

# LIMITS- EXERCISE

1.  $\lim_{x \rightarrow 0} \frac{\sin(x^{1/3}) \ln(1+3x)}{(\tan^{-1} \sqrt{x})^2 (e^{5x^{1/3}} - 1)} =$

- (1)  $\frac{3}{5}$       (2)  $\frac{1}{5}$       (3)  $\frac{2}{5}$       (4)  $\frac{5}{3}$

2.  $\lim_{x \rightarrow 0} \left( \frac{x - \int_0^x \cos t^2 dt}{x^3 - 6x} \right)$  equals

- (1) 0      (2)  $\frac{1}{2}$       (3) -2      (4)  $-\frac{1}{6}$

3.  $\lim_{x \rightarrow 0} \left( \frac{5}{2 + \sqrt{9+x}} \right)^{1/\sin x}$  is equal to

- (1)  $e^{-\frac{1}{3}}$       (2)  $e^{\frac{1}{3}}$       (3)  $e^{-30}$       (4)  $e^{-\frac{1}{30}}$

4. If  $\lim_{x \rightarrow a} (f(x) + g(x))$  exists finitely then

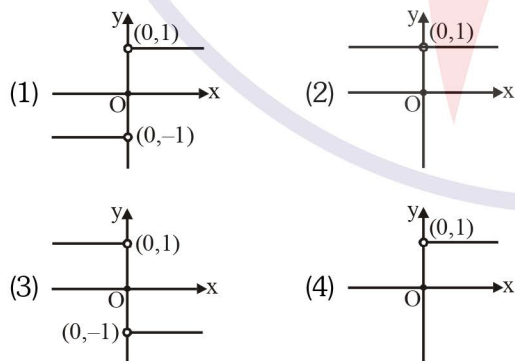
- (1) Only  $\lim_{x \rightarrow a} f(x)$  exist  
 (2) Only  $\lim_{x \rightarrow a} g(x)$  exist  
 (3) Both must be exist  
 (4) Both may not be exist

5.  $\lim_{n \rightarrow \infty} \left( \frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \dots + n \text{ terms} \right)$  equals

- (1)  $\frac{1}{2}$       (2)  $\frac{1}{3}$       (3)  $\frac{1}{10}$       (4)  $\frac{1}{6}$

6. Which of the following is the graph of the function

$$f(x) = \lim_{n \rightarrow \infty} \frac{2}{\pi} \tan^{-1}(nx)$$



7.  $\lim_{x \rightarrow 2} \frac{(\cos \alpha)^x + (\sin \alpha)^x - 1}{x - 2} =$

- (1)  $(\cos^2 \alpha) \ln \cos \alpha + (\sin^2 \alpha) \ln \sin \alpha$   
 (2) 1  
 (3)  $\ln((\cos \alpha)(\sin \alpha))$   
 (4)  $\ln((\sin^2 \alpha) \times (\cos^2 \alpha))$

8.  $\lim_{x \rightarrow \frac{\pi}{2}^+} e^{[\cot x]}$  is equal to

(where [.] is greatest integer function)

- (1) e      (2) 1      (3) 0      (4)  $\frac{1}{e}$

9.  $\lim_{x \rightarrow a^+} \frac{|x|^3}{a} - \left[ \frac{x}{a} \right]^3$  (a > 0); is equal to

(where [x] is greatest integer function and |x| is modulus function)

- (1)  $a^2 - 3$       (2)  $a^2 - 1$   
 (3)  $a^2$       (4) Does not exist

10. Let a = Minimum  $\{x^2 + 2x + 3, x \in \mathbb{R}\}$

and  $b = \lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta^2}$  The value of  $\sum_{r=0}^n a^r \cdot b^{n-r}$  is

- (1)  $\frac{2^{n+1} - 1}{3 \cdot 2^n}$       (2)  $\frac{2^{n+1} + 1}{3 \cdot 2^n}$   
 (3)  $\frac{4^{n+1} - 1}{3 \cdot 2^n}$       (4) None of these

11. If  $\lim_{x \rightarrow 0} \phi(x) = a^3$ , (a ≠ 0); then  $\lim_{x \rightarrow 0} \phi\left(\frac{x}{a}\right)$  is equal to

- (1)  $\frac{1}{a^3}$       (2)  $a^3$       (3)  $a^2$       (4)  $\frac{1}{a^2}$

12. If  $f(x) = \text{Sgn}(\text{Sgn}(\text{Sgn}(x)))$ , then  $\lim_{x \rightarrow 0} f(x)$  is equal to

- (1) 1      (2) -1  
 (3) 0      (4) does not exist

13.  $\lim_{x \rightarrow 0} \left( \frac{a^x + b^x + c^x}{3} \right)^{2/x}$  is equal to

- (1)  $a^{2/3} + b^{2/3} + c^{2/3}$       (2) abc  
 (3)  $(abc)^{2/3}$       (4) 1

14. If  $L = \lim_{x \rightarrow 0} \frac{2 - \sqrt{4 - x^2} - \frac{x^2}{4}}{x^4}$  then L =

- (1) 1/2      (2) 1/4      (3) 1/8      (4) 1/64

15.  $\lim_{x \rightarrow 0} \frac{(3 \sin x - \sin 3x)^4}{(\sec x - \cos x)^6}$  is equal to

- (1) 96      (2) 144      (3) 216      (4) 256

16. If the value of  $\lim_{x \rightarrow 0^+} \left( \frac{(3/x) + 1}{(3/x) - 1} \right)^{1/x}$  can be expressed

in the form of  $e^{p/q}$ , where p and q are relative prime then (p + q) is equal to

