1. Find the value of p for which the funtion

$$f(x) = \begin{cases} \dfrac{(4^x-1)^3}{\sin\left(\dfrac{x}{p}\right)\log\left(1+\dfrac{x^2}{3}\right)}, & x \neq 0 \\ 12(\log 4)^3, & x=0 \end{cases}$$

is continuous at x = 0

(1) 1 (2) 2 (3) 3 (4) 4

2. If $f(x) = \begin{cases} \dfrac{x^3+x^2-16x+20}{(x-2)^2}, & x \neq 2 \\ k, & x=2 \end{cases}$

is continuous for all values of x, then the value of k is-

(1) 5 (2) 6 (3) 7 (4) 8

3. If the function $f(x) = \begin{cases} 1, & x \leq 2 \\ ax+b, & 2<x<4 \\ 7, & x \geq 4 \end{cases}$

is continuous at x = 2 and 4, then the values of a and b are-

(1) 3, 5 (2) 3, –5
(3) 0, 3 (4) 0, 5

4. If $f(x) = \begin{cases} \dfrac{1-\sin^3 x}{3\cos^2 x}, & x < \pi/2 \\ a, & x = \pi/2 \\ \dfrac{b(1-\sin x)}{(\pi-2x)^2}, & x>\pi/2 \end{cases}$

is continuous at x = π/2, then value of a and b are-

(1) 1/2, 1/4 (2) 2, 4
(3) 1/2, 4 (4) 1/4, 2

5. If $f(x) = \begin{cases} (1+|\sin x|)^{a/|\sin x|}, & -\pi/6 < x < 0 \\ b, & x=0 \\ e^{\tan 2x/\tan 3x}, & 0 < x < \pi/6 \end{cases}$

is continuous at x = 0, then value of a, b are-

(1) 2/3, $e^{2/3}$ (2) 1/3, $e^{1/3}$
(3) 2/3, 1/3 (4) None of these

6. If $f(x) = \begin{cases} \dfrac{\sin[x]}{[x]+1}, & x > 0 \\ \dfrac{\cos\dfrac{x}{2[x]}}{[x]}, & x<0 \\ k, & x=0 \end{cases}$

(where [x] = greatest integer ≤ x) is continuous at x = 0, then k is equal to-

(1) 0 (2) 1
(3) –1 (4) Indeterminate

7. Number of points of discontinuity of $f(x)=[2x^3-5]$ in [1,2), is equal to-
(where [x] denotes greatest integer less than or equal to x)

(1) 14 (2) 13 (3) 10 (4) 8

8. If $f(x) = \begin{cases} \dfrac{x(1+a\cos x)-b\sin x}{x^3}, & x \neq 0 \\ 1, & x=0 \end{cases}$

then f is continuous for values of a and b given by-

(1) $\dfrac{5}{2}, \dfrac{3}{2}$ (2) $\dfrac{5}{2}, \dfrac{-3}{2}$

(3) $\dfrac{-5}{2}, \dfrac{-3}{2}$ (4) $\dfrac{-5}{2}, \dfrac{3}{2}$

9. The value of f(0), so that function $f(x) = \dfrac{1-\cos(1-\cos x)}{x^4}$ is continuous everywhere is:

(1) $\dfrac{1}{8}$ (2) $\dfrac{1}{2}$

(3) $\dfrac{1}{4}$ (4) none of these

10. If $f(x) = \dfrac{1}{(x-1)(x-2)}$ and $g(x) = \dfrac{1}{x^2}$, then points of discontinuity of $f\{g(x)\}$ are -

(1) $\left\{-1,0,1,\dfrac{1}{\sqrt{2}}\right\}$ (2) $\left\{-\dfrac{1}{\sqrt{2}},-1,0,1,\dfrac{1}{\sqrt{2}}\right\}$

(3) {0,1} (4) $\left\{0,1,\dfrac{1}{\sqrt{2}}\right\}$

11. If $x = \exp\left(\tan^{-1}\left(\frac{y-x^2}{x^2}\right)\right)$, then $\frac{dy}{dx}$ equals -

(1) x [1 + tan (log x) + sec² x]
(2) 2x [1 + tan (log x)] + sec² x
(3) 2x [1 + tan (log x)] + sec x
(4) 2x + x[1 + tan(logx)]²

12. $\frac{d}{dx}\left\{\sin^2\left(\cot^{-1}\sqrt{\frac{1+x}{1-x}}\right)\right\} =$

(1) $-\frac{1}{2}$ (2) 0 (3) $\frac{1}{2}$ (4) –1

13. Obtain differential coefficient of $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$

with respect to $\cos^{-1}\sqrt{\frac{1+\sqrt{1+x^2}}{2\sqrt{1+x^2}}}$

(1) 1 (2) 2 (3) 3 (4) 4

14. If g is inverse of f and f(x) = 2x + sinx; then g'(x) equals:

(1) $-\frac{3}{x^2}+\frac{1}{\sqrt{1-x^2}}$ (2) 2 + $\sin^{-1}x$

(3) 2 + cos g(x) (4) $\frac{1}{2+\cos(g(x))}$

15. If f(x) = $x.\left(\frac{e^{1/x}-e^{-1/x}}{e^{1/x}+e^{-1/x}}\right)$, x ≠ 0, f(0) = 0 then

(1) f is differentiable at x = 0
(2) f is not differentiable at x = 0
(3) f is not continuous at x = 0
(4) None of these

