $3 + 4\sqrt{3}$ (4) $1 + 6\sqrt{2}$

33. There are 50 apple trees in an orchard. Each tree produces 800 apples. For each additional tree planted in the orchard, the output per additional tree drops by 10 apples. Number of trees that should be added to the existing orchard for maximising the output of the trees, is

(1) 5 (2) 10 (3) 15 (4) 20

* **Marked Question is multiple answer**

ANSWER KEY Exercise-I

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	**1**	**4**	**4**	**4**	**3**	**4**	**4**	**3**	**1**	**2**
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	**3**	**3**	**3**	**1**	**1**	**4**	**2**	**2**	**1,3**	**3**
Que.	21	22	23	24	25	26	27	28	29	30
Ans.	**2**	**2**	**3**	**3**	**3**	**3**	**1**	**1**	**2**	**1**
Que.	31	32	33							
Ans.	**2**	**1**	**3**							