- (1) 2      (2) 3      (3) 4      (4) 5

17.  $\lim_{x \rightarrow \infty} 2^{x-1} \left( \sin \frac{\pi}{2^x} + \tan \frac{\pi}{2^x} \right)$  is equal to  
 (1) 0 (2) 1 (3)  $\pi$  (4)  $\frac{\pi}{2}$
18.  $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\int_0^{\sqrt{2} \sec x} f(t) dt}{x^2 - \frac{\pi^2}{16}}$  equals  
 (1)  $\frac{8}{\pi} f(2)$  (2)  $\frac{2}{\pi} f(2)$  (3)  $\frac{2}{\pi} f(\sqrt{2})$  (4)  $\frac{4}{\pi} f(2)$
19. If  $L = \lim_{x \rightarrow 0} \frac{x(1 - \cos x) - ax^2 \sin x}{x^5}$  exists finitely, then a & L equals  
 (1)  $a = 1, L = \frac{1}{12}$  (2)  $a = \frac{1}{2}, L = \frac{1}{24}$   
 (3)  $a = \frac{1}{3}, L = \frac{1}{24}$  (4)  $a = \frac{1}{4}, L = \frac{1}{12}$
20. The value of  $\lim_{x \rightarrow 0} \left( \left[ \frac{100x}{\sin x} \right] + \left[ \frac{99 \sin x}{x} \right] \right)$  is  
 (where [.] is greatest integer function)  
 (1) 199 (2) 198  
 (3) 0 (4) does not exist
21.  $\lim_{x \rightarrow 0} \frac{10^x - 2^x - 5^x + 1}{x \tan x} =$   
 (1)  $\ln 7$  (2)  $(\ln 5)(\ln 2)$   
 (3)  $\ln 10$  (4)  $\ln \left( \frac{5}{2} \right)$
22. For  $x > 0$ ,  $\lim_{x \rightarrow 0} \left( (\sin x)^{\frac{1}{x}} + \left( \frac{1}{x} \right)^{\sin x} \right)$  is equal to  
 (1) 0 (2) -1 (3) 1 (4) 2
23.  $\lim_{x \rightarrow -\infty} \frac{\sqrt{3x^2 - 1} - \sqrt{2x^2 - 1}}{4x + 3}$  equals  
 (1)  $\frac{\sqrt{2} - \sqrt{3}}{4}$  (2)  $\frac{\sqrt{3} - \sqrt{2}}{4}$   
 (3) 0 (4) 1

24.  $\lim_{x \rightarrow \pi/4} \frac{1 - \cot^3 x}{2 - \cot x - \cot^3 x}$  equals  
 (1)  $\frac{3}{4}$  (2)  $-\frac{3}{4}$  (3)  $\frac{4}{3}$  (4) 3
25.  $\lim_{x \rightarrow 0} \left( \frac{1 + 2^x + 2^{2x}}{3} \right)^{\frac{1}{x}}$  equals  
 (1)  $\sqrt{2}$  (2)  $\frac{1}{2}$  (3) 2 (4)  $5^{\frac{1}{3}}$
26. If  $f(x+y) = f(x) + f(y)$ ;  $\forall x, y \in \mathbb{R}$  and  $f(1) = 1$ , then  $\lim_{x \rightarrow 0} \frac{2^{f(\tan x)} - 2^{f(\sin x)}}{x^2 f(\sin x)}$  equals  
 (1)  $\frac{1}{2} \ln 2$  (2)  $-\frac{1}{2} \ln 2$  (3)  $\ln 2$  (4)  $\frac{1}{2}$
27. The value of  $\lim_{n \rightarrow \infty} n^2 \left\{ \sqrt{\left(1 - \cos \frac{1}{n}\right)} \sqrt{\left(1 - \cos \frac{1}{n}\right)} \sqrt{\left(1 - \cos \frac{1}{n}\right)} \dots \infty \right\}$  is  
 (1) 1 (2) 2 (3) 0 (4)  $1/2$
28. If  $f(n+1) = \frac{1}{2} \left\{ f(n) + \frac{9}{f(n)} \right\}$ ;  $n \in \mathbb{N}$  and  $f(n) > 0$  for all  $n \in \mathbb{N}$  then  $\lim_{n \rightarrow \infty} f(n)$  is equal to  
 (1) 3 (2) -3 (3)  $\frac{1}{2}$  (4) 0
29. Let  $f: (1, \infty) \rightarrow (0, \infty)$  be a continuous decreasing function with  $\lim_{x \rightarrow \infty} \frac{f(4x)}{f(8x)} = 1$  then  $\lim_{x \rightarrow \infty} \frac{f(6x)}{f(8x)}$  is equal to  
 (1)  $\frac{4}{8}$  (2)  $\frac{4}{6}$  (3)  $\frac{6}{8}$  (4) 1
30. If  $f(x)$  is differentiable function at every where.  
 $\lim_{x \rightarrow 0} \left( \frac{f(x) + f\left(\frac{x}{2}\right) + f\left(\frac{x}{2^2}\right) + \dots + \text{upto } \infty \text{ terms}}{x} \right)$   
 is equal to : (where  $f(0) = 0$  &  $f'(0) = 1$ )  
 (1) 3 (2) 1 (3) 0 (4) 2

ANSWER KEY

Exercise-I

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	1	1	4	4	4	1	1	4	2	3
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	2	4	3	4	4	4	3	4	2	2
Que.	21	22	23	24	25	26	27	28	29	30
Ans.	2	3	1	1	3	1	4	1	4	4