16. If $f(x) = \begin{cases} \frac{x-1}{2x^2-7x+5}, & \text{for } x \neq 1 \\ -\frac{1}{3}, & \text{for } x = 1 \end{cases}$, then f' (1) =

(1) $-\frac{1}{9}$ (2) $-\frac{2}{9}$ (3) $-\frac{1}{3}$ (4) $\frac{1}{3}$

17. If $y = \tan^{-1}\left(\frac{a\sin x+b\cos x}{a\cos x-b\sin x}\right)$, $-\pi/2 < x < \pi/2$,

$\frac{b}{a}\tan x < 1$, then $\frac{dy}{dx}$ equals

(1) 1 (2) –1
(3) 0 (4) None of these

18. If $\sin^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right) = \log a$, then $\frac{dy}{dx}$ is equal to

(1) $\frac{x^2-y^2}{x^2+y^2}$ (2) $\frac{y}{x}$ (3) $\frac{y}{x^2}$ (4) $\frac{x}{y}$

19. If f(x) = | cosx – sinx |, then f'(π/4) is equal to

(1) $\sqrt{2}$ (2) $-\sqrt{2}$
(3) 0 (4) Does not exist

20. If $\sqrt{(1-x^6)}+\sqrt{(1-y^6)} = a^3(x^3-y^3)$, then $\frac{dy}{dx} =$

(1) $\frac{x^2}{y^2}\sqrt{\left(\frac{1-x^6}{1-y^6}\right)}$ (2) $\frac{y^2}{x^2}\sqrt{\left(\frac{1-y^6}{1-x^6}\right)}$

(3) $\frac{x^2}{y^2}\sqrt{\left(\frac{1-y^6}{1-x^6}\right)}$ (4) none of these

21. Let $3f(x)-2f\left(\frac{1}{x}\right)=x$, then f'(2) is equal to :

(1) $\frac{2}{7}$ (2) $\frac{1}{2}$ (3) 2 (4) $\frac{7}{2}$

22. If $y = x^2 + \cfrac{1}{x^2+\cfrac{1}{x^2+\cfrac{1}{x^2+\ldots\infty}}}$, then $\frac{dy}{dx}$ is :

(1) $\frac{2xy}{2y-x^2}$ (2) $\frac{xy}{y+x^2}$

(3) $\frac{xy}{y-x^2}$ (4) $\frac{2xy^2}{2y+x^2}$

23. The left-hand derivative of f(x) =[x] sin(πx) at x = k, (k is an integer and [x] = greatest integer ≤ x, is, K ∈ I) :

(1) $(-1)^k\ (k-1)\pi$ (2) $(-1)^{k-1}\ (k-1)\pi$
(3) $(-1)^k\ k\pi$ (4) $(-1)^{k-1}\ k\pi$

24. Suppose that f is a differentiable function with the property that $f(x + y) = f(x) + f(y) + xy$ and $\lim_{h\to 0} \frac{1}{h} f(h) = 3$ then

(1) f is a linear function (2) $f(x) = 3x + x^2$
(3) $f(x) = 3x + x^2/2$ (4) None of these

*__25.__ The function $f(x) = \max. \{1 - x), (1 + x), 2\}$, $x \in (-\infty, \infty)$, is

(1) continuous at all points
(2) differentiable at all points
(3) differentiable at all points except at $x = 1$ and $x = -1$
(4) continuous at all points except at $x = 1$ and $x = -1$, where it is discontinuous.

26. Let R be the set of real numbers and $f : R \to R$, be a differentiable function such that $|f(x) - f(y)| \le |x - y|^3 \ \forall\ x,y \in R$. If $f(10) = 100$, then the value of $f(20)$ is equal to

(1) 0 (2) 10
(3) 20 (4) 100

27. If $y = (A + Bx) e^{mx} + (m - 1)^{-2} e^x$ then $\frac{d^2y}{dx^2} - 2m \frac{dy}{dx} + m^2y$ is equal to -

(1) e^x (2) e^{mx}
(3) e^{-mx} (4) $e^{(1 - m)x}$

28. If $\frac{d^2x}{dy^2}\left(\frac{dy}{dx}\right)^3 + \frac{d^2y}{dx^2} = K$ then the value of K is equal to

(1) 1 (2) –1 (3) 2 (4) 0

29. Let $f(x) = x + \sin x$. Suppose g denotes the inverse function of f. The value of $g'\left(\frac{\pi}{4} + \frac{1}{\sqrt{2}}\right)$ has the value equal to

(1) $\sqrt{2} - 1$ (2) $\frac{\sqrt{2}+1}{\sqrt{2}}$
(3) $2 - \sqrt{2}$ (4) $\sqrt{2} + 1$

30. Let $f(x) = |x - a|\ \phi(x)$, where ϕ is a continuous function and $\phi(a) \ne 0$. Then

(1) $f'(a+) = \phi'(a)$
(2) f is differentiable at x = a
(3) $f'(a) = \phi'(a)$
(4) $f'(a-) = -\phi(a)$

* **Marked Question is multiple answer**

ANSWER KEY										Exercise-I
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	4	3	2	3	1	4	2	3	1	2
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	4	1	1	4	2	2	1	2	4	3
Que.	21	22	23	24	25	26	27	28	29	30
Ans.	2	1	1	3	1,3	4	1	4	3	4